1. Differentiate the following

   a. $y = \cos(x^2)$

   b. $y = \cos^2(x)$

   c. $y = 3\sin(2x)$

   d. $y = 6x\sin(3x^2)$

2. Differentiate the following:

$$y(t) = 2^{\sqrt{t}}$$

$$f(x) = \frac{e^x}{x^2 + 2}$$

$$d(t) = \frac{e^t}{1 - e^{2t}}$$

$$f(y) = y^3 + y5^{\sqrt{y}}$$

3. Differentiate the following:

$$y(t) = \sqrt{1 + \cos t + \sin^2 t}$$

$$f(x) = \frac{x^2}{2 - \cos(\pi x)} \quad .$$

$$f(y) = y^2 \cos\left(3y^3\right)$$

4. Differentiate the following:

$$y(x) = \ln(x^2 + 2x - 1)$$

$$y(x) = \ln\left(\frac{1}{x}\right)$$

$$y(x) = \ln(\ln x)$$

$$y(x) = (\ln x)^2$$

$$f(x) = \log_2\left(\frac{1}{x}\right)$$

$$f(x) = \log_3\left(1 + x\ln 3\right)$$

$$f(x) = \log e^x$$

$$f(x) = x\ln x - x$$

5. Solve for x if $\frac{(x+1)(x-2)}{2x-3} > 0$

6. Simplify $\frac{6(x+5)^2(3x-2) - 3(3x-2)^2(x+5)}{(x+5)^6}$

7. Determine the equation of the line normal to $g(x) = \frac{2x^2 - x - 3}{x^2 - 1}$ at $x = 0$.

8. Differentiate each of the following rational functions. Simplify the derivative.

$f(x) = \frac{2x+5}{3x-1}$

$g(x) = \frac{x^3 - 3}{1 + 4x^2}$

$h(x) = \frac{x^2}{(x-2)(x^4 - x^2)}$

9. Find the minimum and maximum point(s) of the function $f(x) = x\sin x + \cos x$ on the interval $(-\pi/4, \pi)$

10. Find the equation of the tangent line to $f(x) = x\sin(2x)$ at the point where $x = \pi/4$.

11. What angle does the tangent line to the curve $y = f(x)$ at the origin make with the x-axis if y is given by the equation $y = \frac{1}{\sqrt{3}}\sin 3x$.

12. Find the equation of the tangent line to the curve $y = 1 + xe^{2x}$ at $x = 0$.

13. On what interval is $y = e^{-x^2}$ concave down?

14. Find the intervals of increase/decrease for the function $f(x) = x^2e^{-x}$

15. The curve $y(x) = \frac{1}{1+x^2}$ is called a "**witch of Maria Agnesi**". Find the equation of the tangent line at the point (-1, ½).

16. The curve $y(x) = \frac{x}{1+x^2}$ is called a **serpentine** curve. Find the equation of the tangent line to this curve at the point (3, 0.3).

17. The curve $y(x) = \frac{|x|}{\sqrt{2 - x^2}}$ is called a **bullet nose** curve. Find the equation of the tangent line to this curve at the point (1,1).

Different Types of Qs

18. Given the following information for $y = f(x)$ and $y = g(x)$ ➔ $f(3) = 4$; $g(3) = 2$; $f'(3) = -6$; $g'(3) = 5$. Find the following:

a. $(f+g)(3)$ b. $(fg)'(3)$ c. $(f/g)'(3)$ d. $\left(\dfrac{f}{f-g}\right)'(3)$

19. If $F = f \times g \times h$, find an equation for the derivative of F.

20. If $f(x)$ is a differentiable function, find an expression for the derivative of each of the following functions:

(a) $y = x^2 f(x)$ (b) $y = \dfrac{f(x)}{x^2}$ (c) $y = \dfrac{1 + xf(x)}{\sqrt{x}}$

21. If $F(x) = f(x)g(x)$ where $f(x)$ and $g(x)$ are twice differentiable functions,

a. show that $F'' = f''g + 2f'g' + f g''$

b. Find similar formulas for F''' and $F^{(4)}$.

c. Make a conjecture for the formula for $F^{(n)}$

22. A differential equation is simply an equation that has derivatives in it. A solution to a differential equation is simply any function which can then be differentiated and thus makes the original differential equation true.

a. Show that the equation $y = Ae^{-x} + Bxe^{-x}$ is a solution to the differential equation $y'' + 2y' + y = 0$.

b. For what values of r does the function $y = e^{rx}$ satisfy the differential equation $y'' + 5y' - 6y = 0$?

23. A particle moves along a straight line with a displacement of $s(t)$, velocity of $v(t)$ and acceleration of $a(t)$. Show that $a(t) = v(t)\dfrac{dv}{ds}$. Explain the difference between the meanings of the derivatives $\dfrac{dv}{dt}$ and $\dfrac{dv}{ds}$

Derivatives of New Functions

24. Using the techniques you have now learned, determine the derivative of:
$$\begin{gathered} y = \csc(kx) \\ y = \sec(kx) \\ y = \cot(kx) \end{gathered}$$

25. Using the chain rule, determine the derivative of:
$$\begin{gathered} y = \ln(kx) \\ y = \log_B(kx) \end{gathered}.$$

26. Using the chain rule, determine the derivative of:
$$\begin{gathered} y = \sin^{-1}(x) \\ y = \cos^{-1}(x). \\ y = \tan^{-1}(x) \end{gathered}$$

27. Likewise, use the chain rule, determine the derivative of:
$$\begin{gathered} y = \sin^{-1}(kx) \\ y = \cos^{-1}(kx). \\ y = \tan^{-1}(kx) \end{gathered}$$