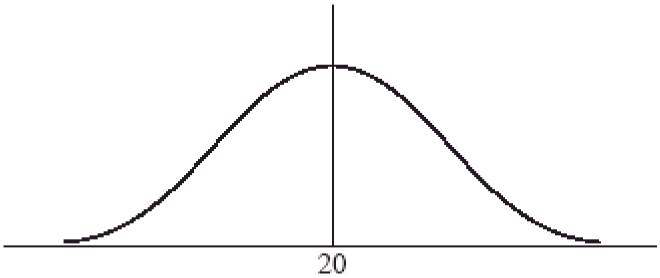
1. A random variable X is distributed normally with a mean of 20 and variance 9.

(a) Find $P(X \leq 24.5)$.

(3)

(b) Let $P(X \leq k) = 0.85$.

(i) Represent this information on the following diagram.



(ii) Find the value of k.

(5)
(Total 8 marks)

2. In a country called *Tallopia*, the height of adults is normally distributed with a mean of 187.5 cm and a standard deviation of 9.5 cm.

(a) What percentage of adults in *Tallopia* have a height greater than 197 cm?

(3)

(b) A standard doorway in *Tallopia* is designed so that 99% of adults have a space of at least 17 cm over their heads when going through a doorway. Find the height of a standard doorway in *Tallopia*. Give your answer to the nearest cm.

(4)
(Total 7 marks)

3. The mass of packets of a breakfast cereal is normally distributed with a mean of 750 g and standard deviation of 25 g.

(a) Find the probability that a packet chosen at random has mass

(i) less than 740 g;

(ii) at least 780 g;

(iii) between 740 g and 780 g.

(5)

(b) Two packets are chosen at random. What is the probability that both packets have a mass which is less than 740 g?

(2)

(c) The mass of 70% of the packets is more than x grams. Find the value of x.

(2)
(Total 9 marks)

4. Intelligence Quotient (IQ) in a certain population is normally distributed with a mean of 100 and a standard deviation of 15.

(a) What percentage of the population has an IQ between 90 and 125?

(2)

(b) If two persons are chosen at random from the population, what is the probability that both have an IQ greater than 125?

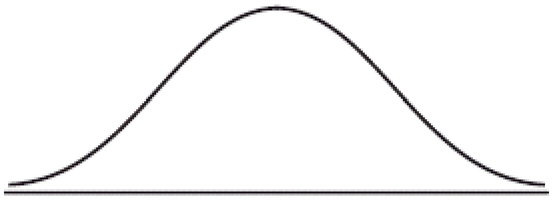
(3)

(c) The mean IQ of a random group of 25 persons suffering from a certain brain disorder was found to be 95.2. Is this sufficient evidence, at the 0.05 level of significance, that people suffering from the disorder have, on average, a lower IQ than the entire population? State your null hypothesis and your alternative hypothesis, and explain your reasoning.

(4)
(Total 9 marks)

5. Let X be normally distributed with mean 100 cm and standard deviation 5 cm.

(a) On the diagram below, shade the region representing $P(X > 105)$.



(2)

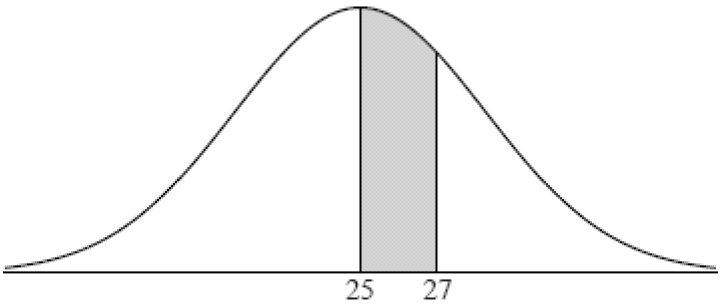
(b) Given that $P(X < d) = P(X > 105)$, find the value of d.

(2)

(c) Given that $P(X > 105) = 0.16$ (correct to two significant figures), find $P(d < X < 105)$.

(2)
(Total 6 marks)

6. Let the random variable X be normally distributed with mean 25, as shown in the following diagram.



The shaded region between 25 and 27 represents 30 % of the distribution.

(a) Find $P(X > 27)$.

(2)

(b) Find the standard deviation of X.

(5)
(Total 7 marks)

7. A random variable X is distributed normally with mean 450 and standard deviation 20.

(a) Find $P(X \le 475)$.

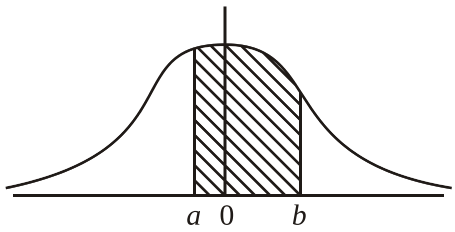
(2)

(b) Given that $P(X > a) = 0.27$, find a.

(4)
(Total 6 marks)

8. The lifespan of a particular species of insect is normally distributed with a mean of 57 hours and a standard deviation of 4.4 hours.

(a) The probability that the lifespan of an insect of this species lies between 55 and 60 hours is represented by the shaded area in the following diagram. This diagram represents the standard normal curve.



(i) Write down the values of a and b.

(2)

(ii) Find the probability that the lifespan of an insect of this species is

(a) more than 55 hours;

(1)

(b) between 55 and 60 hours.

(2)

(b) 90% of the insects die after t hours.

(i) Represent this information on a standard normal curve diagram, similar to the one given in part (a), indicating clearly the area representing 90%.

(2)

(ii) Find the value of t.

(3)
(Total 10 marks)

9. Bags of cement are labelled 25 kg. The bags are filled by machine and the actual weights are normally distributed with mean 25.7 kg and standard deviation 0.50 kg.

(a) What is the probability a bag selected at random will weigh less than 25.0 kg?

(2)

In order to reduce the number of underweight bags (bags weighing less than 25 kg) to 2.5% of the total, the mean is increased without changing the standard deviation.

(b) Show that the increased mean is 26.0 kg.

(3)

It is decided to purchase a more accurate machine for filling the bags. The requirements for this machine are that only 2.5% of bags be under 25 kg and that only 2.5% of bags be over 26 kg.

(c) Calculate the mean and standard deviation that satisfy these requirements.

(3)

The cost of the new machine is $5000. Cement sells for $0.80 per kg.

(d) Compared to the cost of operating with a 26 kg mean, how many bags must be filled in order to recover the cost of the new equipment?

(3)
(Total 11 marks)

10. A box contains a large number of biscuits. The weights of biscuits are normally distributed with mean 7 g and standard deviation 0.5 g.

(a) One biscuit is chosen at random from the box. Find the probability that this biscuit

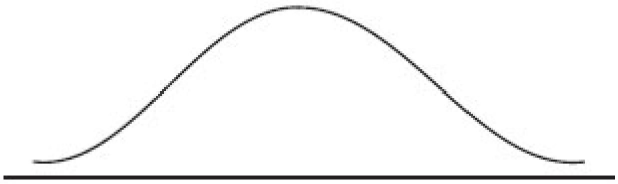
(i) weighs less than 8 g;

(ii) weighs between 6 g and 8 g.

(4)

(b) Five percent of the biscuits in the box weigh less than d grams.

(i) Copy and complete the following normal distribution diagram, to represent this information, by indicating d, and shading the appropriate region.



(ii) Find the value of d.

(5)

(c) The weights of biscuits in another box are normally distributed with mean μ and standard deviation 0.5 g. It is known that 20% of the biscuits in this second box weigh less than 5 g.

Find the value of μ.

(4)
(Total 13 marks)

11. A van can take either Route A or Route B for a particular journey.

If Route A is taken, the journey time may be assumed to be normally distributed with mean 46 minutes and a standard deviation 10 minutes.

If Route B is taken, the journey time may be assumed to be normally distributed with mean μ minutes and standard deviation 12 minutes.

(a) For Route A, find the probability that the journey takes **more** than 60 minutes.

(2)

(b) For Route B, the probability that the journey takes **less** than 60 minutes is 0.85. Find the value of μ.

(3)

(c) The van sets out at 06:00 and needs to arrive before 07:00.

(i) Which route should it take?

(ii) Justify your answer.

(3)

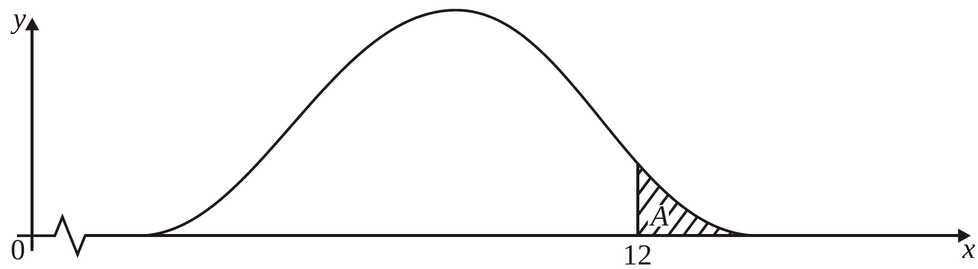
(d) On five consecutive days the van sets out at 06:00 and takes Route B. Find the probability that

(i) it arrives before 07:00 on all five days;

(ii) it arrives before 07:00 on at least three days.

(5)
(Total 13 marks)

12. The graph shows a normal curve for the random variable X, with mean μ and standard deviation σ.



It is known that $p\ (X \geq 12) = 0.1$.

(a) The shaded region A is the region under the curve where $x \geq 12$. Write down the area of the shaded region A.

(1)

It is also known that $p(X \leq 8) = 0.1$.

(b) Find the value of μ, explaining your method in full.

(5)

(c) Show that $\sigma = 1.56$ to an accuracy of three significant figures.

(5)

(d) Find $p(X \leq 11)$.

(5)
(Total 16 marks)

13. The weights of players in a sports league are normally distributed with a mean of 76.6 kg, (correct to three significant figures). It is known that 80 % of the players have weights between 68 kg and 82 kg. The probability that a player weighs less than 68 kg is 0.05.

(a) Find the probability that a player weighs more than 82 kg.

(2)

(b) (i) Write down the standardized value, z, for 68 kg.

(ii) Hence, find the standard deviation of weights.

(4)

To take part in a tournament, a player's weight must be within 1.5 standard deviations of the mean.

(c) (i) Find the set of all possible weights of players that take part in the tournament.

(ii) A player is selected at random. Find the probability that the player takes part in the tournament.

(5)

Of the players in the league, 25 % are women. Of the women, 70 % take part in the tournament.

(d) Given that a player selected at random takes part in the tournament, find the probability that the selected player is a woman.

(4)
(Total 15 marks)