HL1 - Lesson 44 DAY 2: Antiderivatives as Integrals | Topic 6 – Calculus

A. Lesson Context

<table>
<tr><td>BIG PICTURE of this UNIT:</td><td colspan="3">

- How do we measure “change” in a function or function model?
- Why do we measure “change” in a function?
- How do we analytically analyze a function or function model – beyond a simple preCalculus & visual/graphic level?

</td></tr>
<tr><td>CONTEXT of this LESSON:</td><td>Where we’ve been

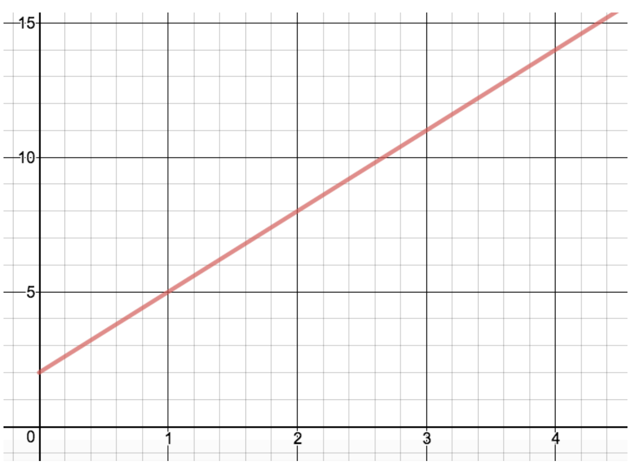
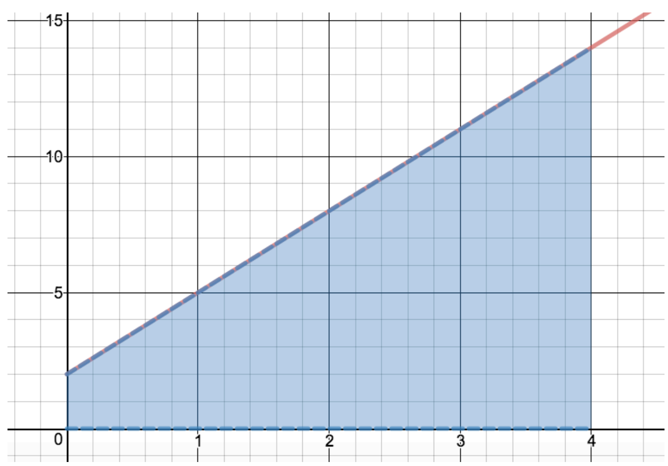
We have connected two ideas through our motion work ➔ antiderivatives and area under the curve</td><td>Where we are

Can we now carry forward the area under the curve idea to nonlinear functions and once again use an antiderivative?</td><td>Where we are heading

We will explore the concept of integration</td></tr>
</table>

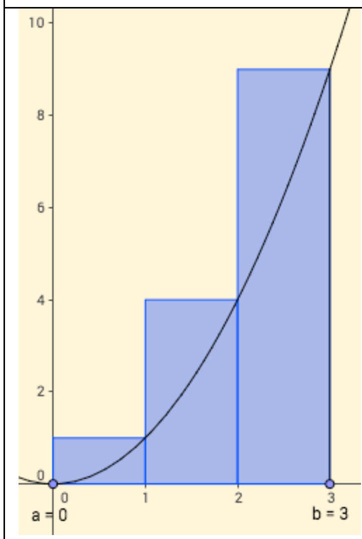
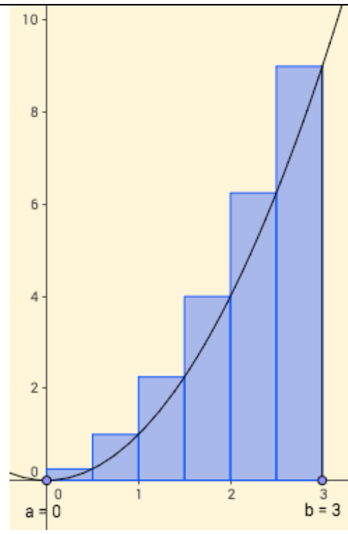
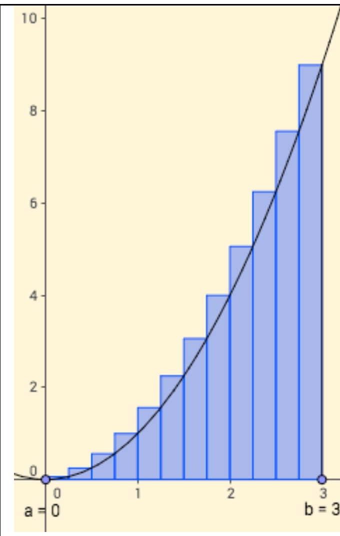
B. Recap:

We have connected two fundamental ideas in our last lesson ➔ idea #1: that of area under a curve (in our simplest cases wherein we looked at a velocity-time graph) to find a total distance travelled (or displacement) AND then idea #2: where we can use an antiderivative (i.e the position function) to find the same answer of distance traveled (or displacement).

<table>
<tr><td colspan="2">An object travels with a velocity defined by the function $v(t) = 3t + 2$ for 4 seconds. Its starting position was $s(0) = 2$. How far did it travel?</td></tr>
<tr><td>Velocity-Time graph
</td><td>Area under the curve of V-T graph

Total distance travelled is (2x4) + 0.5(4)(12) = 32 m</td></tr>
<tr><td colspan="2">Using the antiderivatives, the position function is $s(t) = 1.5t^2 + 2t + 2$. So we can also use this position function to determine $s(4) – s(0) = (1.5x4^2 + 2x4 + 2) – 2 = 24 + 8 + 2 – 2 = 32$ and come up with the same 32 m of distance traveled.</td></tr>
</table>

C. **But what if ……. (Go to Geogebra link) ➔ https://www.geogebra.org/m/CfwjsmHx**

But what happens when the velocity function is NOT linear, but rather a curve? How do we now find an area under the curve?

So, let's estimate the area under the curve of f(x) = x^2, between x = 0 and x = 3 ➔ how? ➔ let's make rectangles		
		
$A_T = A_1 + A_2 + A_3$ $AT = h_1w + h_2w + h_3w$ $A_T = f(1)\cdot 1 + f(2)\cdot 1 + f(3)\cdot 1$	$A_T = A_1 + A_2 + A_3 + A_4 + A_5 + A_6$ $AT = h_1w + h_2w + h_3w + h_4w + h_5w + h_6w$ $A_T = f(0.5)\cdot \frac{1}{2} + f(1)\cdot \frac{1}{2} + \ldots\ldots + f(2.5)\cdot \frac{1}{2} + f(3)\cdot \frac{1}{2}$	$A_T = A_1 + A_2 + A_3 + \ldots\ldots\ A_7 + A_8 + A_9$ $AT = h_1w + h_2w + \ \ldots.\ + h_8w + h_9w$ $A_T = f\left(\frac{1}{3}\right)\cdot \frac{1}{3} + f\left(\frac{2}{3}\right)\cdot \frac{1}{3} + \ldots\ldots + f\left(\frac{8}{3}\right)\cdot \frac{1}{3} + f(3)\cdot \frac{1}{3}$

So what are we seeing? A summation of a the areas of rectangles (and the dimensions of each rectangle are determined by the function "height" multiplied by a width)

$$A_T = A_1 + A_2 + A_3 + \ldots\ldots\ldots\ldots = \sum_{i=1}^{n} A_i$$

$$A_T = f(x_1)\cdot \Delta x + f(x_2)\cdot \Delta x + f(x_3)\cdot \Delta x + \ldots\ldots = \sum_{i=1}^{n} f(x_i)\cdot \Delta x$$

But how many rectangles do we need? ➔ how about an infinite number!!! ➔ hence that limit idea again …….

$$A_T = f(x_1)\cdot \Delta x + f(x_2)\cdot \Delta x + f(x_3)\cdot \Delta x + \ldots\ldots = \sum_{i=1}^{n} f(x_i)\cdot \Delta x$$

$$A_T = \lim_{n \to} \sum_{i=1}^{n} f(x_i)\cdot \Delta x$$

So how about a new symbol? $A_T = \lim_{n\to} \sum_{i=1}^{n} f(x_i)\cdot \Delta x$ will now be represented/replaced by $A_T = \int_a^b f(x)dx$, where *a* and *b* are the two x-value "boundaries" along the x-axis that form the area we are after.

What calculus approach do we use to find these areas? ➔ Antiderivatives!!

So what does the "integral" symbol ask us to "perform"? ➔ determine an antiderivative!!

D. <u>Definite Integrals as Area under the curve using antiderivatives: EXAMPLES</u>

1. Evaluate the following definite integral ➔ $\int_1^5 (x-1)dx$. Verify using a GDC
2. Evaluate the following definite integral ➔ $\int_{-5}^1 (x-1)dx$. Verify using a GDC
3. Evaluate the following definite integral ➔ $\int_{-5}^5 (x-1)dx$. Verify using a GDC

What point(s) is/are being made by these three examples?

4. Evaluate the following definite integral ➔ $\int_1^3 \left(x^2+2x\right) dx$. Verify using a GDC
5. Evaluate the following definite integral ➔ $\int_1^4 \left(\frac{1}{x}\right) dx$. Verify using a GDC
6. Evaluate the following definite integral ➔ $\int_1^3 4x^2(x+1)\, dx$. Verify using a GDC
7. Evaluate the following definite integral ➔ $\int_0^{\pi/2} \sin(x)\, dx$. Verify using a GDC
8. Evaluate the following definite integral ➔ $\int_0^{\frac{3\pi}{2}} \cos(x-\pi)\, dx$. Verify using a GDC

http://hahnmath.weebly.com/uploads/5/7/6/3/57632417/worksheet - definite integrals.pdf

E. Indefinite Integrals as Antiderivatives

Notice that these two columns are really asking the SAME question, asking the same thing of you

(a) Find the antiderivative of x^4	(a) Find the indefinite integral $\int x^4 dx$
(b) Find the antiderivative of $x^{\frac{2}{5}}$	(b) Find the indefinite integral $\int x^{\frac{2}{5}} dx$
(c) Find the antiderivative of $\frac{1}{\sqrt[5]{x}}$	(c) Find the indefinite integral $\int \left(\frac{1}{\sqrt[5]{x}}\right) dx$

Find the following indefinite integrals:

i. $\int x^3 dx$ and then $\int 2x^3 dx$ and then $\int -4x^3 dx$ and then $\int \frac{1}{\sqrt[3]{2}} x^3 dx$ ➔ point being?

ii. $\int x^3 dx$ and then $\int 5x^{-2} dx$ and then $\int 4 dx$ and then finally $\int (x^3 + 5x^{-2} + 4) dx$ ➔ point being?

iii. $\int x^3 dx$ and then $\int (x+2)^3 dx$ and then $\int (x-4)^3 dx$ and then $\int (x-\pi)^3 dx$ ➔ point being?

iv. $\int \frac{1}{x} dx$ and then $\int \frac{2}{x} dx$ and then $\int e^x dx$ and then $\int -4e^x dx$ ➔ point being?

v. $\int \frac{1}{2x} dx$ and then $\int \frac{1}{x+2} dx$ and then $\int \frac{1}{3x+2} dx$ and then $\int \frac{1}{4-3x} dx$ ➔ point being?

vi. $\int \sin(x) dx$ and then $\int 2\cos(x) dx$ and then $-\int \sin(x) dx$ and then $\int \cos(x) dx$ ➔ point being?

vii. $\int \sin(x+2) dx$ and then $\int \sin(x-2) dx$ and then $\int \sin(3x) dx$ and then $\int \sin(3x-5) dx$ ➔ point being?

http://www.personal.kent.edu/~bosikiew/Math11012/Section5-1.pdf

F. Indefinite Integrals: PRACTICE

1. $\int (x^3 - 4x + 5)dx$
2. $\int \frac{1}{x^3}dx$
3. $\int (3u^2 - 2)^2 du$
4. $\int (\sqrt{x} + \frac{1}{2\sqrt{x}})dx$
5. $\int y^2(1+3y)dy$
6. $\int (\sqrt[4]{x^3} - 4x)dx$
7. $\int (x+1)(3x-2)dx$
8. $\int \frac{1}{u^2\sqrt{u}}du$
9. $\int \frac{x^2+2x-3}{x^4}dx$
10. $\int \left(\frac{3}{y^3} - \frac{5}{y^2} + 2y \right)dy$

G. Definite Integrals: PRACTICE

1. $\int_1^4 (2x+5)dx$
2. $\int_{-2}^2 (x^2 - 1)dx$
3. $\int_1^4 \frac{3}{\sqrt{u}}du$
4. $\int_{-2}^{-1} (x - \frac{1}{x^2})dx$
5. $\int_1^8 (t^{\frac{1}{3}} + 4)dt$
6. $\int_2^4 (y^2 - \frac{3}{y^2})dy$
7. $\int_1^3 (3u^2 + 5u - 4)du$
8. $\int_0^2 (x+3)^2 dx$

H. Definite Integrals: PRACTICE

So, looking for patterns ➔ here's one ➔ Evaluate the following 4 definite integrals:

First $\int_0^5 (x^2 - 4)\, dx$

and then $\int_0^2 (x^2 - 4)\, dx + \int_2^5 (x^2 - 4)\, dx$ (why did I select x = 2 to "split up" the integral?

and then $\int_0^5 \left| x^2 - 4 \right| dx$ and then $\int_0^2 \left| x^2 - 4 \right| dx + \int_2^5 \left| x^2 - 4 \right| dx$