LESSON 84 – INTERSECTION OF 3 PLANES

HL2 Math - Santowski

LEARNING GOALS

- Recognize geometrically intersection three planes
- Solve for the point and/or line of intersection of three planes

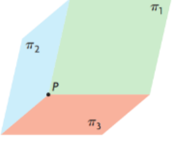
CASES WITH ONE OR MORE POINTS OF INTERSECTION OF 3 PLANES

- There is just one solution to the corresponding system of equations.
- This is a single point. The coordinates of the point of intersection will satisfy each of the three equations.

The planes intersect at a point.
There is exactly one solution.

The normals are not parallel and not coplanar.

Case 1: P is the point of intersection of three planes.



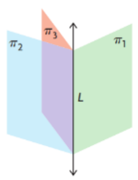
CASES WITH ONE OR MORE POINTS OF INTERSECTION OF 3 PLANES

- There are an infinite number of solutions to the related system of equations. Geometrically, this corresponds to a lie, and the solution is given in terms of one parameter.
- The tree planes intersect along a lie and are mutually non-coincident.

The planes intersect in a line. There are an infinite number of solutions.

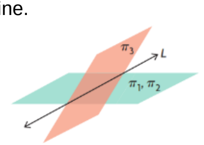
The normals are coplanar, but not parallel.

Case 2a: L is the line of intersection of three planes.



CASES WITH ONE OR MORE POINTS OF INTERSECTION OF 3 PLANES

- There are an infinite number of solutions to the related system of equations. Geometrically, this corresponds to a lie, and the solution is given in terms of one parameter.
- The two planes are coincident, and the third plane cuts through these two planes intersecting along a line.



Case 2b: L is the line of intersection of two coincident planes and a third plane not parallel to the coincident planes.

CASES WITH ONE OR MORE POINTS OF INTERSECTION OF 3 PLANES

- Three planes are coincident, and there are an infinite number of solutions to the related system of equations. The number of solutions corresponds to the infinite number of points on a plane and the solution is given in terms of two parameters.
- In this case, there are three coincident planes that have identical equations or can be reduced to three equivalent equations.

The planes are coincident. There are an infinite number of solutions.

The normals are parallel.

Case 3: The plane of intersection of three coincident planes is the plane itself.

Using elementary operations to solve a system of three equations in three unknowns

Determine the intersection of the three planes with the equations $x - y + z = -2$, $2x - y - 2z = -9$, and $3x + y - z = -2$.

Selecting a strategy to determine the intersection of three planes

Determine the solution to the following system of equations:

① $2x - y + z = 1$

② $3x - 5y + 4z = 3$

③ $3x + 2y - z = 0$

More on solving a consistent system of equations

Determine the solution to the following system of equations:

① $2x + y + z = 1$

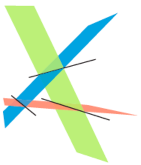
② $4x - y - z = 5$

③ $8x - 2y - 2z = 10$

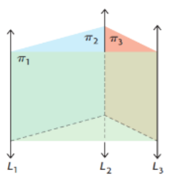
CASES WITH NO POINTS OF INTERSECTIONS OF 3 PLANES

Three planes form a triangular prism as shown. This means that, if you consider any two of the three planes, they intersect in a line and each of these three lines is parallel to the others.

Case 1: A triangular prism is formed by three parallel lines.



The normals are coplanar but not parallel.



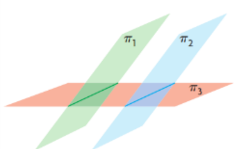
CASES WITH NO POINTS OF INTERSECTIONS OF 3 PLANES

Each of the parallel planes has a line of intersection with the third plane,
But there is no intersection between all three planes.

Two planes are parallel and distinct.
The third plane is not parallel.

Two of the normals are parallel.

Case 2: Two parallel planes intersect a third plane.



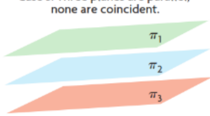
CASES WITH NO POINTS OF INTERSECTIONS OF 3 PLANES

- All three planes do not have any points of intersection.

The three planes are parallel and at least two are distinct.

The normals are parallel.

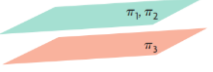
Case 3: Three planes are parallel; none are coincident.



CASES WITH NO POINTS OF INTERSECTIONS OF 3 PLANES

- All three planes do not have any points of intersection.

Case 4: Two planes are coincident; a third plane is parallel to and non-coincident with the first two planes.



Selecting a strategy to solve an inconsistent system of equations

Determine the solution to the following system of equations:

① $x - y + z = 1$

② $x + y + 2z = 2$

③ $x - 5y - z = 1$

Reasoning about an inconsistent system of equations

Determine the solution to the following system of equations:

① $x - y + 2z = -1$

② $x - y + 2z = 3$

③ $x - 3y + z = 0$

Identifying coincident and parallel planes in an inconsistent system

Solve the following system of equations:

① $x + y + z = 5$

② $x + y + z = 4$

③ $x + y + z = 5$