1. The probability distribution of a discrete random variable X is given by

$$P(X = x) = \frac{x^2}{14},\ x \in \{1, 2, k\}, \text{ where } k > 0.$$

(a) Write down P(X = 2).

(1)

(b) Show that k = 3.

(4)

(c) Find E(X).

(2)
(Total 7 marks)

2. In a game a player rolls a biased four-faced die. The probability of each possible score is shown below.

Score	1	2	3	4
Probability	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{1}{10}$	x

(a) Find the value of x.

(2)

(b) Find E(X).

(3)

(c) The die is rolled twice. Find the probability of obtaining two scores of 3.

(2)
(Total 7 marks)

3. A factory makes calculators. Over a long period, 2 % of them are found to be faulty. A random sample of 100 calculators is tested.

(a) Write down the expected number of faulty calculators in the sample.

(1)

(b) Find the probability that three calculators are faulty.

(2)

(c) Find the probability that more than one calculator is faulty.

(3)
(Total 6 marks)

4. Paula goes to work three days a week. On any day, the probability that she goes on a red bus is $\frac{1}{4}$.

(a) Write down the expected number of times that Paula goes to work on a red bus in one week.

(2)

In one week, find the probability that she goes to work on a red bus

(b) on exactly two days;

(2)

(c) on at least one day.

(3)
(Total 7 marks)

5. A multiple choice test consists of ten questions. Each question has five answers.
Only one of the answers is correct. For each question, Jose randomly chooses one of the five answers.

(a) Find the expected number of questions Jose answers correctly.

(1)

(b) Find the probability that Jose answers exactly three questions correctly.

(2)

(c) Find the probability that Jose answers more than three questions correctly.

(3)
(Total 6 marks)

6. The following table shows the probability distribution of a discrete random variable X.

x	-1	0	2	3
$\mathrm{P}(X = x)$	0.2	$10k^2$	0.4	$3k$

(a) Find the value of k.

(4)

(b) Find the expected value of X.

(3)
(Total 7 marks)

7. Jan plays a game where she tosses two fair six-sided dice. She wins a prize if the sum of her scores is 5.

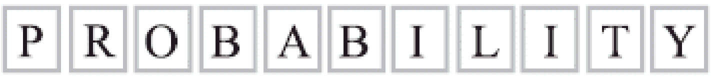
(a) Jan tosses the two dice once. Find the probability that she wins a prize.

(3)

(b) Jan tosses the two dice 8 times. Find the probability that she wins 3 prizes.

(2)
(Total 5 marks)

8. The letters of the word PROBABILITY are written on 11 cards as shown below.



Two cards are drawn at random without replacement.
Let A be the event the first card drawn is the letter A.
Let B be the event the second card drawn is the letter B.

(a) Find $\mathrm{P}(A)$.

(1)

(b) Find $\mathrm{P}(B \mid A)$.

(2)

(c) Find $\mathrm{P}(A \cap B)$.

(3)
(Total 6 marks)

9. The following table gives the examination grades for 120 students.

Grade	Number of students	Cumulative frequency
1	9	9
2	25	34
3	35	p
4	q	109
5	11	120

(a) Find the value of

(i) p;

(ii) q.

(4)

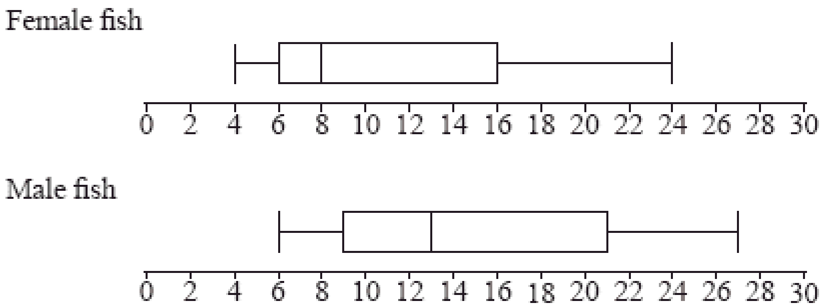
(b) Find the mean grade.

(2)

(c) Write down the standard deviation.

(1)
(Total 7 marks)

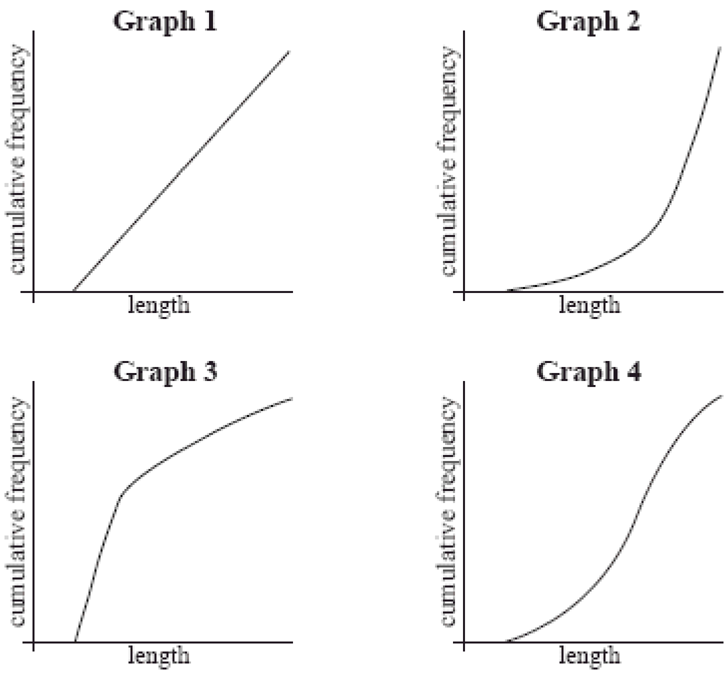
10. A scientist has 100 female fish and 100 male fish. She measures their lengths to the nearest cm. These are shown in the following box and whisker diagrams.



(a) Find the range of the lengths of **all** 200 fish.

(3)

(b) Four cumulative frequency graphs are shown below.



Which graph is the best representation of the lengths of the **female** fish?

(2)
(Total 5 marks)

11. The following frequency distribution of marks has mean 4.5.

Mark	1	2	3	4	5	6	7
Frequency	2	4	6	9	x	9	4

(a) Find the value of x.

(4)

(b) Write down the standard deviation.

(2)
(Total 6 marks)

12. In any given season, a soccer team plays 65 % of their games at home.
When the team plays at home, they win 83 % of their games.
When they play away from home, they win 26 % of their games.

The team plays one game.

(a) Find the probability that the team wins the game.

(4)

(b) If the team does not win the game, find the probability that the game was played at home.

(4)
(Total 8 marks)

13. A company uses two machines, A and B, to make boxes. Machine A makes 60 % of the boxes.

80 % of the boxes made by machine A pass inspection.
90 % of the boxes made by machine B pass inspection.

A box is selected at random.

(a) Find the probability that it passes inspection.

(3)

(b) The company would like the probability that a box passes inspection to be 0.87.
Find the percentage of boxes that should be made by machine B to achieve this.

(4)
(Total 7 marks)

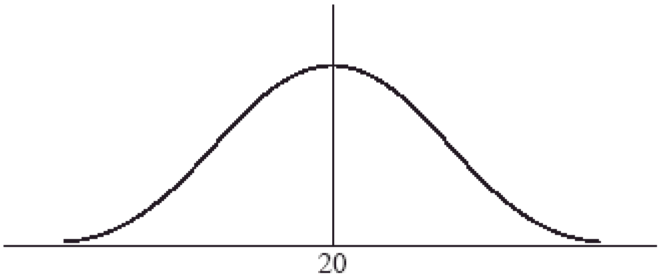
14. A random variable X is distributed normally with a mean of 20 and variance 9.

(a) Find $P(X \leq 24.5)$.

(3)

(b) Let $P(X \leq k) = 0.85$.

(i) Represent this information on the following diagram.



(ii) Find the value of k.

(5)
(Total 8 marks)

15. A box holds 240 eggs. The probability that an egg is brown is 0.05.

(a) Find the expected number of brown eggs in the box.

(2)

(b) Find the probability that there are 15 brown eggs in the box.

(2)

(c) Find the probability that there are at least 10 brown eggs in the box.

(3)
(Total 7 marks)

16. Evan likes to play two games of chance, A and B.

For game A, the probability that Evan wins is 0.9. He plays game A seven times.

(a) Find the probability that he wins exactly four games.

(2)

For game B, the probability that Evan wins is p. He plays game B seven times.

(b) Write down an expression, in terms of p, for the probability that he wins exactly four games.

(2)

(c) Hence, find the values of p such that the probability that he wins exactly four games is 0.15.

(3)
(Total 7 marks)

17. Two standard six-sided dice are tossed. A diagram representing the sample space is shown below.

		Score on second die					
		1	2	3	4	5	6
	1	•	•	•	•	•	•
	2	•	•	•	•	•	•
Score on first die	3	•	•	•	•	•	•
	4	•	•	•	•	•	•
	5	•	•	•	•	•	•
	6	•	•	•	•	•	•

Let X be the sum of the scores on the two dice.

(a) Find

(i) $P(X = 6)$;

(ii) $P(X > 6)$;

(iii) $P(X = 7 \mid X > 5)$.

(6)

(b) Elena plays a game where she tosses two dice.

If the sum is 6, she wins 3 points.
If the sum is greater than 6, she wins 1 point.
If the sum is less than 6, she **loses** k points.

Find the value of k for which Elena's expected number of points is zero.

(7)
(Total 13 marks)

18. The probability of obtaining heads on a biased coin is $\frac{1}{3}$.

(a) Sammy tosses the coin three times. Find the probability of getting

(i) three heads;

(ii) two heads and one tail.

(5)

(b) Amir plays a game in which he tosses the coin 12 times.

(i) Find the expected number of heads.

(ii) Amir wins \$ 10 for each head obtained, and loses \$ 6 for each tail. Find his expected winnings.

(5)
(Total 10 marks)

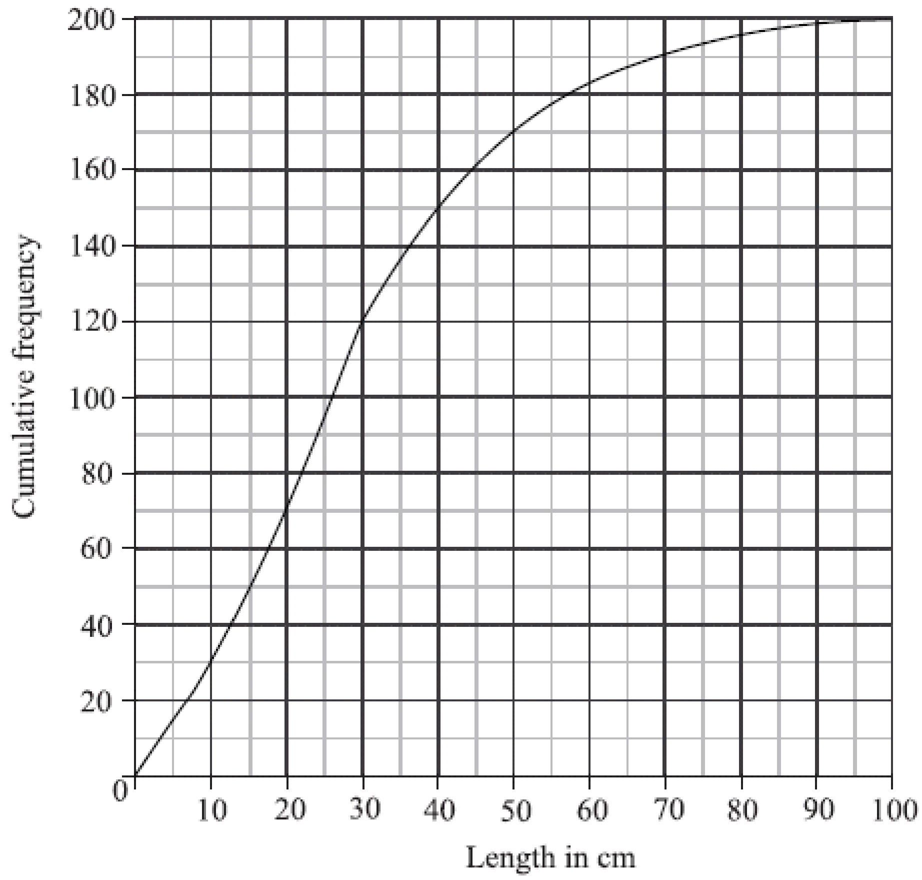
19. A fisherman catches 200 fish to sell. He measures the lengths, *l* cm of these fish, and the results are shown in the frequency table below.

Length *l* cm	$0 \le l < 10$	$10 \le l < 20$	$20 \le l < 30$	$30 \le l < 40$	$40 \le l < 60$	$60 \le l < 75$	$75 \le l < 100$
Frequency	30	40	50	30	33	11	6

(a) Calculate an estimate for the standard deviation of the lengths of the fish.

(3)

(b) A cumulative frequency diagram is given below for the lengths of the fish.



Use the graph to answer the following.

(i) Estimate the interquartile range.

(ii) Given that 40 % of the fish have a length more than *k* cm, find the value of *k*.

(6)

In order to sell the fish, the fisherman classifies them as small, medium or large.

Small fish have a length less than 20 cm.
Medium fish have a length greater than or equal to 20 cm but less than 60 cm.
Large fish have a length greater than or equal to 60 cm.

(c) Write down the probability that a fish is small.

(2)

The cost of a small fish is \$4, a medium fish \$10, and a large fish \$12.

(d) Copy and complete the following table, which gives a probability distribution for the cost \$*X*.

Cost \$*X*	4	10	12
$P(X = x)$		0.565	

(2)

(e) Find $E(X)$.

(2)
(Total 15 marks)

20. In a class of 100 boys, 55 boys play football and 75 boys play rugby. Each boy must play at least one sport from football and rugby.

(a) (i) Find the number of boys who play both sports.

(ii) Write down the number of boys who play only rugby.

(3)

(b) One boy is selected at random.

(i) Find the probability that he plays only one sport.

(ii) Given that the boy selected plays only one sport, find the probability that he plays rugby.

(4)

Let A be the event that a boy plays football and B be the event that a boy plays rugby.

(c) Explain why A and B are **not** mutually exclusive.

(2)

(d) Show that A and B are **not** independent.

(3)
(Total 12 marks)

21. Bag A contains 2 red balls and 3 green balls. Two balls are chosen at random from the bag without replacement. Let X denote the number of red balls chosen. The following table shows the probability distribution for X.

X	0	1	2
$\mathrm{P}(X = x)$	$\frac{3}{10}$	$\frac{6}{10}$	$\frac{1}{10}$

(a) Calculate $\mathrm{E}(X)$, the mean number of red balls chosen.

(3)

Bag B contains 4 red balls and 2 green balls. Two balls are chosen at random from bag B.

(b) (i) Draw a tree diagram to represent the above information, including the probability of each event.

(ii) Hence find the probability distribution for Y, where Y is the number of red balls chosen.

(8)

A standard die with six faces is rolled. If a 1 or 6 is obtained, two balls are chosen from bag A, otherwise two balls are chosen from bag B.

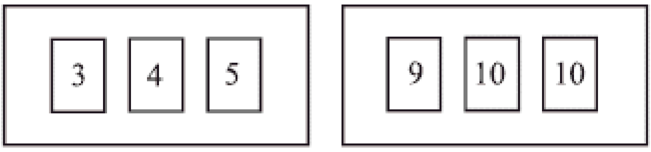
(c) Calculate the probability that two red balls are chosen.

(5)

(d) Given that two red balls are obtained, find the conditional probability that a 1 or 6 was rolled on the die.

(3)
(Total 19 marks)

22. Two boxes contain numbered cards as shown below.



Two cards are drawn at random, one from each box.

(a) Copy and complete the table below to show all nine equally likely outcomes.

3, 9		
3, 10		
3, 10		

(2)

Let *S* be the sum of the numbers on the two cards.

(b) Write down all the possible values of *S*.

(2)

(c) Find the probability of each value of *S*.

(2)

(d) Find the expected value of *S*.

(3)

(e) Anna plays a game where she wins \$50 if S is even and loses \$30 if S is odd.
Anna plays the game 36 times. Find the amount she expects to have at the end of the 36 games.

(3)
(Total 12 marks)

23. A test has five questions. To pass the test, at least three of the questions must be answered correctly.

The probability that Mark answers a question correctly is $\frac{1}{5}$. Let X be the number of questions that Mark answers correctly.

(a) (i) Find $E(X)$.

(ii) Find the probability that Mark passes the test.

(6)

Bill also takes the test. Let Y be the number of questions that Bill answers correctly.
The following table is the probability distribution for Y.

y	0	1	2	3	4	5
$\mathbf{P(Y = y)}$	0.67	0.05	$a + 2b$	$a - b$	$2a + b$	0.04

(b) (i) Show that $4a + 2b = 0.24$.

(ii) Given that $E(Y) = 1$, find a and b.

(8)

(c) Find which student is more likely to pass the test.

(3)
(Total 17 marks)

24. The weights of players in a sports league are normally distributed with a mean of 76.6 kg, (correct to three significant figures). It is known that 80 % of the players have weights between 68 kg and 82 kg. The probability that a player weighs less than 68 kg is 0.05.

(a) Find the probability that a player weighs more than 82 kg.

(2)

(b) (i) Write down the standardized value, z, for 68 kg.

(ii) Hence, find the standard deviation of weights.

(4)

To take part in a tournament, a player's weight must be within 1.5 standard deviations of the mean.

(c) (i) Find the set of all possible weights of players that take part in the tournament.

(ii) A player is selected at random. Find the probability that the player takes part in the tournament.

(5)

Of the players in the league, 25 % are women. Of the women, 70 % take part in the tournament.

(d) Given that a player selected at random takes part in the tournament, find the probability that the selected player is a woman.

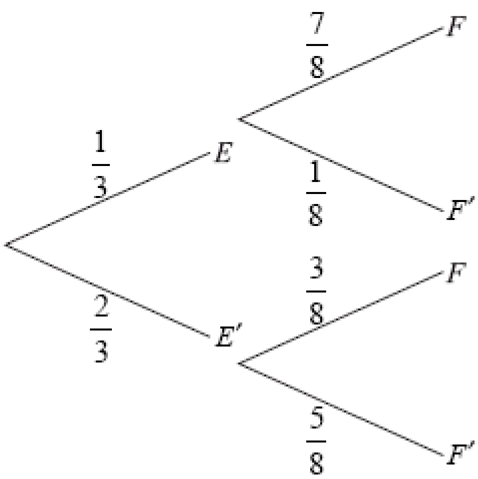
(4)
(Total 15 marks)

25. José travels to school on a bus. On any day, the probability that José will miss the bus is $\frac{1}{3}$.

If he misses his bus, the probability that he will be late for school is $\frac{7}{8}$.

If he does not miss his bus, the probability that he will be late is $\frac{3}{8}$.

Let E be the event "he misses his bus" and F the event "he is late for school".
The information above is shown on the following tree diagram.



(a) Find

(i) P($E \cap F$);

(ii) P(F).

(4)

(b) Find the probability that

(i) José misses his bus and is not late for school;

(ii) José missed his bus, given that he is late for school.

(5)

The cost for each day that José catches the bus is 3 euros. José goes to school on Monday and Tuesday.

(c) **Copy** and complete the probability distribution table.

***X* (cost in euros)**	0	3	6
P (*X*)	$\frac{1}{9}$		

(3)

(d) Find the expected cost for José for both days.

(2)
(Total 14 marks)

26. Two fair 4-sided dice, one red and one green, are thrown. For each die, the faces are labelled 1, 2, 3, 4. The score for each die is the number which lands face down.

(a) List the pairs of scores that give a sum of 6.

(3)

The probability distribution for the sum of the scores on the two dice is shown below.

Sum	2	3	4	5	6	7	8
Probability	p	q	$\frac{3}{16}$	$\frac{4}{16}$	$\frac{3}{16}$	r	$\frac{1}{16}$

(b) Find the value of p, of q, and of r.

(3)

Fred plays a game. He throws two fair 4-sided dice four times. He wins a prize if the sum is 5 on three or more throws.

(c) Find the probability that Fred wins a prize.

(6)
(Total 12 marks)