

**A. Skills Review:** → Techniques of Integration (Review the 4 we know)

## 1. Integration by Substitution

1.  $\int x^3 \sqrt{4 + x^4} dx$

2.  $\int \frac{dx}{x \ln x}$

3.  $\int \frac{(x + 5) dx}{\sqrt{x + 4}}$

 4. In each integral below, find the integer  $n$  that allows for an integration by **substitution**. Then perform the integration.

(a)  $\int x^n \sqrt{1 - x^4} dx$

(b)  $\int \frac{x^n}{\sqrt{1 - x^4}} dx$  (there are two very natural choices for  $n$ ).

(c)  $\int \frac{x^n}{1 + x^{10}} dx$  (there are two very natural choices for  $n$ ).



## 2. Integration by Parts

2.  $\int x^2 e^{-x/10} dx.$

3.  $\int x^2 \ln x dx$

5.  $\int x^2 \sin x dx$

9.  $\int \sin^{-1} x dx$

10.  $\int (\sin^{-1} x)^2 dx$

8. Assume that  $\int f(x) dx = g(x)$ , that  $\int g(x) dx = h(x)$  and compute

(a)  $\int x^3 f(x^2) dx$

(b)  $\int x^{2n-1} f(x^n) dx$

### 3. Trigonometric Substitutions

1.  $\int \frac{\sqrt{9-x^2}}{x^2} dx$

2.  $\int \frac{dx}{x\sqrt{1-x^2}}$

3.  $\int \frac{dx}{x\sqrt{a^2+x^2}}$

### 4. Partial Fractions

1.  $\int \frac{5x-3}{x^2-2x-3} dx$

2.  $\int \frac{6x+7}{(x+2)^2} dx$

3.  $\int \frac{2x^3-4x^2-x-3}{x^2-x^2-3} dx$

B. Asd
**8.7 ADDITIONAL EXERCISES**

These problems require the techniques of this chapter, and are in no particular order. Some problems may be done in more than one way.

1.  $\int (t + 4)^3 dt \Rightarrow$

2.  $\int t(t^2 - 9)^{3/2} dt \Rightarrow$

3.  $\int (e^{t^2} + 16)te^{t^2} dt \Rightarrow$

4.  $\int \sin t \cos 2t dt \Rightarrow$

5.  $\int \tan t \sec^2 t dt \Rightarrow$

6.  $\int \frac{2t + 1}{t^2 + t + 3} dt \Rightarrow$

7.  $\int \frac{1}{t(t^2 - 4)} dt \Rightarrow$

8.  $\int \frac{1}{(25 - t^2)^{3/2}} dt \Rightarrow$

9.  $\int \frac{\cos 3t}{\sqrt{\sin 3t}} dt \Rightarrow$

10.  $\int t \sec^2 t dt \Rightarrow$

11.  $\int \frac{e^t}{\sqrt{e^t + 1}} dt \Rightarrow$

12.  $\int \cos^4 t dt \Rightarrow$

13.  $\int \frac{1}{t^2 + 3t} dt \Rightarrow$

14.  $\int \frac{1}{t^2 \sqrt{1 + t^2}} dt \Rightarrow$

15.  $\int \frac{\sec^2 t}{(1 + \tan t)^3} dt \Rightarrow$

16.  $\int t^3 \sqrt{t^2 + 1} dt \Rightarrow$

17.  $\int e^t \sin t dt \Rightarrow$

18.  $\int (t^{3/2} + 47)^3 \sqrt{t} dt \Rightarrow$

19.  $\int \frac{t^3}{(2 - t^2)^{5/2}} dt \Rightarrow$

20.  $\int \frac{1}{t(9 + 4t^2)} dt \Rightarrow$

21.  $\int \frac{\arctan 2t}{1 + 4t^2} dt \Rightarrow$

22.  $\int \frac{t}{t^2 + 2t - 3} dt \Rightarrow$

23.  $\int \sin^3 t \cos^4 t dt \Rightarrow$

24.  $\int \frac{1}{t^2 - 6t + 9} dt \Rightarrow$

25.  $\int \frac{1}{t(\ln t)^2} dt \Rightarrow$

26.  $\int t(\ln t)^2 dt \Rightarrow$

27.  $\int t^3 e^t dt \Rightarrow$

28.  $\int \frac{t + 1}{t^2 + t - 1} dt \Rightarrow$

Compute  $\int f(x) dx$  for  $f(x) =$

1.  $\frac{1}{\sqrt[3]{3x}}$

2.  $\frac{x}{\sqrt{2x^2 + 1}}$

3.  $\frac{x}{2x^2 + 1}$

4.  $\frac{\cos(x)}{\sqrt[3]{\sin(x)}}$

5.  $\ln(1 + x)$

6.  $\frac{e^{\sqrt{x}}}{\sqrt{x}}$

7.  $e^{\sqrt{x}}$

8.  $\frac{1}{x^3 + x}$

9.  $\frac{1}{x^3 - x^2}$

10.  $\frac{x^2 + 1}{x^2 - 1}$

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<http://web.mit.edu/jorloff/www/18.03-esg/integ-review.pdf>

[https://airacademy.asd20.org/Teachers/Amy\\_Cofield/AP%20Calculus/Chapter%206/Ch%206%20WS%20solutions.pdf](https://airacademy.asd20.org/Teachers/Amy_Cofield/AP%20Calculus/Chapter%206/Ch%206%20WS%20solutions.pdf)

## Mixed Integration – All

1. 
$$\int \frac{2x^{\frac{2}{3}} + 5x^2 \sqrt[3]{x} - 9x}{3x^2} dx$$

2. 
$$\int \frac{\tan^2(x) + 1}{\tan(x)} dx$$

3. 
$$\int (4xe^{3x^2-7} + 4x^3) dx$$

4. 
$$\int \frac{3^{\ln(2x+1)}}{6x+3} dx$$

5. 
$$\int \sec^4(2x) \tan(2x) dx$$

6. 
$$\int \frac{18x-15}{3x^2-5x+2} dx$$

7. 
$$\int \frac{-5e^{\sin(2x)}}{3e^{\sin(2x)} \sec(2x)} dx$$

8. 
$$\int (4x^3 - 8x^{-2} + 6x^{-1}) dx$$

9. 
$$\int \frac{2x^2 + 3x - 35}{x-3} dx$$

10. 
$$\int \frac{-5e^x}{1+e^{2x}} dx$$

11. 
$$\int \frac{2x}{4x^4 + 5} dx$$

12. 
$$\int \frac{5x}{\sqrt{25-2x^2}} dx$$