

Lesson Objectives

- 1. Define the terms explicit and implicit equations
- > 2. Implicitly differentiate implicitly defined equations
- ▶ 3. Determine the equation of tangents and normals of implicitly defined equations
- 4. Apply implicit differentiation to real world problems

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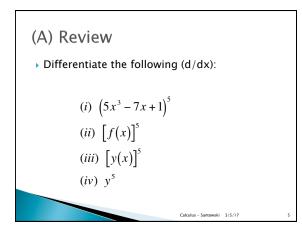
Fast Five

- 1. Isolate *y* from $x^2 + y^2 = 25$
- 2. Isolate *y* from 3x 2y + 10 = 0
- 3. Isolate *y* from $y^2 4x + 7 = 0$
- 4. Isolate *y* from $3x^2 2y^3 = 1$
- 5. Isolate y from $2x^5 + x^4y + y^5 = 36$
- > 6. Differentiate $36 = 2x^5 + x^4y + y^5$ on Wolframalpha

(A) Review

- ▶ Up to this point, we have always defined functions by expressing one variable *explicitly* in terms of another i.e. y = f(x) = x² - 1/x + x
- > In other courses, we have also seen functions expressed *implicitly* i.e. in terms of both variables i.e. $x^2 + y^2 = 25$
- In simple implicit functions, we can always isolate the *y* term to rewrite the equation in *explicit* terms • i.e. $y = \pm \sqrt{(25 - x^2)}$
- In other cases, rewriting an implicit relation is not so easy i.e. $2x^5 + x^4y + y^5 = 36$

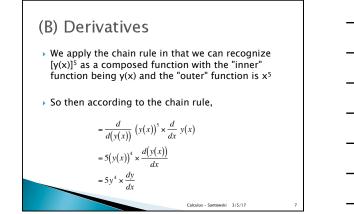
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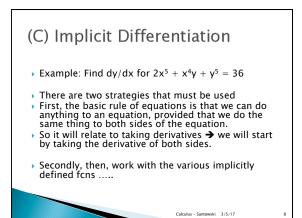


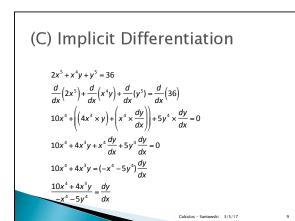
(A) Review

- We need to agree on one convention → when we see a y term in an implicitly (or explicitly defined equation), we will understand that we are really saying y(x) → i.e. that y(x) is a differentiable function in x
- Therefore, if we see y⁵, then we will interpret this expression as (y(x))⁵ → it is therefore differentiable in x.

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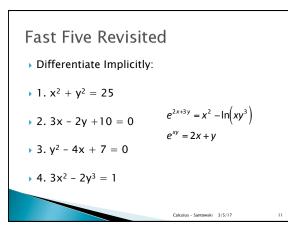


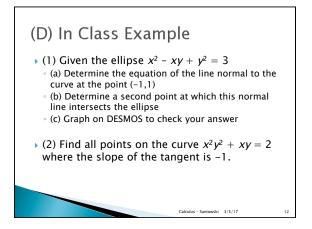
(D) In Class Examples

- ex 2. Find dy/dx if $x + \sqrt{y} = x^2y^3 + 5$
- > ex 3. Find the slope of the tangent line drawn to $x^2 + 2xy + 3y^2 = 27$ at x = 0.
-) ex 4. Determine the equation of the tangent line to the ellipse $4x^2 + y^2 8x + 6y = 12$ at x = 3.

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• ex 5. Find d^2y/dx^2 of $x^3 + y^3 = 6$





"Level 67" Level Questions

- > 1. Find the equation of the lines that are tangent to the ellipse $x^2 + 4y^2 = 16$ AND also pass through the point (4,6)
- 2. Prove that the curves defined by x² y² = k and xy = p intersect orthogonally for all values of of the constants k and p. Illustrate with a sketch

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13

"Level 67" Level Questions

Find the equation of the tangent line at the point (a,b) on the curve $x^{2/3} + y^{2/3} = 1$. Hence, show that the distance between the x-and y-intercepts of the tangent line is independent of the point of tangency

Resources

- > You can watch the following ppt/pdfs:
- http://mrsantowski.tripod.com/2014MathHL/Resources/ Implicit_Diff_Part_1.pdf
- http://mrsantowski.tripod.com/2014MathHL/Resources/ Implicit_Diff_Part_2.pdf
- https://www.youtube.com/watch?v=anq_8ARu08g
- https://www.youtube.com/watch?v=rtZqpBSowJU
- https://www.youtube.com/watch?v=1scXr6g7HdA
