A. Lesson Context

CONTEXT of this LESSON:	Where we've been We know how to determine the equations of derivative functions of polynomial functions using the Power Rule	Where we are Can we now work with and analyze polynomials functions (and polynomial models) using derivatives	Where we are heading Can I find generalized methods/techniques for working with instantaneous RoC for ANY function types?

B. Opening (REVIEW) Example

a. Ex 1: Differentiate the following:

i. $b(x) = 4x^3 + 2x^2 - x$

ii. $g(x) = 5x - \frac{10}{x^{-2}} + \frac{1}{2}x^3 - 3x^4 + 1$

iii. $b(x) = 0.1x^3 + 2x^6 - 0.05x^{10}$

iv. $g(x) = 3x^4 - 3x^2 + 4 - \frac{1}{x}$

b. Find the second derivatives of :

i. $f(x) = 5 + x^2$

ii. $g(x) = x^3 + x^2$

iii. $b(x) = 4x^3 + 2x^2 - x$

C. Examples: Working with Polynomial Functions

1. Find the equation of the line which is orthogonal to the curve $y = x^2 - 2x + 4$ at $x = 3$.

2. Given the parabola $y = 2x^2 + 6x + 5$, find the point at which the line $y = -4x+b$ is tangent to the parabola. Hence, find the value of b.

3. Find the point(s) on the graph of $f(x) = x^2 + 3x - 7$ at which the slope of the tangent line is equal to 4.

4. Find the values of x where $y = x^3$ and $y = x^2 + 5x$ have parallel tangent lines.

5. Find all x such that the tangent line to $y = 4x^2 + 11x + 2$ is steeper than the tangent line to $y = x^3$.

6. IB 67 Q. Given an external point $A(-4,0)$ and a parabola $f(x) = x^2 - 2x + 4$, find the equations of the 2 tangents to $f(x)$ that pass through A.

D. Examples: Working with Polynomial Models

1. A ball is dropped from the top of the Empire State building to the ground below. The height in feet, $h(t)$, of the ball above the ground is given as a function of time, t, in seconds since release is given by the model $h(t) = 1250 - 16t^2$

   a. Determine the velocity of the ball 5 seconds after release.
   b. How fast is the ball going when it hits the ground?
   c. what is the acceleration of the ball?

2. Suppose that the total cost in hundreds of dollars of producing x thousands of barrels of oil is given by the function $C(x) = 4x^2 + 100x + 500$. Determine the following.

   a. the cost of producing 5000 barrels of oil
   b. the cost of producing 5001 barrels of oil
   c. the cost of producing the 5001st barrelof oil
   d. C `(5000) = the **marginal cost** at a production level of 5000 barrels of oil. Interpret.
   e. The production level that minimizes the average cost ➔ where $AC(x) = \frac{C(x)}{x}$.

Revenue functions: A demand function, $p = f(x)$, relates the number of units of an item that consumers are willing to buy and the price of the item. Therefore, the revenue of selling these items is then determined by the amount of items sold, x, and the demand (# of items). Thus, $R(x) = x \times p(x)$

3. The demand function for a certain product is given by $p(x) = \frac{50{,}000 - x}{20{,}000}$.

   a. Determine the **marginal revenue** when the production level is 15,000 units.
   b. If the cost function is given by $C(x) = 2100 - 0.25x$, determine the **marginal profit** at the same production level.
   c. How many items should be produced to maximize profits?

E. Further Polynomial Function Analysis: Intervals of Increase & Decrease

RQ #1 ➔ A function is said to be increasing if …….

RQ #2 ➔ A function is said to be decreasing if ….

Concept Question ➔ How does CALCULUS (i.e. DERIVATIVES) help us determine where a function is increasing or decreasing?

RQ #3 ➔ Define the term "extrema" as it relates to functions (like polynomial functions)

Concept Question ➔ How does CALCULUS (i.e. DERIVATIVES) help us determine where a function has extrema?

All these ideas can be summarized in a "test" called the **First Derivative Test**" ➔ Research the First Derivative Test and write out the statement here ➔

1. Given the following polynomial functions, use calculus to determine: (and then your TI-84 to verify)

   $(a)\ f(x) = x^3 + 6x^2 + 9x + 2$

   $(b)\ g(x) = x^4 - 4x^3 - 8x^2 - 1$

   a. the co-ordinates of the extrema
   b. the intervals of increase & decrease

2. Use the FDT to classify all extrema of the function $f(x) = x^4 - 4x^3 + 4x^2$

3. Find and classify all extrema using the FDT given the function $f(x) = 3x^5 - 25x^3 + 60x$.

4. At which x-values does the function $f(x) = x^4 - 4x^3$ have extrema?

5. Determine the co-ordinates of the extrema of the function $g(x) = x^5 - 16x$.

F. More on Extreme Values

RQ #4 ➔ Explain the difference between a GLOBAL (or ABSOLUTE) maximum versus a LOCAL (or RELATIVE) maximum.

Concept Q ➔ Find out what the Extreme Value Theorem states and write it here.

1. Determine the absolute extrema for the following functions given the interval/domain:

   (i) $g(x) = 3x^4 - 4x^3 - 12x^2 + 2$ on $x \in R$

   (ii) $g(x) = 2x^3 + 3x^2 - 12x + 4$ on $[-4,2]$