Lesson 57 – Linear Differential Equations

IBHL - Calculus - Santowski

Lesson Objectives

- Review the previous types of First Order Diff Eqns that we already know how to solve
- Introduce linear Diff Eqns and then solving using an “integration factor”

(A) Linear Diff Eqns

- Consider the following Diff Eqns. Two are linear Diff Eqns and two are not

$$(a)\quad \frac{dy}{dx}+2xy=x^2$$
$$(b)\quad \frac{dy}{dx}+2xy^2=\tan x$$
$$(c)\quad \frac{dy}{dx}-\frac{1}{x}y=\cos x$$
$$(d)\quad \frac{dy}{dx}+x\sqrt{y}=2x$$

3 Calculus - Santowski 8/26/17

(A) Linear ODEs

A differential equation is linear, if

1. **dependent variable and its derivatives are of degree one,**
2. **coefficients of a term does not depend upon dependent variable.**

A linear first order equation is an equation that can be expressed in the form

$$\frac{dy}{dx}+P(x)y=Q(x)$$

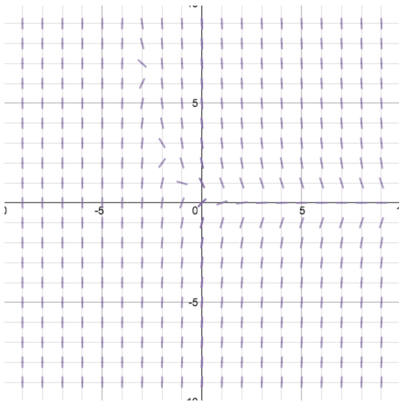
Where P and Q are functions of x

(B) Example to Discuss

- Solve the linear Diff Eqn

$$\frac{dy}{dx} + 3y = e^{-x}$$

- Here is our graphical solution (slope field diagram)



(B) Example to Discuss

- Solve the linear Diff Eqn $\frac{dy}{dx} + 3y = e^{-x}$

- Multiply through by ***e^3x***
- ?? ??

- So we get $\underbrace{e^{3x}\frac{dy}{dx} + 3e^{3x}y}_{NOTICE.....??} = e^{-x} \times e^{3x}$

(B) Example to Discuss

- Solve the linear Diff Eqn

$$\frac{dy}{dx} + 3y = e^{-x}$$

- Multiply through by ***e*^3x^** …. So we get

$$e^{3x}\frac{dy}{dx} + 3e^{3x}y = e^{-x} \times e^{3x}$$

$$\frac{d}{dx}\left(ye^{3x}\right) = e^{2x}$$

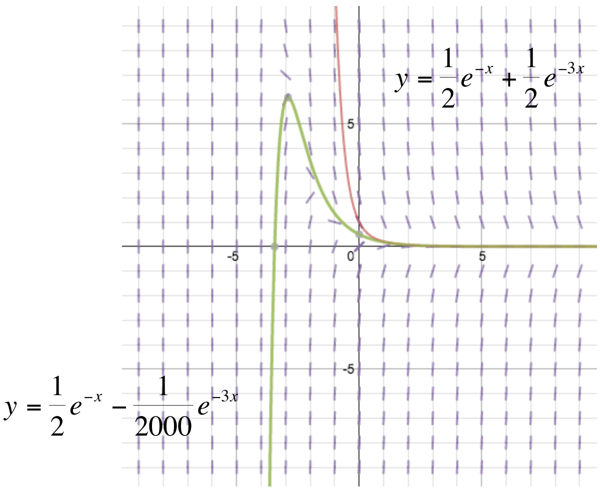
$$\int \frac{d}{dx}\left(ye^{3x}\right) = \int e^{2x}$$

$$ye^{3x} + k = \frac{1}{2}e^{2x} + c$$

$$ye^{3x} = \frac{1}{2}e^{2x} + C$$

$$y = \frac{1}{2}e^{-x} + Ce^{-3x}$$

(B) Example to Discuss



(B) Example to Discuss

- Solve the linear Diff Eqn $\frac{dy}{dx} + 3y = e^{-x}$
- Multiply through by ***e*^3x^**
-
- So HOW did we get ***e*^3x^** as the "integrating factor"?
- We use $IF = e^{\int P(x)dx}$
- (To see WHY, read through Paul Dawkins notes …. Link also at end of ppt)

9 Calculus - Santowski 8/26/17

(C) Examples for Practice

- Solve the following linear Diff Eqns …..

$$(a)\ \frac{dv}{dt} = 9.8 - 0.196v$$

$$(b)\ \frac{dy}{dx} = -4y$$

$$(c)\ \frac{dy}{dx} - 2y = x$$

$$(d)\ \frac{dy}{dx} + 3x^2 y = x^5$$

10 Calculus - Santowski 8/26/17

(D) Further Examples for Practice

- Solve the following linear Diff Eqns …..

$$(a)\ \frac{1}{\mathrm{x}}y' - \frac{2}{x^2}y = x\cos x \quad \text{for } x > 0$$

$$(b)\ \frac{dx}{dt} - x\cos(t) = \cos(t)$$

$$(c)\ y' + y\tan(x) = \cos(x^2) \ \text{ and } \ y(0) = 2$$

$$(d)\ \frac{dy}{dx} = \cos^2(x) - y\sec(x)$$

(D) Further Examples for Practice

- Solve the following linear Diff Eqns …..

$$(a)\ \frac{dy}{dx} + 4xy = x$$

$$(b)\ y' - 2y = t^2e^{2t}$$

$$(c)\ (1+\mathrm{t}^2)\mathrm{y}' + 4ty = \frac{1}{(1+t^2)^2}$$

Further Practice ..

- http://www.cse.salford.ac.uk/physics/gsmcdonald/H-Tutorials/ordinary-differential-equations-integrating-factor.pdf

(G) Resources

- Detailed Notes & explanations/derivations from Paul Dawkins
- http://tutorial.math.lamar.edu/Classes/DE/Linear.aspx

- From patrickJMT:
- https://www.youtube.com/watch?v=Et4Y41ZNyao
- https://www.youtube.com/watch?v=RnYzatmp-_s

- From Mathispower4u
- https://www.youtube.com/watch?v=HAb9JbBD2ig
- https://www.youtube.com/watch?v=zN0TmKEXFh8