HL1 - Lesson 44 DAY 1: Motion – Introducing Antiderivatives | Topic 6 – Calculus

A. Lesson Context

BIG PICTURE of this UNIT:	• How do we measure "change" in a function or function model? • Why do we measure "change" in a function? • How do we analytically analyze a function or function model – beyond a simple preCalculus & visual/graphic level?		
CONTEXT of this LESSON:	Where we've been We understand how differentiate a position function in order to generate a velocity (and acceleration) function	Where we are Can we now use this motion concept & work backwards from a velocity function to a position function?	Where we are heading We will use the idea of motion to introduce and explore integration

B. Derivatives: The Concept of "Family of Curves"

1. For the following functions, graph them and then take their derivatives. What observation(s) do you make about (i) the functions and (ii) the derivatives? Explain what the term "family of curves" might mean.

   (a) $f(x) = x^2$ (b) $f(x) = x^2 + 3$ (c) $f(x) = x^2 - 5$ (d) $f(x) = x^2 + e$ (e) $f(x) = x^2 - \pi$

2. What could be **one** function that was differentiated to create a **derivative** of 4x? (i.e. $dy/dx = 4x$)

3. A function whose derivative is equal to 4x passes through the point (0,3). What was this function?

C. Bringing Motion into it??

1. The acceleration of an object is determined by the equation $a(t) = 12t$ m/s^2. It is known that the object's initial velocity was 5 m/s and its initial position was 6 m to the left of an "origin".

   i. Determine the velocity of the object as a function of *t*.
   ii. Determine the position of the object as a function of *t*.
      (a) Determine the position of object at t = 4 s.
      (b) Determine the distance traveled in the first 4 seconds.

2. The velocity of an object is determined by the equation $v(t) = 3\sqrt{t} - 6$ m/s. It is known that the object's initial position was 16 m to the left of an "origin".

   i. Determine the acceleration of the object as a function of *t*.
   ii. Determine the position of the object as a function of *t*.
      (a) Determine the position of the object at (i) t = 4 s and also at (ii) t = 8
      (b) Determine the total distance traveled in the (i) first 4 seconds and then (ii) in the first 8 seconds
      (c) Determine the displacement of the object in the (i) first 4 seconds and then (ii) in the first 8 seconds

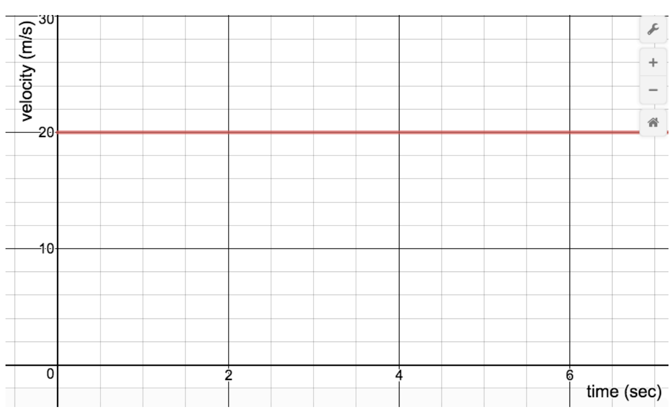
D. Antiderivatives

1. What could be one function that was differentiated to create a **derivative** of $-8x$? (i.e. $dy/dx = -8x$)
2. A function, whose derivative is equal to $11x$, passes through the point (0,3). What was this function?
3. Determine an antiderivative of $y = x^3$ if the antiderivative passes through (2,5).
4. Determine the antiderivative of $y = \sin(x)$, knowing that this antiderivative passes through the point (0,3)
5. Determine the equation of $y = f(x)$, knowing that $f'(x) = 6e^{3x}$, given that $f(0) = 0$
6. Determine an antiderivative of $y = \dfrac{1}{x}$.

E. Connecting Ideas – Example #1

1. GRADE 9 SCIENCE: An object travels at a constant velocity of 20 m/s for 5 seconds. Determine how far the object traveled in those 5 seconds.

2. GRADE 9 SCIENCE/INTRO PHYSICS: The velocity-time graph of a object is shown below. Use the graph to determine how far the object traveled in the first 5 seconds of its motion.



3. INTRO CALCULUS: The velocity function of an object is $v(t) = 20$, where v is in m/s and t is time in seconds. If the initial position of the obect was 5 m to the right of the “origin” (i.e. $s(0) = 5$), determine (i) the distance function and (ii) **hence**, the total distance travelled by the object in the first five seconds

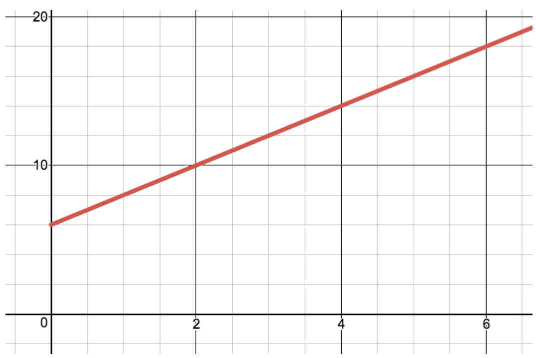
4. Explain how these three questions are really all asking the same thing!!

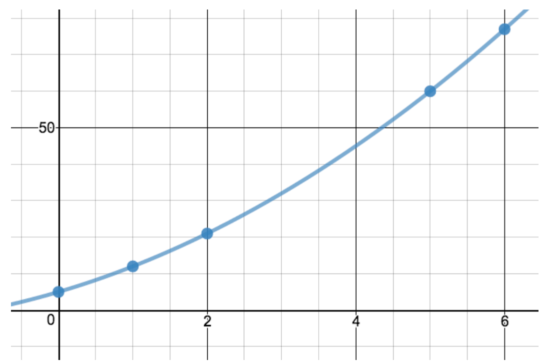
F. <u>Connecting Ideas – Example #2</u>

1. The acceleration of an object is determined by the equation $a(t) = 2$ m/s^2. It is known that the object's initial velocity was 6 m/s to the right and its initial position was 5 m to the right of an "origin".

   i. Determine the velocity of the object as a function of *t*.

   ii. Use the *v*(*t*) function and graph to determine the total distance travelled by the object in the first 6 seconds as well as the displacement of the object after the first 6 seconds.





   iii. Determine the position of the object as a function of *t*.

   iv. Use the *s*(*t*) function as well as the graph to determine the total distance travelled by the object in the first 6 seconds

   v. Use the *s*(*t*) function as well as the graph to determine the displacement of the object after the first 6 seconds of motion.

2. Since raindrops grow as they fall, their surface area increases and therefore the resistance to their falling increases. A raindrop has an an initial downward speed of 10 m/s and its downward acceleration, *a*, is given by

$$a = \begin{cases} 9 - 0.9t & 0 \le t \le 10 \\ 0 & t > 10 \end{cases}.$$

   i. What is the velocity of the raindrop after (a) 1 second and then (b) after 10 seconds?

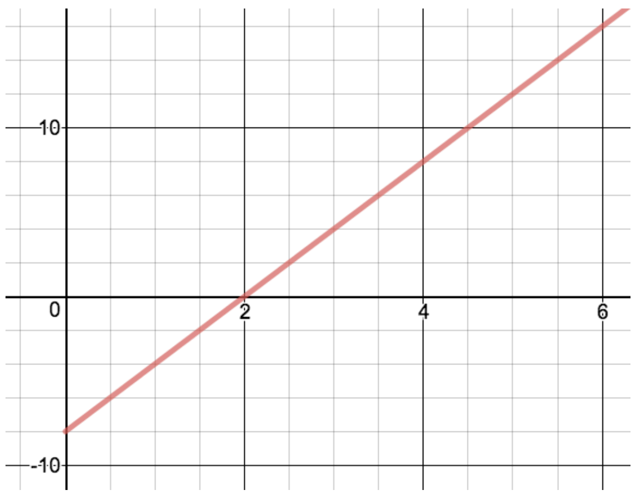
   ii. How far does the raindrop fall in the first (a) 10 seconds and then (b) in the first 12 seconds

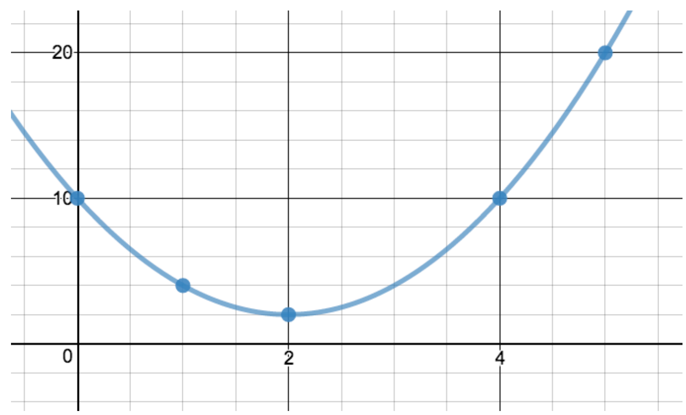
   iii. If the raindrop is initially 600 m above the ground, how long does it take to fall?

3. The acceleration of an object is determined by the equation $a(t) = 4$ m/s^2. It is known that the object's initial velocity was 8 m/s to the left and its initial position was 10 m to the right of an "origin".

   i. Determine the velocity of the object as a function of *t*.

   ii. Use the *v*(*t*) function and graph to determine the total distance travelled by the object in the first 5 seconds as well as the displacement of the object after the first 5 seconds.





   iii. Determine the position of the object as a function of *t*.

   iv. Use the *s*(*t*) function and graph to determine the total distance travelled by the object in the first 5 seconds

   v. Use the *s*(*t*) function and graph to determine the displacement of the object after the first 5 seconds of motion.

G. **Practice with Antiderivatives** ➔ Find the general antiderivatives of:

$f(x) = \frac{2}{x^7} + \frac{x^5}{2}$	$\frac{d}{dx} f(x) = \sqrt{x} + \sqrt[3]{x}$	$f'(x) = \sqrt{x} - \sqrt{1-x}$
$y = \frac{1}{x} - \frac{1}{1-x}$	$f(x) = 4e^{2x} + 6e^{-\frac{1}{2}x}$	$g(x) = \sin(2x) + 2\cos(x)$
$f(x) = -4\cos(2-x)$	$f'(x) = \frac{2}{1+x^2}$	$f(x) = -\sec x \tan x$
$\frac{dy}{dx} = -\tan(x)$	$y = xe^{x^2}$	$f(x) = \sin^2(x)\cos(x)$

http://www.ms.uky.edu/~heidegl/Ma113/RecWorksheets/RWS23.pdf