

Lesson 54 – Slope Fields – Graphical Solutions to DE

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Fast Five

- ▶ 1. Solve the DE $y' = 2x - 5$
- ▶ 2. Determine the tangent slopes of $y(x)$ from Q1 for the points on the table below:
- ▶ 3. Solve the DE $y' = x + y$
- ▶ 4. Determine the tangent slopes of $y(x)$ from Q3 for the points on the table below:

(-2,2)	(-1,2)	(0,2)	(1,2)	(2,2)
(-2,1)	(-1,1)	(0,1)	(1,1)	(2,1)
(-2,0)	(-1,0)	(0,0)	(1,0)	(2,0)
(-2,-1)	(-1,-1)	(0,-1)	(1,-1)	(2,-1)
(-2,-2)	(-1,-2)	(0,-2)	(1,-2)	(2,-2)

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Lesson Objectives

- ▶ 1. Sketch a slope field for a given differential equation and use the given boundary conditions to identify a specific solution curve on their slope field.
- ▶ 2. Provide a geometric interpretation of differential equations using slope fields.
- ▶ 3. Explain the relationship between slope fields and solution curves for differential equations.

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(A) Intro to Slope Fields

- ▶ There is a **geometric/graphical representation** of the solutions to a differential equation that can assist us with understanding the solution even if we do not know the algebraic expression of the solution.
- ▶ This representation is called a slope field.

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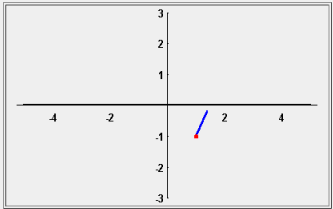
(A) Intro to Slope Fields

- ▶ Consider the following example: $dy/dx = -2xy$
- ▶ **Key point: we can get information about slope, of the solution curve, at any point directly from the DE (without solving it).**
- ▶ The slope, $y'(x)$, of the solutions $y(x)$, is determined once we know the values for x and y .
- ▶ e.g., if $x=1$ and $y=-1$, then the slope of the solution $y(x)$ passing through the point $(1,-1)$ will be $(-2)(1)(-1) = 2$.
- ▶ If we graph $y(x)$ in the x - y plane, it will have slope 2, given $x=1$ and $y=-1$.
- ▶ We indicate this graphically by inserting a small line segment at the point $(1,-1)$ of slope 2.
- ▶ See next slide for a graphic representation of this slope segment

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(A) Intro to Slope Fields

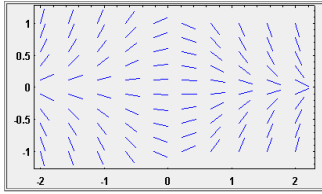
- ▶ Thus, the solution of the DE $dy/dx = -2xy$ with the initial condition $y(1)=-1$ will look similar to this line segment as long as we stay close to $x=1$.
- ▶ Hence, **we can draw small line segments with slope $f(x_i, y_i)$ at any desired point (x_i, y_i)**



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(A) Intro to Slope Fields

- ▶ Of course, doing this at just one point does not give much information about the solutions. We want to do this simultaneously at many points in the x - y plane.

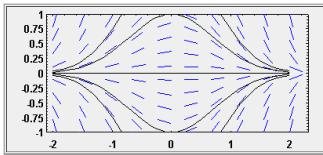


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(A) Intro to Slope Fields

- ▶ We can get an idea as to the form of the differential equation's solutions by "connecting the dots." So far, we have graphed little pieces of the tangent lines of our solutions. The "true" solutions should not differ very much from those tangent line pieces!

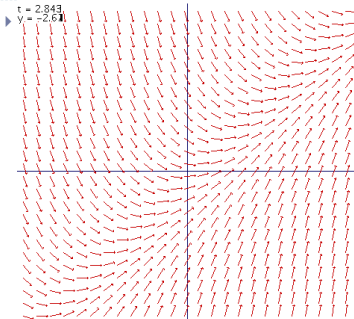


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(B) Slope Fields: Drawing Specific Solutions to DE

- ▶ We can get an idea as to the visual form of the differential equation's solutions by "connecting the dots." → So let's practice:

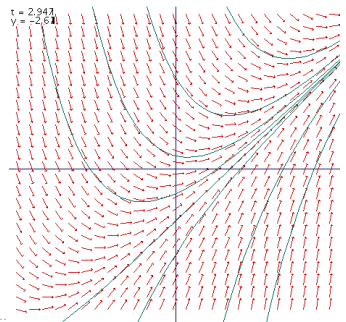


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(B) Slope Fields: Drawing Specific Solutions to DE

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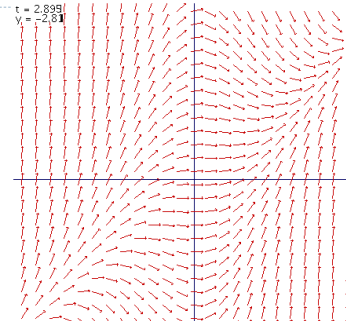


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(B) Slope Fields: Drawing Specific Solutions to DE

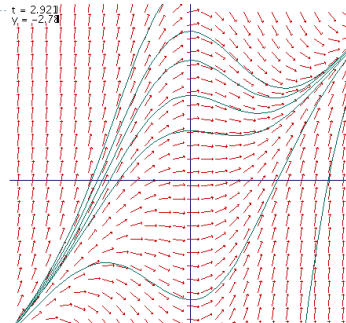
▶ We can get an idea as to the visual form of the differential equation's solutions by "connecting the dots." → So let's practice:



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(B) Slope Fields: Drawing Specific Solutions to DE

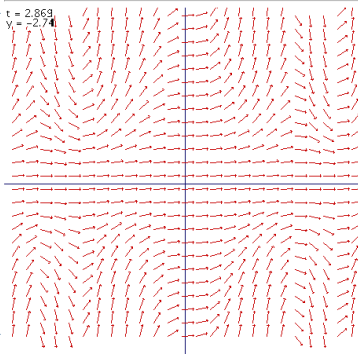
▶ We can get an idea as to the visual form of the differential equation's solutions by "connecting the dots." → So let's practice:



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(B) Slope Fields: Drawing Specific Solutions to DE

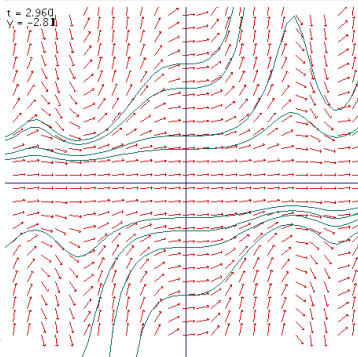
- ▶ We can get an idea as to the visual form of the differential equation's solutions by "connecting the dots." → So let's practice:



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(B) Slope Fields: Drawing Specific Solutions to DE

- ▶ We can get an idea as to the visual form of the differential equation's solutions by "connecting the dots." → So let's practice:



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(C) Drawing Slope Fields w/o Technology

- ▶ It is great to use a computer, but sometimes one is required to sketch a slope field by hand.
- ▶ To do this we use a version of a T-table and then use small line segments to make the sketch.
- ▶ **Example: Sketch a few representative slopes of the slope field** $y' = x - y/2$

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(C) Drawing Slope Fields w/o Technology

- ▶ We will sketch the integer points for $-2 \leq x \leq 2$ and $-2 \leq y \leq 2$.

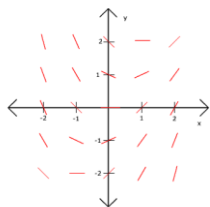
Point	(-2,-2)	(-2,-1)	(-2,0)	(-2,1)	(-2,2)
Slope	-1	-1.5	-2	-2.5	-3
Point	(-1,-2)	(-1,-1)	(-1,0)	(-1,1)	(-1,2)
Slope	0	-0.5	-1	-1.5	-2
Point	(0,-2)	(0,-1)	(0,0)	(0,1)	(0,2)
Slope	1	0.5	0	-0.5	-1
Point	(1,-2)	(1,-1)	(1,0)	(1,1)	(1,2)
Slope	2	1.5	1	0.5	0
Point	(2,-2)	(2,-1)	(2,0)	(2,1)	(2,2)
Slope	3	2.5	2	1.5	1

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(C) Drawing Slope Fields w/o Technology

- ▶ We now draw the slope field by plotting the small line segments centered at the given point with the given slope. The slope field is shown below.



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(D) Drawing Slope Fields - Examples

Draw the slope field for $\frac{dy}{dx} = 2y$



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(D) Drawing Slope Fields - Examples

Draw the slope field for $\frac{dy}{dx} = x + y$

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(D) Drawing Slope Fields - Examples

Draw the slope field for $\frac{dy}{dx} = y - 1$

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(D) Drawing Slope Fields - Examples

Draw the slope field for $\frac{dy}{dx} = \frac{y}{x}$

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(E) Matching Slope Fields to DE

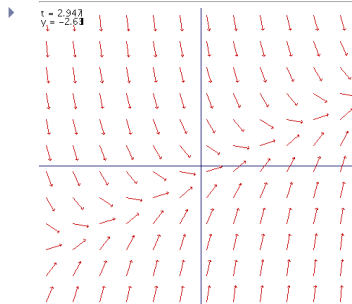
- ▶ Finally, we should be able to match DE with slope field diagrams:

$$y' = 2x - y$$

$$y' = 2x + y$$

$$y' = x - 2y$$

$$y' = x + 2y$$



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(F) Internet Links

- ▶ (3) <http://tutorial.math.lamar.edu/Classes/DE/DirectionFields.aspx>
- ▶ http://apcentral.collegeboard.com/apc/public/repository/ap08_calculus_slope_fields_worksheet.pdf
- ▶ <http://www.mrsk.ca/AP/Korpi7.1slopeFields.pdf>
- ▶ <http://www.sandi.net/cms/lib/CA01001235/Centricity/Domain/8052/61%20-%20Slope%20Fields%20Worksheet.pdf>
- ▶ <http://designatedderiver.wikispaces.com/file/view/slope+fields+%231.pdf>

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