Lesson 25 – Addition & Subtraction Identities

HL Math - Santowski

Fast Five

- True or False (and justify your responses algebraically, graphically & numerically – but without the use of a calculator)

$$\text{(a)}\quad \sin\left(x+\frac{\pi}{4}\right)=\sin(x)+\sin\left(\frac{\pi}{4}\right)$$

$$\text{(b)}\quad \cos\left(\frac{\pi}{2}-x\right)=\cos\left(\frac{\pi}{2}\right)+\cos(x)$$

(A) Six New Identities (GASP!!)

- Here are six new identities that we call the addition & subtraction identities

$$\sin(A+B) = \sin A \cos B + \sin B \cos A$$
$$\sin(A-B) = \sin A \cos B - \sin B \cos A$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$
$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$
$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

(B) Proving Cosine Subtraction Identity

- We will prove

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

- We will use our unit circle to do so ..... http://www.cut-the-knot.org/triangle/SinCosFormula.shtml

(C) Proving Sine Addition Identity

- We will prove

$$\sin(A+B)=\sin A\cos B+\sin B\cos A$$

- We will use right triangle trig to do so

- http://www.cut-the-knot.org/triangle/SinCosFormula.shtml

(D) Proving Tan Addition Identity

- You will prove

$$\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$$

- You will use fundamental trig identities to do so

(E) Using the Addition/Subtraction Identities

- Evaluate each expression
- 1) sin (75°) ➔ using sin (45° + 30°)
- 2) sin (75°) ➔ using sin (120° – 45°)
- 3) cos (345°)
- 4) $\tan\left(\frac{11\pi}{12}\right)$

(E) Using the Addition/Subtraction Identities

- Determine the exact value of sin(15°)
- Determine the exact value of cos(-195°)
- Determine the exact value of $\sec\left(\frac{5\pi}{12}\right)$
- Determine the exact value of tan(255°)

Pop Quiz

- Find the exact value of

$$\sec\left(\frac{13\pi}{12}\right)$$

(E) Using the Addition/Subtraction Identities

- Find each of the following numbers:
- If $\sin A = \frac{12}{13}$, for $\frac{\pi}{2} < A < \pi$ and $\cos B = -\frac{8}{17}$, for $\pi < B < \frac{3\pi}{2}$
- 1) Evaluate sin (A + B)
- 2) Evaluate cos (A – B)
- 3) Evaluate tan (A + B)
- 4) If sin(a) = -4/5 for $180^\circ \leq a \leq 270^\circ$ and if cos(b) = -5/13 for $90^\circ \leq b \leq 180^\circ$, evaluate tan(a+b)

(E) Using the Addition/Subtraction Identities

- Simplify the following:

$$(1)\quad \cos(270° - x)$$

$$(2)\quad \sin\left(x + \frac{\pi}{2}\right)$$

$$(3)\quad \cos(x + \pi)$$

(E) Using the Addition/Subtraction Identities

- We can use the new identities to develop new identities:
- Prove the following: (describe each identity from a transformations perspective as well as a unit circle perspective)

$$\text{(a)}\quad \cos(\pi + x) = -\cos x$$

$$\text{(b)}\quad \cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\text{(c)}\quad \sin(\pi - \theta_{ref}) = \sin(\theta_{ref})$$

(E) Using the Addition/Subtraction Identities

- Find the exact value of:

(a) $\sin 13^\circ \cos 17^\circ + \sin 17^\circ \cos 13^\circ$

(b) $\cos 25^\circ \cos 35^\circ - \sin 25^\circ \sin 35^\circ$

(E) Using the Addition/Subtraction Identities

- Prove the following:

(a) $\cos(x+y)\cos y + \sin(x+y)\sin y = \cos x$

(b) $1 + \cot x \tan y = \dfrac{\sin(x+y)}{\sin x \cos y}$

Solve for xER

55. $\sin\left(x+\frac{\pi}{3}\right)+\sin\left(x-\frac{\pi}{3}\right)=1$

56. $\cos\left(x+\frac{\pi}{4}\right)-\cos\left(x-\frac{\pi}{4}\right)=1$

57. $\tan(x+\pi)+2\sin(x+\pi)=0$

58. $2\sin\left(x+\frac{\pi}{2}\right)+3\tan(\pi-x)=0$

Solve for xER

59. $\cos\left(x+\frac{\pi}{4}\right)+\cos\left(x-\frac{\pi}{4}\right)=1$

60. $\sin\left(x+\frac{\pi}{2}\right)-\cos\left(x+\frac{3\pi}{2}\right)=0$

61. $\tan(x+\pi)-\cos\left(x+\frac{\pi}{2}\right)=0$

62. $\tan(\pi-x)+2\cos\left(x+\frac{3\pi}{2}\right)=0$

(E) Using the Addition/Subtraction Identities

- We can use the new identities to develop new identities:
- Develop a new identity for:
- (a) sin(2x)
- (b) cos(2x)
- (c) tan(2x)

Sinusoidals: f(x) = Asin(x) + Bcos(x)

- (a) Graph the function $f(x)=\sqrt{3}\sin x+\cos x$
- (b) This function can be rewritten in the form $g(x)=K\sin(x+\alpha)$. Determine the values of K and α.
- (c) Determine the value of $\sin(\alpha)$ and $\cos(\alpha)$

Sinusoidals: f(x) = Asin(x) + Bcos(x)

- (a) Graph the function $f(x)=2\sin x-2\cos x$
- (b) This function can be rewritten in the form $g(x)=K\sin(x+\alpha)$. Determine the values of K and α.
- (c) Determine the value of $\sin(\alpha)$ and $\cos(\alpha)$

Sinusoidals: f(x) = Asin(x) + Bcos(x)

The function $f(x)=A\sin x+B\cos x$ can be rewritten in the form $g(x)=K\sin(x+\alpha)$

Predict the values of K and α, based upon the connections you established in your previous work on the previous 2 slides

Test your conjecture by rewriting $f(x)=5\sin x+12\cos x$ as $g(x)=A\sin(x+\alpha)$

Linear Combinations of Sine and Cosine

A **sum of multiples** of two functions is called a **linear combination** of the two functions.

Thus, if $f(x) = a\sin x + b\cos x$, then $f(x)$ is a linear combination of $\sin x$ and $\cos x$.

We are going to see an example of the general principle that every linear combination of $\sin x$ and $\cos x$ is a sinusoidal function.

Let $f(x) = 3\sin x + 4\cos x$. Show that f is a sinusoidal function. Find the amplitude, period and phase shift.

.

The first step is to find the square root of the sum of the squares of the coefficients. In this case, that is $\sqrt{3^2+4^2} = \sqrt{9+16} = \sqrt{25} = 5$. Factor 5 out of the expression to get $f(x) = 5\left(\frac{3}{5}\sin x + \frac{4}{5}\cos x\right)$

Next, regard the coefficient of $\sin x$ as the cosine of some angle ϕ and the coefficient of $\cos x$ as the sine of the same angle ϕ. We can make this assumption since the sum of the squares of the coefficients equals 1 as does the sum of the squares of $\cos\phi$ and $\sin\phi$ for any angle ϕ..

This gives $f(x) = 5(\cos\phi\sin x + \sin\phi\cos x)$

Using the sum formula for sine, this can be re-written $f(x) = 5\sin(x+\phi)$.

The amplitude is 5, the period is 2π and the phase shift is $-\phi$.

What is the value of ϕ ?

Since both the sine and the cosine of ϕ are positive, ϕ is an angle in quadrant I. So we can calculate $\phi = \arccos\left(\frac{3}{5}\right)$ or alternately, $\phi = \arcsin\left(\frac{4}{5}\right)$. So $\phi = 0.9273$ radians or 53.1°.