Lesson 38 - Graphical Differentiation

CALCULUS - SANTOWSKI

Lesson Objectives

1. Given the equation of a function, graph it and then make conjectures about the relationship between the derivative function and the original function

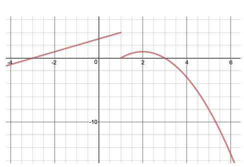
2. From a function, sketch its derivative

3. From a derivative, graph an original function

Fast Five

1. Find f(x) if f'(x) = $-x^2 + 2x$

2. Sketch a graph whose first derivative is always negative

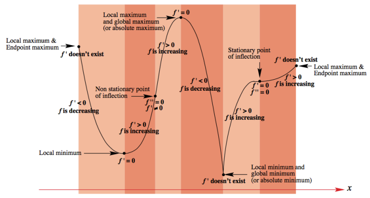


3. Graph the derivative of the function

4. If the graph represented the derivative, sketch the original function

Review of Concepts

A lot of ground has been covered with the many definitions encountered. So, below is a visual summary of the definitions we have covered to date.



690

2/6/17 CALCULUS - SANTOWSKI 4

(A) Important Terms & Derivative Connections

turning point:

maximum:

minimum:

local vs absolute max/min:

"end behaviour"

increase:

decrease:

"concave up"

"concave down"

2/6/17 CALCULUS - SANTOWSKI 5

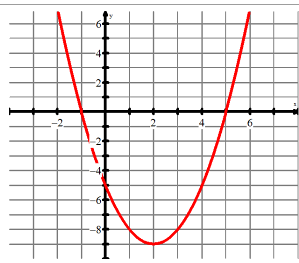
(B) Functions and Their Derivatives

In order to "see" the connection between a graph of a function and the graph of its derivative, we will use graphing technology to generate graphs of functions and simultaneously generate a graph of its derivative

Then we will connect concepts like max/min, increase/decrease, concavities on the original function to what we see on the graph of its derivative

2/6/17 CALCULUS - SANTOWSKI 6

(C) Example #1



(C) Example #1

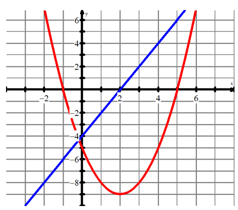
Points to note:

(1) the fcn has a minimum at x=2 and the derivative has an x-intercept at x=2

(2) the fcn decreases on (-∞,2) and the derivative has negative values on (-∞, 2)

(3) the fcn increases on (2,+∞) and the derivative has positive values on (2,+∞)

(4) the fcn changes from decrease to increase at the min while the derivative values change from negative to positive

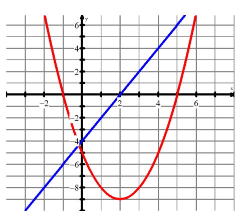


(C) Example #1

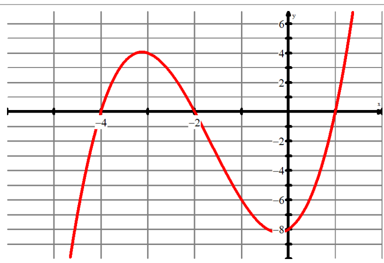
Points to note:

(5) the function is concave up and the derivative fcn is an increasing fcn

(6) The second derivative of f(x) is positive



(D) Example #2



(D) Example #2

f(x) has a max. at x = -3.1 and f `(x) has an x-intercept at x = -3.1

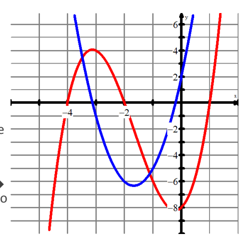
f(x) has a min. at x = -0.2 and f `(x) has a root at –0.2

f(x) increases on (-∞, -3.1) & (-0.2, ∞) and on the same intervals, f `(x) has positive values

f(x) decreases on (-3.1, -0.2) and on the same interval, f `(x) has negative values

At the max (x = -3.1), the fcn changes from being an increasing fcn to a decreasing fcn → the derivative changes from positive values to negative values

At a the min (x = -0.2), the fcn changes from decreasing to increasing → the derivative changes from negative to positive



(D) Example #2

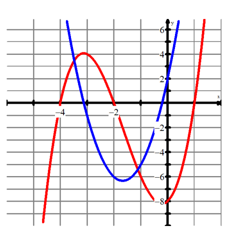
At the max (x = -3.1), the fcn changes from being an increasing fcn to a decreasing fcn → the derivative changes from positive values to negative values

At a the min (x = -0.2), the fcn changes from decreasing to increasing → the derivative changes from negative to positive

f(x) is concave down on (-∞, -1.67) while f `(x) decreases on (-∞, -1.67)

f(x) is concave up on (-1.67, ∞) while f `(x) increases on (-1.67, ∞)

The concavity of f(x) changes from CD to CU at x = -1.67, while the derivative has a min. at x = -1.67

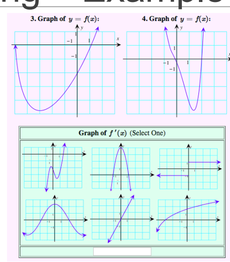


(E) Matching Graph of Derivatives to the Graph of a Function

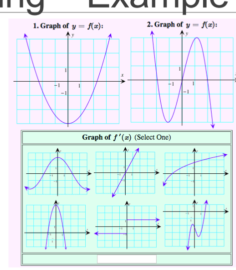
Now, we will build upon this thorough analysis of a function & its connection to the derivative in order to (i) predict what derivatives of more complicated functions look like and (ii) work in REVERSE (given a derivative, sketch the original fcn

To further visualize the relationship between the graph of a function and the graph of its derivative function, we can run through some exercises wherein we are given the graph of a function and we are being asked to match it to the graph of its derivative.

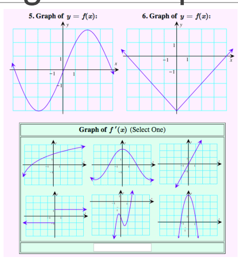
Matching – Example #1



Matching – Example #2



Matching – Example #3

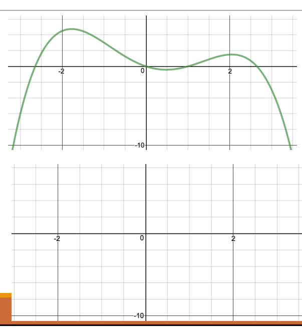


(E) Sketching Graph of Derivatives from the Graph of a Function

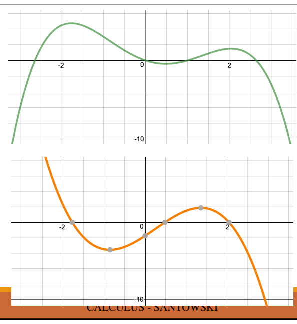
Now, we will build upon this thorough analysis of a function & its connection to the derivative in order to (i) predict what derivatives of more complicated functions look like and (ii) work in REVERSE (given a derivative, sketch the original fcn

To further visualize the relationship between the graph of a function and the graph of its derivative function, we can run through some exercises wherein we are given the graph of a function → can we draw a graph of the derivative and vice versa

(E) Sketching Graph of Derivatives from the Graph of a Function

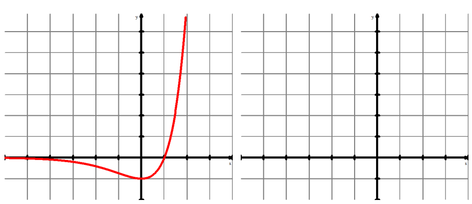


(E) Sketching Graph of Derivatives from the Graph of a Function



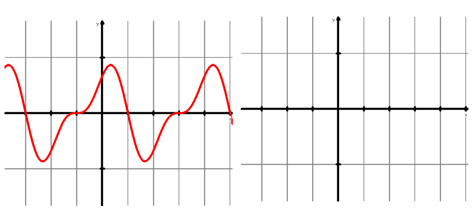
(E) Sketching Graph of Derivatives from the Graph of a Function

Y = F(X) DERIVATIVE = ?



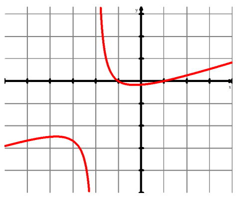
(E) Sketching Graph of Derivatives from the Graph of a Function

Y = F(X) DERIVATIVE = ?

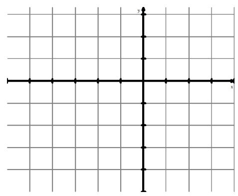


(E) Sketching Graph of Derivatives from the Graph of a Function

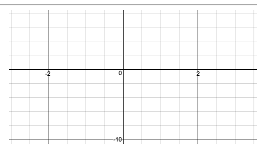
Y = F(X)

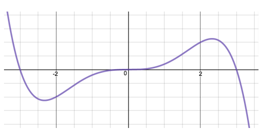


DERIVATIVE = ?

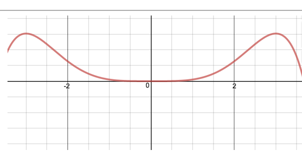


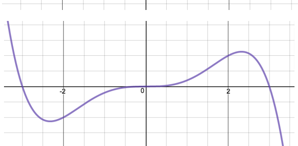
(F) Sketching a Function from the Graph of its Derivative





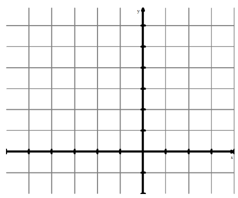
(F) Sketching a Function from the Graph of its Derivative



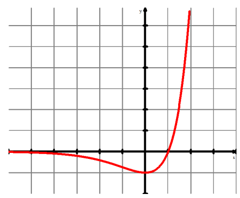


(F) Sketching a Function from the Graph of its Derivative

F(X) = ?

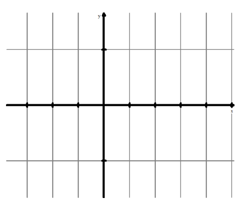


DERIVATIVE

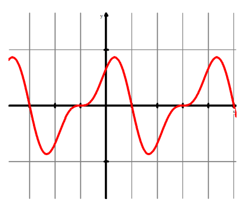


(F) Sketching a Function from the Graph of its Derivative

F(X) = ?

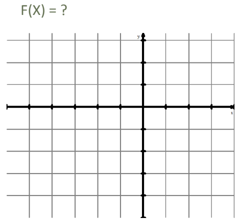


DERIVATIVE

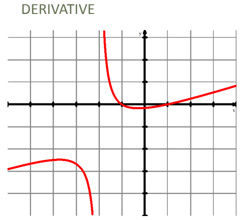


(F) Sketching a Function from the Graph of its Derivative

F(X) = ?

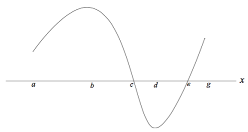


DERIVATIVE



Working With Derivative Graphs

(9) The domain of a function f is $[a, g]$. Below is a sketch of the graph of the *derivative* of f.

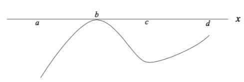


(a) The largest intervals on which f is increasing are (____ , ____) and (____ , ____) .
(b) f has local minima at $x =$ ____ and $x =$ ____ .
(c) The largest intervals on which f is concave up are (____ , ____) and (____ , ____) .
(d) f has points of inflection at: $x =$ ____ and $x =$ ____ .

2/6/17 CALCULUS - SANTOWSKI 28

Working With Derivative Graphs

(11) The domain of a function f is $[a, d]$. Below is a sketch of the graph of the *derivative* of f.



(a) The largest interval on which f is decreasing is (____ , ____) .
(b) f has a local maximum at $x =$ ____ .
(c) The largest intervals on which f is concave up are (____ , ____) and (____ , ____) .
(d) f has points of inflection at: $x =$ ____ and $x =$ ____ .

2/6/17 CALCULUS - SANTOWSKI 29

(G) Matching Function Graphs and Their Derivative Graphs - Internet Links

Work through these interactive applets from maths online Gallery - Differentiation 1 wherein we are given graphs of functions and also graphs of derivatives and we are asked to match a function graph with its derivative graph

(http://www.univie.ac.at/future.media/moe/galerie/diff1/diff1.html)

http://www.univie.ac.at/moe/tests/diff1/ablerkennen.html

2/6/17 CALCULUS - SANTOWSKI 30

Links

https://www.khanacademy.org/math/differential-calculus/taking-derivatives/visualizing-derivatives-tutorial/e/derivative_intuition

http://webspace.ship.edu/msrenault/GeoGebraCalculus/derivative_try_to_graph.html

http://webspace.ship.edu/msrenault/GeoGebraCalculus/derivative_matching.html

http://webspace.ship.edu/msrenault/GeoGebraCalculus/derivative_app_1_graph_AD.html

http://webspace.ship.edu/msrenault/GeoGebraCalculus/derivative_matching_antiderivative.html