Lesson 73 – Polar Form of Complex Numbers

HL2 Math - Santowski

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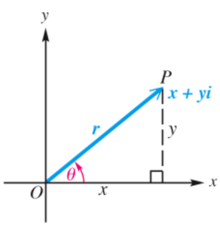
Relationships Among *x, y, r,* and θ

- $x = r\cos\theta$
  $y = r\sin\theta$
  $r = \sqrt{x^2 + y^2}$
  $\tan\theta = \frac{y}{x}, \quad \text{if } x \neq 0$



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Polar Form of a Complex Number

- The expression $r(\cos\theta + i\sin\theta)$

  is called the **polar form** (or **trigonometric form**) of the complex number *x* + *yi*. The expression cos θ + *i* sin θ is sometimes abbreviated cis θ.

Using this notation $r(\cos\theta + i\sin\theta)$ is written r cis θ.

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Example

- Express 2(cos 120° + *i* sin 120°) in rectangular form.

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Example

- Express 2(cos 120° + *i* sin 120°) in rectangular form.

$$\cos 120^\circ = -\frac{1}{2} \qquad 2(\cos 120^\circ + i\sin 120^\circ) = 2\left(-\frac{1}{2}, i\frac{\sqrt{3}}{2}\right)$$

$$\sin 120^\circ = \frac{\sqrt{3}}{2} \qquad = -1 + i\sqrt{3}$$

- Notice that the real part is negative and the imaginary part is positive, this is consistent with 120 degrees being a quadrant II angle.

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Converting from Rectangular to Polar Form

- Step 1 Sketch a graph of the number $x + yi$ in the complex plane.
- Step 2 Find r by using the equation $r = \sqrt{x^2 + y^2}$.
- Step 3 Find θ by using the equation $\tan\theta = \frac{y}{x}, x \neq 0$ choosing the quadrant indicated in Step 1.

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Example

- Example: Find trigonometric notation for –1 – *i*.

Example

- Example: Find trigonometric notation for –1 – *i*.
- First, find *r*.

$$r = \sqrt{a^2 + b^2} \qquad \sin\theta = \frac{-1}{\sqrt{2}} = \frac{-\sqrt{2}}{2} \qquad \cos\theta = \frac{-1}{\sqrt{2}} = \frac{-\sqrt{2}}{2}$$

$$r = \sqrt{(-1)^2 + (-1)^2} \qquad \theta = \frac{5\pi}{4}$$

$$r = \sqrt{2}$$

- Thus, $-1 - i = \sqrt{2}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)$ or $\sqrt{2}\ \text{cis}\ \frac{5\pi}{4}$

Product of Complex Numbers

- Find the product of $4(\cos 50^\circ + i\sin 50^\circ)$ and $2(\cos 10^\circ + i\sin 10^\circ)$.

Product Theorem

- If $r_1 = (\cos\theta_1 + i\sin\theta_1)$ and $r_2(\cos\theta_2 + i\sin\theta_2)$, are any two complex numbers, then

$$\left[r_1(\cos\theta_1 + i\sin\theta_1)\right]\cdot\left[r_2(\cos\theta_2 + i\sin\theta_2)\right]$$
$$= r_1 r_2\left[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)\right].$$

- In compact form, this is written

$$(r_1 \text{ cis } \theta_1)(r_2 \text{ cis } \theta_2) = r_1 r_2 \text{ cis}(\theta_1 + \theta_2).$$

Example: Product

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$$\left[4(\cos 50^\circ + i\sin 50^\circ)\right]\cdot\left[2(\cos 10^\circ + i\sin 10^\circ)\right]$$
$$= 4\cdot 2\left[\cos(50^\circ + 10^\circ) + i\sin(50^\circ + 10^\circ)\right]$$
$$= 8(\cos 60^\circ + i\sin 60^\circ)$$
$$= 8\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$$
$$= 4 + 4i\sqrt{3}$$

Quotient of Complex Numbers

- Find the quotient.

$16(\cos 70^{\circ} + i\sin 70^{\circ})$ and $4(\cos 40^{\circ} + i\sin 40^{\circ})$

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Quotient Theorem

- If $r_1 = (\cos\theta_1 + i\sin\theta_1)$ and $r_2(\cos\theta_2 + i\sin\theta_2)$ are any two complex numbers, where

$r_2(\cos\theta_2 + i\sin\theta_2),\ r_2 \neq 0,$

then

$$\frac{r_1(\cos\theta_1 + i\sin\theta_1)}{r_2(\cos\theta_2 + i\sin\theta_2)} = \frac{r_1}{r_2}\left[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)\right].$$

In compact form, this is written

$$\frac{r_1 \text{ cis } \theta_1}{r_2 \text{ cis } \theta_2} = \frac{r_1}{r_2} \text{ cis}(\theta_1 - \theta_2)$$

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Example: Quotient

- Find the quotient.

$16(\cos 70^{\circ} + i\sin 70^{\circ})$ and $4(\cos 40^{\circ} + i\sin 40^{\circ})$

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Example: Quotient

- Find the quotient.

$16(\cos 70^\circ + i\sin 70^\circ)$ and $4(\cos 40^\circ + i\sin 40^\circ)$

$$\frac{16(\cos 70^\circ + i\sin 70^\circ)}{4(\cos 40^\circ + i\sin 40^\circ)} = \frac{16}{4}\left(\cos(70^\circ - 40^\circ) + i\sin(70^\circ - 40^\circ)\right)$$
$$= 4\cos 30^\circ + i\sin 30^\circ$$
$$= 4\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)$$
$$= 2\sqrt{3} + 2i$$

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If $z = 4\left(\cos 40^\circ + i\sin 40^\circ\right)$ **and** $w = 6\left(\cos 120^\circ + i\sin 120^\circ\right)$,
find: (a) zw **(b)** z/w

If $z = 4\left(\cos 40^\circ + i\sin 40^\circ\right)$ and $w = 6\left(\cos 120^\circ + i\sin 120^\circ\right)$

find: (a) zw (b) z/w

$$zw = \left[4\left(\cos 40^\circ + i\sin 40^\circ\right)\right]\left[6\left(\cos 120^\circ + i\sin 120^\circ\right)\right]$$
$$= \left[(4\cdot 6)\left(\cos(40^\circ + 120^\circ) + i\sin(40^\circ + 120^\circ)\right)\right]$$

multiply the moduli

add the arguments (the *i* sine term will have same argument)

$$= 24\left(\cos 160^\circ + i\sin 160^\circ\right)$$
$$= 24(-0.93969 + 0.34202i)$$
$$= -22.55 + 8.21i$$

If you want the answer in rectangular coordinates simply compute the trig functions and multiply the 24 through.

$$\frac{z}{w} = \frac{4\left(\cos 40^\circ + i\sin 40^\circ\right)}{6\left(\cos 120^\circ + i\sin 120^\circ\right)}$$

$$= \frac{4}{6}\left[\cos\left(40^\circ - 120^\circ\right) + i\sin\left(40^\circ - 120^\circ\right)\right]$$

divide the moduli

subtract the arguments

$$= \frac{2}{3}\left[\cos\left(-80^\circ\right) + i\sin\left(-80^\circ\right)\right]$$

$$= \frac{2}{3}\left[\cos\left(280^\circ\right) + i\sin\left(280^\circ\right)\right]$$

In polar form we want an angle between 0 and 360° so add 360° to the -80°

In rectangular coordinates: $= \frac{2}{3}\left(0.1736 - 0.9848i\right) = 0.12 - 0.66i$

Example

$divide\ \frac{z_1}{z_2}$

$Where\ \ z_1 = 3\sqrt{2} + 3\sqrt{2}i = 6\left(\cos 45 + i\sin 45\right)$

$z_2 = 2\sqrt{3} + 2i = 4\left(\cos 30 + i\sin 30\right)$

Rectangular form

Trig form

Example

$divide\ \frac{z_1}{z_2}$

$Where\ \ z_1 = 3\sqrt{2} + 3\sqrt{2}i = 6\left(\cos 45 + i\sin 45\right)$

$z_2 = 2\sqrt{3} + 2i = 4\left(\cos 30 + i\sin 30\right)$

Rectangular form

$$\frac{3\sqrt{2} + 3\sqrt{2}i}{2\sqrt{3} + 2i}$$

$$\left(\frac{3\sqrt{2} + 3\sqrt{2}i}{2\sqrt{3} + 2i}\right)\left(\frac{2\sqrt{3} - 2i}{2\sqrt{3} - 2i}\right)$$

$$\frac{6\sqrt{6} - 6\sqrt{2}i + 6\sqrt{6}i - 6\sqrt{2}i^2}{12 - 4i^2}$$

$$\frac{6\sqrt{6} - 6\sqrt{2}i + 6\sqrt{6}i + 6\sqrt{2}}{12 + 4}$$

$$\frac{\left(6\sqrt{6} + 6\sqrt{2}\right)}{16} + \frac{\left(6\sqrt{6} - 6\sqrt{2}\right)i}{16}$$

Trig form

$$\frac{6\left(\cos 45 + i\sin 45\right)}{4\left(\cos 30 + i\sin 30\right)}$$

$$\frac{6}{4}\left(\cos\left(45 - 30\right) + i\sin\left(45 - 30\right)\right)$$

$$\frac{3}{2}\left(\cos 15 + i\sin 15\right)$$

$$r = \sqrt{\left(\frac{\left(6\sqrt{6} + 6\sqrt{2}\right)}{16}\right)^2 + \left(\frac{\left(6\sqrt{6} - 6\sqrt{2}\right)}{16}\right)^2} \qquad \theta = \tan^{-1}\left(\frac{\frac{\left(6\sqrt{6} - 6\sqrt{2}\right)}{16}}{\frac{\left(6\sqrt{6} + 6\sqrt{2}\right)}{16}}\right) = 15$$

$$r = \sqrt{\frac{216 + 72\sqrt{12} + 72 + 216 - 72\sqrt{12} + 72}{256}}$$

$$r = \sqrt{\frac{576}{256}} = \sqrt{\frac{9}{4}} = \frac{3}{2}$$

DeMoivre's Theorem

Taking Complex Numbers to Higher Powers

$z = r(\cos\theta + i\sin\theta)$

$z^2 = ??$

$z = r(\cos\theta + i\sin\theta)$

$z^2 = r(\cos\theta + i\sin\theta)r(\cos\theta + i\sin\theta)$

$= r^2(\cos^2\theta + 2i\cos\theta\sin\theta + i^2\sin\theta)$

$= r^2(\cos^2\theta - \sin^2\theta + i2\sin\theta\cos\theta)$

$= r^2(\cos 2\theta + i\sin 2\theta)$

$\boxed{z^2 = r^2(\cos 2\theta + i\sin 2\theta)}$

Nice ☺

What about z^3?

$z = r(\cos\theta + i\sin\theta),\ z^2 = r^2(\cos 2\theta + i\sin 2\theta)$

$z^3 = z \cdot z^2 =$

$r^3(\cos 2\theta + i\sin 2\theta)(\cos\theta + i\sin\theta) =$

$r^3(\cos 2\theta\cos\theta + i\cos 2\theta\sin\theta + i\sin 2\theta\cos\theta + i^2\sin 2\theta\sin\theta) =$

$r^3(\cos 2\theta\cos\theta - \sin 2\theta\sin\theta + i\cos 2\theta\sin\theta + i\sin 2\theta\cos\theta) =$

$r^3\left[\cos 2\theta\cos\theta - \sin 2\theta\sin\theta + i(\sin 2\theta\cos\theta + \cos 2\theta\sin\theta)\right] =$

$r^3\left[\cos(2\theta + \theta) + i(\sin(2\theta + \theta))\right] = r^3(\cos 3\theta + i\sin 3\theta)$

$\boxed{z^3 = r^3(\cos 3\theta + i\sin 3\theta)}$

☺ Hooray!! ☺

We saw…

We saw…

$z^2 = r^2(\cos 2\theta + i \sin 2\theta)$ *and*

$z^3 = r^3(\cos 3\theta + i \sin 3\theta)$

Similarly…

$z^4 = r^4(\cos 4\theta + i \sin 4\theta)$

$z^5 = r^5(\cos 5\theta + i \sin 5\theta)$

$\vdots$

$$\boxed{z^n = r^n(\cos n\theta + i \sin n\theta)}$$

Hooray DeMoivre and his incredible theorem

Powers of Complex Numbers

This is horrible in rectangular form.

$(a+bi)^n$

$(a+bi)(a+bi)(a+bi)\ldots(a+bi)$

The best way to expand one of these is using Pascal's triangle and binomial expansion.

You'd need to use an i-chart to simplify.

It's much nicer in trig form. You just raise the r to the power and multiply theta by the exponent.

$z = r(\cos\theta + i \sin\theta)$

$z^n = r^n(\cos n\theta + i \sin n\theta)$

Example

$z = 5(\cos 20 + i \sin 20)$

$z^3 = 5^3(\cos 3 \cdot 20 + i \sin 3 \cdot 20)$

$z^3 = 125(\cos 60 + i \sin 60)$

De Moivre's Theorem

- If $r_1 = (\cos\theta_1 + i\sin\theta_1)$ is a complex number, and if *n* is any real number, then

$$\left[r\left(\cos\theta_1 + i\sin\theta_1\right)\right]^n = r^n\left(\cos n\theta + i\sin n\theta\right).$$

- In compact form, this is written

$$\left[r\ \text{cis}\theta\right]^n = r^n\left(\text{cis}\ n\theta\right).$$

Example: Find $(-1 - i)^5$ and express the result in rectangular form.

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- First, find trigonometric notation for $-1 - i$

$$-1 - i = \sqrt{2}\left(\cos 225 + i\sin 225\right)$$

- Theorem $\left(-1-i\right)^5 = \left[\sqrt{2}\left(\cos 225^\circ + i\sin 225^\circ\right)\right]^5$

$$= \left(\sqrt{2}\right)^5\left[\cos(5\cdot 225^\circ) + i\sin(5\cdot 225^\circ)\right]$$

$$= 4\sqrt{2}\left(\cos 1125^\circ + i\sin 1125^\circ\right)$$

$$= 4\sqrt{2}\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)$$

$$= 4 + 4i$$

Example

- Let $z = 1 - i$ Find z^{10}

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Example

Example 8.3.2* Let $z = 1 - i$. Find z^{10}.

Solution: First write z in polar form.

$|z| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$

$\arg(z) = -\frac{\pi}{4}$ (or $\frac{7\pi}{4}$)

Polar Form : $z = \sqrt{2}\left(\cos(-\frac{\pi}{4}) + i\sin(-\frac{\pi}{4})\right)$.

Applying de Moivre's Theorem gives :

$$\begin{aligned} z^{10} &= (\sqrt{2})^{10}\left(\cos(10 \times (-\frac{\pi}{4})) + i\sin(10 \times (-\frac{\pi}{4}))\right) \\ &= 2^5\left(\cos(-\frac{10\pi}{4}) + i\sin(-\frac{10\pi}{4})\right) \\ &= 32\left(\cos(-\frac{5\pi}{2}) + i\sin(-\frac{5\pi}{2})\right) \\ &= 32\left(\cos(-\frac{5\pi}{2} + 2\pi) + i\sin(-\frac{5\pi}{2} + 2\pi)\right) \\ &= 32\left(\cos(-\frac{\pi}{2}) + i\sin(-\frac{\pi}{2})\right) \\ &= 32(0 + i(-1)) \\ &= -32i \end{aligned}$$

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Further Examples

In Exercises 35–44, use DeMoivre's Theorem to find the indicated powers of the given complex number. Express the result in standard form.

35. $(1 + i)^4$

36. $(2 + 2i)^6$

37. $(-1 + i)^{10}$

38. $(\sqrt{3} + i)^7$

39. $(1 - \sqrt{3}i)^3$

40. $\left[5\left(\cos\frac{\pi}{9} + i\sin\frac{\pi}{9}\right)\right]^3$

41. $\left[3\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)\right]^4$

42. $\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)^{10}$

43. $\left[2\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)\right]^8$

44. $\left[5\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)\right]^4$

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*n*th Roots

- For a positive integer *n*, the complex number $a + bi$ is an *n*th root of the complex number $x + yi$ if $(a+bi)^n = x + yi.$

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*n*th Root Theorem

- If *n* is any positive integer, *r* is a positive real number, and θ is in degrees, then the nonzero complex number $r(\cos\theta + i\sin\theta)$ has exactly *n* distinct *n*th roots, given by
- where $\sqrt[n]{r}\left(\cos\alpha + i\sin\alpha\right)$ or $\sqrt[n]{r}$ cis α,

$$\alpha = \frac{\theta + 360^\circ \cdot k}{n} \text{ or } \alpha = \frac{\theta}{n} + \frac{360^\circ \cdot k}{n}, \quad k = 0,1,2,\ldots,n-1.$$

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Example: Square Roots

- Find the square roots of $1 + \left(\sqrt{3}\right)i$

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Example: Square Roots

- Find the square roots of $1+\left(\sqrt{3}\right)$
- Polar notation: $1+\left(\sqrt{3}\right)=2\left(\cos 60+i\sin 60\right)$

$$\left[2\left(\cos 60+i\sin 60\right)\right]^{\frac{1}{2}}=2^{\frac{1}{2}}\left[\cos\left(\frac{60}{2}+k\cdot\frac{360}{2}\right)+i\sin\left(\frac{60}{2}+k\cdot\frac{360}{2}\right)\right]$$
$$=\sqrt{2}\left[\cos\left(30+k\cdot 180\right)+i\sin\left(30+k\cdot 180\right)\right]$$

- For k = 0, root is $\sqrt{2}\left(\cos 30+i\sin 30\right)$
- For k = 1, root is $\sqrt{2}\left(\cos 210+i\sin 210\right)$

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Example: Fourth Root

- Find all fourth roots of $-8+8i\sqrt{3}$.

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Example: Fourth Root

- Find all fourth roots of $-8+8i\sqrt{3}$. Write the roots in rectangular form.
- Write in polar form. $-8+8i\sqrt{3}=16\text{ cis }120^\circ$
- Here r = 16 and θ = 120°. The fourth roots of this number have absolute value

$$\sqrt[4]{16}=2.$$

$$\alpha=\frac{120^\circ}{4}+\frac{360^\circ\cdot k}{4}=30^\circ+90^\circ\cdot k$$

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Example: Fourth Root continued

- There are four fourth roots, let *k* = 0, 1, 2 and 3.

$$k=0 \qquad \alpha = 30^\circ + 90^\circ \cdot 0 = 30^\circ$$
$$k=1 \qquad \alpha = 30^\circ + 90^\circ \cdot 1 = 120^\circ$$
$$k=2 \qquad \alpha = 30^\circ + 90^\circ \cdot 2 = 210^\circ$$
$$k=3 \qquad \alpha = 30^\circ + 90^\circ \cdot 3 = 300^\circ$$

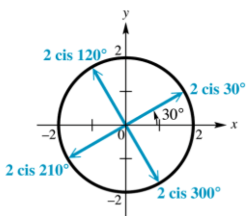
- Using these angles, the fourth roots are

$2 \text{ cis } 30^\circ$, $2 \text{ cis } 120^\circ$, $2 \text{ cis } 210^\circ$, $2 \text{ cis } 300^\circ$

Example: Fourth Root continued

- Written in rectangular form

$$\sqrt{3} + i$$
$$-1 + i\sqrt{3}$$
$$-\sqrt{3} - i$$
$$1 - i\sqrt{3}$$



- The graphs of the roots are all on a circle that has center at the origin and radius 2.

Find the complex fifth roots of $-2\sqrt{3} + 2i$

Find the complex fifth roots of $-2\sqrt{3}+2i$

$$-2\sqrt{3}+2i = 4\left(-\frac{\sqrt{3}}{2}+\frac{1}{2}i\right) = 4(\cos 150^\circ + i\sin 150^\circ)$$

The five complex roots are:

$$z_k = \sqrt[5]{4}\left[\cos\left(\frac{150^\circ}{5}+\frac{360^\circ k}{5}\right)+i\sin\left(\frac{150^\circ}{5}+\frac{360^\circ k}{5}\right)\right]$$

$$z_k = \sqrt[5]{4}\left[\cos(30^\circ + 72^\circ k)+i\sin(30^\circ + 72^\circ k)\right]$$

for $k = 0, 1, 2, 3, 4$.

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$$z_k = \sqrt[5]{4}\left[\cos(30^\circ + 72^\circ k)+i\sin(30^\circ + 72^\circ k)\right]$$

$$k=0,\quad z_0 = \sqrt[5]{4}\left[\cos(30^\circ)+i\sin(30^\circ)\right]$$

$$k=1,\quad z_1 = \sqrt[5]{4}\left[\cos(102^\circ)+i\sin(102^\circ)\right]$$

$$k=2,\quad z_2 = \sqrt[5]{4}\left[\cos(174^\circ)+i\sin(174^\circ)\right]$$

$$k=3,\quad z_3 = \sqrt[5]{4}\left[\cos(246^\circ)+i\sin(246^\circ)\right]$$

$$k=4,\quad z_4 = \sqrt[5]{4}\left[\cos(318^\circ)+i\sin(318^\circ)\right]$$

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Example

- Find all the complex fifth roots of 32

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$32+0i$

$r=\sqrt{a^2+b^2}=\sqrt{32^2+0^2}=\sqrt{32^2}=32$

$\theta=tan^{-1}\frac{0}{32}\qquad \theta=tan^{-1}0=0°$

$32[cos(0°)+i\,sin(0°)]$

$\sqrt[5]{32}\left[cos\left(\frac{0}{5}+\frac{360k}{5}\right)+i\,sin\left(\frac{0}{5}+\frac{360k}{5}\right)\right]$

$2[cos(72k)+i\,sin(72k)]$

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$2[cos(72k)+i\,sin(72k)]$

$k=0\quad 2[cos(0°)+i\,sin(0°)]=2(1+0i)=2$

$k=1\quad 2[cos(72)+i\,sin(72)]=.62+1.90i$

$k=2\quad 2[cos(144)+i\,sin(144)]=-1.62+1.18i$

$k=3\quad 2[cos(216)+i\,sin(216)]=-1.62-1.18i$

$k=4\quad 2[cos(288)+i\,sin(288)]=.62-1.90i$

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Example Find $\sqrt[3]{i}$

You may assume it is the principle root you are seeking unless specifically stated otherwise.

First express i as a complex number in standard form. $0+i$

Then change to polar form $r=\sqrt{a^2+b^2}\qquad r=1$

$tan\,\theta=\frac{b}{a}\qquad tan\,\theta=\frac{1}{0}\qquad \theta=tan^{-1}\frac{1}{0}\qquad \theta=90°$

$1[cos(90°)+i\,sin(90°)]$

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$$1\left[\cos(90°)+i\sin(90°)\right]$$

Since we are looking for the cube root, use DeMoivre's Theorem and raise it to the $\frac{1}{3}$ power

$$1^{\frac{1}{3}}\left[\cos\left(\frac{1}{3}90°\right)+i\sin\left(\frac{1}{3}90°\right)\right]$$

$$1\left[\cos(30°)+i\sin(30°)\right]$$

$$1\left[\frac{\sqrt{3}}{2}+i\frac{1}{2}\right]=\frac{\sqrt{3}}{2}+\frac{1}{2}i$$

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Example:

Find the 4th root of $-2+i$

$$(-2+i)^{\frac{1}{4}}$$

Change to polar form $\sqrt{5}\left[\cos(153.4)+i\sin(153.4)\right]$

Apply DeMoivre's Theorem

$$\sqrt{5}^{\frac{1}{4}}\left[\cos\left(\frac{1}{4}\right)(153.4)+i\sin\left(\frac{1}{4}\right)(153.4)\right]$$

$$1.22\left[\cos(38.4)+i\sin(38.4)\right]$$

$$.96+.76i$$

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