Lesson 78 – Geometric Vectors

HL Math - Santowski

What is the difference between speed and velocity?

=> Velocity is speed combined with direction

What are Vectors?

- A scalar quantity is simply anything in life that can be described by just a number

  e.g. the temperature, my age etc.

- However, a vector is a quantity that needs a direction as well for it to make sense.

=> Vectors have both magnitude (size) and direction

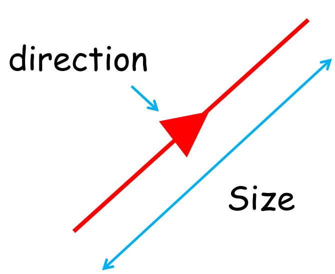
Examples
- Velocity = speed + direction
  e.g. wind velocity is 20kmh East
- Displacement = distance + direction
  e.g. displacement of Leeds from York is 25 miles W

=> Vectors are directed quantities

What do they look like?

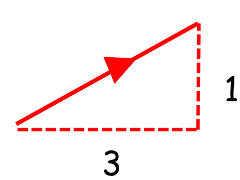
Vectors can be represented as:

-Straight Lines
(directed lines segments)



- Column Vectors

$$\begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

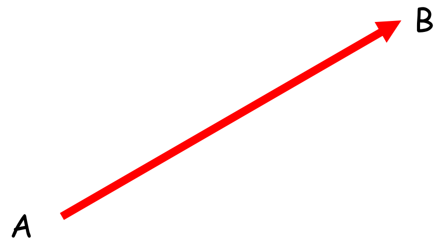


→ Vectors are pathways

What do they look like?

The most common notation for vectors is threefold

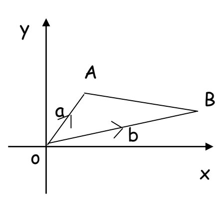
$$\overrightarrow{AB} = \mathbf{a} = \underline{a}$$



Position Vectors

$\overrightarrow{OA}$ is the journey from the origin to the point A. It is known as the position vector written **a**.

$\overrightarrow{OB}$ Is the position vector of the point b, written **b**.



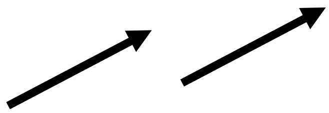
$\overrightarrow{AB}$ = **b** - **a** where **a** and **b** are the position vectors of A and B

<u>Example</u>
If P and Q have coordinates (4,8) and 2,3), respectively, find the components of $\overrightarrow{PQ}$.

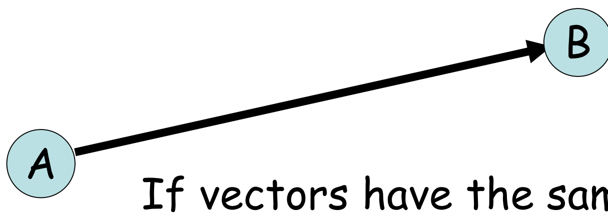
$\overrightarrow{PQ}$ = **q** - **p**

$$\begin{pmatrix} 2 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ 8 \end{pmatrix} = \begin{pmatrix} -2 \\ -5 \end{pmatrix}$$

Properties



Vectors with the same magnitude and direction are equal



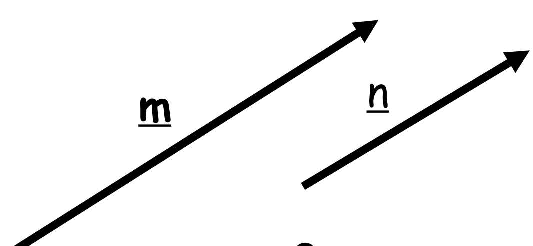
If vectors have the same magnitude and opposite directions then:

$\overrightarrow{AB} = \mathbf{a}$ then $\overrightarrow{BA} = -\mathbf{a}$

Properties

If different sized vectors have the same direction they are scalar multiples of each other

e.g. $\mathbf{m} = k\mathbf{n}$

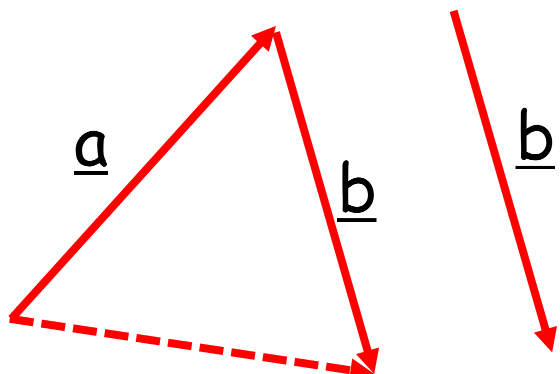


$m = 2n$

Resultants

The resultant is a single vector which is equivalent to a set of vectors

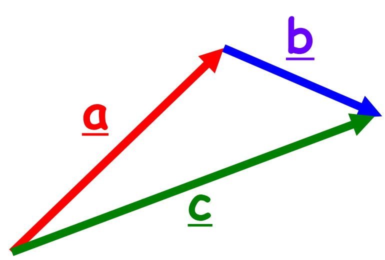
e.g. the result of adding a and b



c = a + b

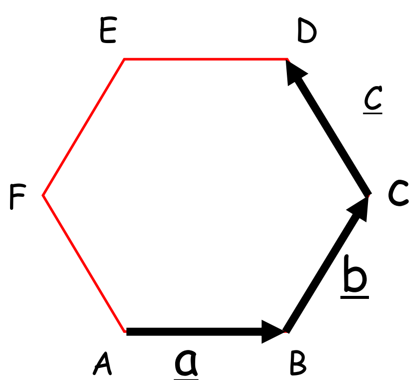
Rewriting Vectors

=> vectors can be rewritten in terms of other vectors



a + b = c

Rewrite the following vectors in terms of $\underline{a}$, $\underline{b}$ and $\underline{c}$



i) $\overrightarrow{BA}$

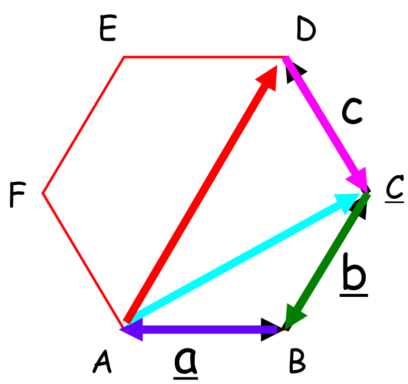
ii) $\overrightarrow{CB}$

iii) $\overrightarrow{DC}$

iv) $\overrightarrow{AC}$

v) $\overrightarrow{AD}$

Rewrite the following vectors in terms of $\underline{a}$, $\underline{b}$ and $\underline{c}$



i) $\overrightarrow{BA} = -\underline{a}$

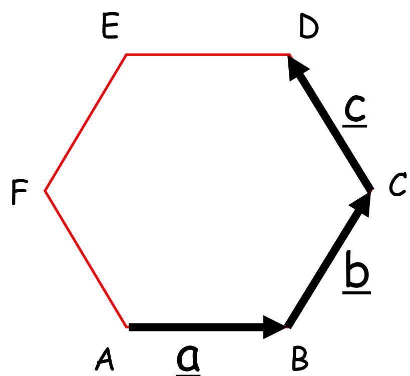
ii) $\overrightarrow{CB} = -\underline{b}$

iii) $\overrightarrow{DC} = -\underline{c}$

iv) $\overrightarrow{AC} = \underline{a} + \underline{b}$

v) $\overrightarrow{AD} = \underline{a} + \underline{b} + \underline{c}$

Rewrite the following vectors in terms of $\underline{a}$, $\underline{b}$ and $\underline{c}$



i) $\overrightarrow{ED}$

ii) $\overrightarrow{FE}$

iii) $\overrightarrow{AF}$

iv) $\overrightarrow{AE}$

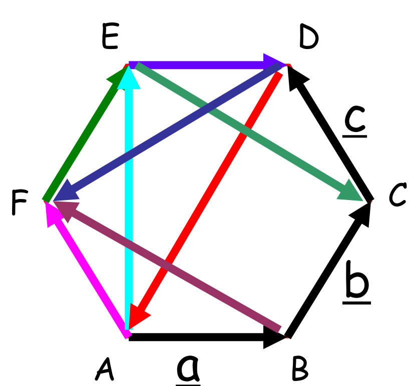
v) $\overrightarrow{DA}$

vi) $\overrightarrow{BF}$

vii) $\overrightarrow{EC}$

viii) $\overrightarrow{DF}$

Rewrite the following vectors in terms of $\underline{a}$, $\underline{b}$ and $\underline{c}$



i) $\overrightarrow{ED} = \underline{a}$

ii) $\overrightarrow{FE} = \underline{b}$

iii) $\overrightarrow{AF} = \underline{c}$

iv) $\overrightarrow{AE} = \underline{c} + \underline{b}$

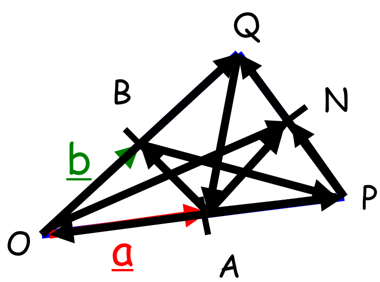
v) $\overrightarrow{DA} = -\underline{c} - \underline{b} - \underline{c}$

vi) $\overrightarrow{BF} = -\underline{a} + \underline{c}$

vii) $\overrightarrow{EC} = \underline{a} - \underline{c}$

viii) $\overrightarrow{DF} = -\underline{a} - \underline{b}$

Rewrite the following vectors in terms of $\underline{a}$ and $\underline{b}$



a. AP =

b. AB =

c. OQ =

d. PO =

e. PQ =

f. PN =

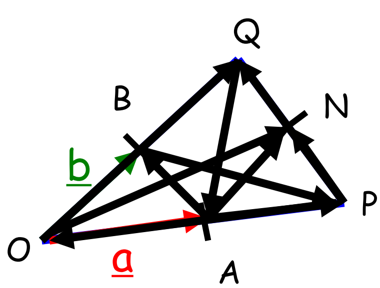
g. ON =

h. AN =

i. BP =

j. QA =

Rewrite the following vectors in terms of $\underline{a}$ and $\underline{b}$



a. AP = $\underline{a}$

b. AB = $\underline{b} - \underline{a}$

c. OQ = $2\underline{b}$

d. PO = $-2\underline{a}$

e. PQ = $2\underline{b} - 2\underline{a}$

f. PN = $\underline{b} - \underline{a}$

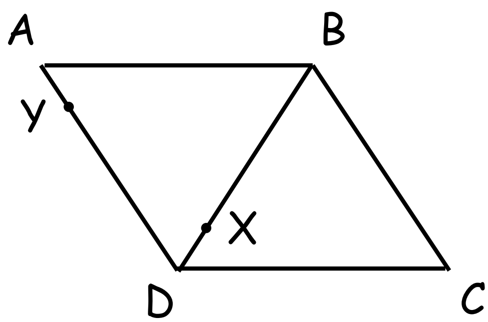
g. ON = $\underline{a} + \underline{b}$

h. AN = $\underline{b}$

i. BP = $2\underline{a} - \underline{b}$

j. QA = $\underline{a} - 2\underline{b}$

Vector Geometry



$\overrightarrow{AB} = \underline{r}$

$\overrightarrow{AD} = \underline{s}$

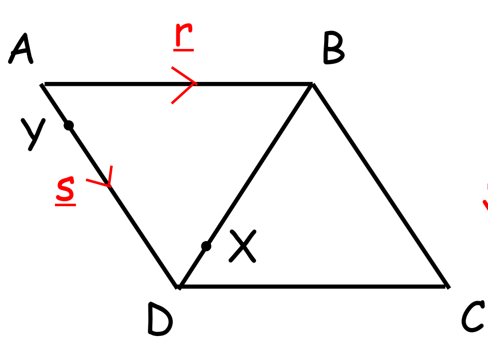
AY:YD = 1:2

DX:XB = 1:2

i) Show that YX is parallel to AC

ii) What is the ratio YX:AC

Solutions



$\overrightarrow{AC} = \underline{r} + \underline{s}$

$\overrightarrow{DB} = \underline{r} - \underline{s}$

$\overrightarrow{YX} = 2/3\underline{r} + 1/3(\underline{r} - \underline{s})$

$\overrightarrow{YX} = 1/3(\underline{r} + \underline{s})$

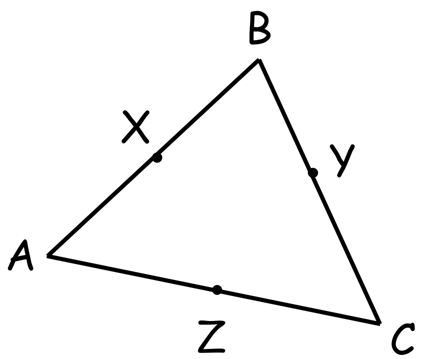
i) AC and YX are scalar multiples => parallel

ii) YX : AC = 1 : 3

i) Show that YX is parallel to AC

ii) What is the ratio YX:AC

Vector Geometry



X, Y and Z are all midpoints

$\overrightarrow{AB} = \underset{\sim}{p}$

$\overrightarrow{AC} = \underset{\sim}{q}$

Solutions

$\overrightarrow{BC} = \underset{\sim}{q} - \underset{\sim}{p}$

$\overrightarrow{XZ} = \frac{1}{2}(\underset{\sim}{q} - \underset{\sim}{p})$

=> Scalar multiples (same direction)

i) Express $\overrightarrow{BC}$ in terms of $\underset{\sim}{p}$ and $\underset{\sim}{q}$

ii) Show that XZ is parallel to BC

Exercises from Cirrito 26

EXAMPLE 26.6 Prove that the line segment joining the mid-points of two sides of a triangle is parallel to the third side and half its length.

EXAMPLE 26.7 Prove that if one pair of opposite sides of a quadrilateral is equal and parallel, then the quadrilateral is a parallelogram.

EXAMPLE 26.8 Find the position vector of the point P which divides the line segment AB in the ratio $m:n$.

EXAMPLE 26.9 Prove that the diagonals of a parallelogram bisect each other.

- Exercises from Cirrito, 26.4