Lesson 27 – Solving Linear Trigonometric Equations

IB Math HL - Santowski

FAST FIVE – Skills/Concepts Review

- EXPLAIN the difference between the following 2 equations:
- (a) Solve sin(x) = 0.75
- (b) Solve $\sin^{-1}(0.75) = x$
- Now, use you calculator to solve for *x* in both equations
- Define "principle angle" and "related acute angle"

FAST FIVE – Skills/Concepts Review

2. Simplify.

(a) $\sin x\left(\frac{1}{\cos x}\right)$ (b) $(\cos x)(\tan x)$ (c) $1 - \cos^2 x$

(d) $1 - \sin^2 x$ (e) $\cos^2 x + \sin^2 x$ (f) $(1 - \sin x)(1 + \sin x)$

(g) $\frac{\tan x}{\sin x}$ (h) $\frac{\frac{\sin x}{\cos x}}{\tan x}$ (i) $\left(\frac{1}{\tan x}\right)\sin x$

(j) $\frac{1 + \tan^2 x}{\tan^2 x}$ (k) $\frac{\sin x \cos x}{1 - \sin^2 x}$ (l) $\frac{1 - \cos^2 x}{\sin x \cos x}$

(m) $\frac{1}{\sin x} + \frac{1}{\cos x}$ (n) $\tan x + \frac{1}{\cos x}$ (o) $\frac{1}{\tan x} + \sin x$

3. Factor each expression.

(a) $1 - \cos^2\theta$ (b) $1 - \sin^2\theta$

(c) $\sin^2\theta - \cos^2\theta$ (d) $\sin\theta - \sin^2\theta$

(e) $\cos^2\theta + 2\cos\theta + 1$ (f) $\sin^2\theta - 2\sin\theta + 1$

(A) Review

- We have two key triangles to work in two key ways ➔ (i) given a key angle, we can determine the appropriate value of the trig ratio & (ii) given a key ratio, we can determine the value(s) of the angle(s) that correspond to that ratio
- We know what the graphs of the two parent functions look like and the 5 key points on each curve

(B) Solving Linear Trigonometric Equations

- We will outline a process by which we come up with the solution to a trigonometric equation ➔ it is important you understand WHY we carry out these steps, rather than simply memorizing them and simply repeating them on a test of quiz

(B) Solving Linear Trigonometric Equations

- Solve $\sin(\theta) = -\frac{\sqrt{3}}{2}$ if $\{\theta \in R \mid -2\pi \le \theta \le 2\pi\}$

(B) Solving Linear Trigonometric Equations

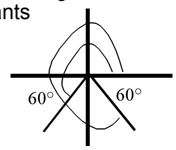
- Work with the example of sin(θ) = -√3/2
- Step 1: determine the related acute angle (RAA) from your knowledge of the two triangles
- Step 2: consider the sign on the ratio (-ve in this case) and so therefore decide in what quadrant(s) the angle must lie
- Step 3: draw a diagram showing the related acute in the appropriate quadrants
- Step 4: from the diagram, determine the principle angles

(B) Solving Linear Trigonometric Equations - Solns

- Work with the example of sin(θ) = -√3/2
- Step 1: determine the related acute angle (RAA) from your knowledge of the two triangles (in this case, simply work with the ratio of √3/2) ➔ θ = 60° or π/3
- Step 2: consider the sign on the ratio (-ve in this case) and so therefore decide in what quadrant the angle must lie ➔ quad. III or IV in this example

(B) Solving Linear Trigonometric Equations

- Step 3: draw a diagram showing the related acute in the appropriate quadrants



- Step 4: from the diagram determine the principle angles ➔ 240° and 300° or 4π/3 and 5π/3 rad.

(B) Solving Linear Trigonometric Equations

- One important point to realize ➔ I can present the same original equation ($\sin(\theta) = -\sqrt{3}/2$) in a variety of ways:
- (i) $2\sin(\theta) = -\sqrt{3}$
- (ii) $2\sin(\theta) + \sqrt{3} = 0$
- (iii) Find the x-intercepts of $f(\theta) = 2\sin(\theta) + \sqrt{3}$
- (iv) Find the zeroes of $f(\theta) = 2\sin(\theta) + \sqrt{3}$

10/26/16 HL Math 10

(B) Solving Linear Trigonometric Equations

- A second important point to realize ➔ I can ask you to solve the equation in a variety of restricted domains (as well as an infinite domain)
- (i) Solve $2\sin(\theta) = -\sqrt{3}$ on $[-2\pi, 2\pi]$
- (ii) Solve $2\sin(\theta) + \sqrt{3} = 0$ on $[-4\pi, \pi/2)$
- (iii) Find the x-intercepts of $f(\theta) = 2\sin(\theta) + \sqrt{3}$ on $(-2\pi, 0)$
- (iv) Find ALL the zeroes of $f(\theta) = 2\sin(\theta) + \sqrt{3}$, given an infinite domain.

10/26/16 HL Math 11

(C) Further Examples

- Solve the following without a calculator

$$2\cos(\theta) + 2 = 3 \quad \text{for } \theta \in (0, 4\pi)$$

$$2\tan(\theta) - \sqrt{2} = 0 \quad \text{for } \theta \in (0, 3\pi)$$

$$\sin(\theta) + 1 = 2 \quad \text{for } \theta \in (-2\pi, 2\pi)$$

10/26/16 HL Math 12

(C) Further Practice

- Solve the following for θ:

$$\sin\theta = 0 \quad \text{for } 0 \le \theta \le 4\pi$$
$$\sin\theta = 1 \quad \text{for } -2\pi \le \theta \le 2\pi$$
$$1+\cos\theta = 0 \quad \text{for } -\pi \le \theta \le 3\pi$$
$$\tan\theta = 0 \quad \text{for } 0 \le \theta \le 3\pi$$

10/26/16 HL Math 13

(C) Further Practice

- Solve without a calculator

$$\sqrt{3} + 3\sin x = 5\sin x \quad \text{for } x \in (0, 4\pi)$$
$$8\cos x + 1 = 2\cos x + 4 \quad \text{for } x \in (-4\pi, 0)$$
$$\sin x - 4 = -2\sin x \quad \text{for } x \in (-2\pi, 2\pi)$$
$$\sin^2 x - 3 = -3\sin^2 x \quad \text{for } x \in (0, 2\pi)$$

10/26/16 HL Math 14

(D) Review – Graphic Solutions

- We know what the graphs of the trigonometric functions look like
- We know that when we algebraically solve an equation in the form of f(x) = 0, then we are trying to find the roots/ zeroes/x-intercepts
- So we should be able to solve trig equations by graphing them and finding the x-intercepts/intersection points

10/26/16 HL Math 15

(E) Examples (with Technology)

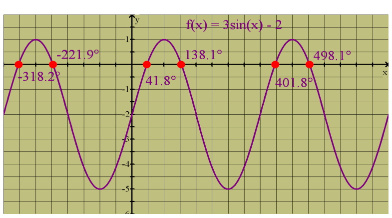
- Solve the equation 3sin(x) – 2 = 0

(E) Examples

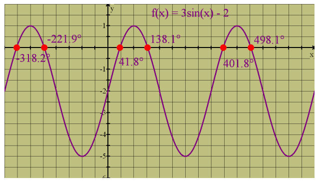
- Solve the equation 3sin(x) – 2 = 0
- The algebraic solution would be as follows:
- We can set it up as sin(x) = 2/3 so x = $\sin^{-1}(2/3)$ giving us 41.8° (and the second angle being 180° - 41.8° = 138.2°
- Note that the ratio 2/3 is not one of our standard ratios corresponding to our “standard” angles (30,45,60), so we would use a calculator to actually find the related acute angle of 41.8°

(E) Examples

- We can now solve the equation 3sin(x) – 2 = 0 by graphing f(x) = 3sin(x) – 2 and looking for the x-intercepts



(E) Examples



- Notice that there are 2 solutions within the limited domain of 0° $\leq$ x $\leq$ 360°
- However, if we expand our domain, then we get two new solutions for every additional period we add
- The new solutions are related to the original solutions, as they represent the positive and negative co-terminal angles
- We can determine their values by simply adding or subtracting multiples of 360° (the period of the given function)

(E) Examples

- Solve the following equations:

$$6\sin\theta + 4 = 0 \quad \text{for } -\pi \leq \theta \leq \pi$$
$$4\sin 2\theta = 7\cos\theta \quad \text{for } -1.5 \leq \theta \leq 3$$
$$2\sin x - 4\sin^2 x = 2\tan\left(\frac{x}{2}\right) \quad \text{for } -8 \leq \theta \leq 0$$

Lesson 25 PART 2 – Solving Linear Trigonometric Equations Involving Angle Changes

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FAST FIVE – Skills/Concepts Review

- EXPLAIN the difference amongst the following 3 functions:

(a) $f(\theta)=\cos(\theta)$

(b) $f(\theta)=\cos(2\theta)$

(c) $f(\theta)=\cos\left(\theta+\frac{\pi}{3}\right)$

10/26/16 HL Math 22

FAST FIVE – Skills/Concepts Review

- EXPLAIN the difference in the solutions to the following 3 equations :

(a) $-\frac{1}{2}=\cos(\theta)$ on $[0,2\pi]$

(b) $-\frac{1}{2}=\cos(2\theta)$ on $[0,2\pi]$

(c) $-\frac{1}{2}=\cos\left(\theta+\frac{\pi}{3}\right)$ on $[0,2\pi]$

10/26/16 HL Math 23

(A) Review

- We have two key triangles to work in two key ways ➔ (i) given a key angle, we can determine the appropriate value of the trig ratio & (ii) given a key ratio, we can determine the value(s) of the angle(s) that correspond to that ratio
- We know what the graphs of the two parent functions look like and the 5 key points on each curve

10/26/16 HL Math 24

(B) Example Set #1

- Without the use of a calculator, prepare an algebraic solution to the following equations:

(a) $\sin(\theta) = 0.5$ on $[\pi, 4\pi]$

(b) $\cos(\theta) + 1 = 0$ on $[-2\pi, 4\pi]$

(c) $3\tan(\theta) = -\sqrt{3}$ on $[-\pi, \pi]$

10/26/16 HL Math 25

(C) Example Set #2

- Use your graphing calculator to prepare a GRAPHIC solution to the following equations:

(a) $\sin(2\theta) = 0.5$ on $[\pi, 4\pi]$

(b) $\cos\left(\theta - \frac{\pi}{4}\right) + 1 = 0$ on $[-2\pi, 4\pi]$

(c) $3\tan\left(2\theta - \frac{\pi}{2}\right) = -\sqrt{3}$ on $[-\pi, \pi]$

10/26/16 HL Math 26

(D) Example Set #3

- Without the use of a calculator, prepare an algebraic solution to the following equations:

(a) $\sin(2\theta) = 0.5$ on $[\pi, 4\pi]$

(b) $\cos\left(\theta - \frac{\pi}{4}\right) + 1 = 0$ on $[-2\pi, 4\pi]$

(c) $3\tan\left(2\theta - \frac{\pi}{2}\right) = -\sqrt{3}$ on $[-\pi, \pi]$

10/26/16 HL Math 27

(D) Example Set #4

- Without the use of a calculator, prepare an algebraic solution to the following equations:

(a) $\sin(2\theta) = 0.5$ on $\theta \in R$

(b) $\cos\left(\theta - \frac{\pi}{4}\right) + 1 = 0$ on $\theta \in R$

(c) $3\tan\left(2\theta - \frac{\pi}{2}\right) = -\sqrt{3}$ on $\theta \in R$

10/26/16 HL Math 28

(E) Further Examples

- 1. Solve the equation

$$\sin(\theta) + \cos\left(\frac{\pi}{2} - \theta\right) - 1 = 0 \text{ for } \theta \in R$$

- 2. Determine the points of intersection of these two functions:

$$f(\theta) = \sin\left(\frac{\theta}{3}\right) \text{ and } g(\theta) = \cos\left(\frac{\theta}{3}\right)$$

10/26/16 HL Math 29

(E) Further Examples

- 3. Find the x-intercepts of

$$h(\theta) = 2 - \sec\left(\frac{\theta}{2}\right) \text{ on } \theta \in R$$

- 4. Determine the zeroes of

$$f(\theta) = \sqrt{2}\csc\left(\frac{\theta}{2} - \pi\right) - 2$$

10/26/16 HL Math 30

(E) Further Examples

- 5. An equation that produces the solutions of $x = 150° + 360°n, n\varepsilon I$ as well as $330° + 360°n, n\varepsilon I$ is …….

(F) Solving Equations with Technology

- The monthly sales of lawn equipment can be modelled by the following function, where S is the monthly sales in thousands of units and t is the time in months, t = 1 corresponds to January.

$$S(t) = 32.4\sin\left(\frac{\pi}{6}\right)t + 53.5$$

- (a) How many units will be sold in August?
- (b) In which month will 70 000 units be sold?
- (c) According to this model, how many times will the company sell 70 000 units over the next ten years?