

Lesson 22 – Graphs of Trigonometric Functions

IB Math HL

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1

Lesson Objectives

- 1. Relate the features of sinusoidal curves to modeling periodic phenomenon
- 2. Transformations of sinusoidal functions and their features

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2

(A) Key Terms

- Define the following key terms that relate to trigonometric functions:
 - (a) period
 - (b) amplitude
 - (c) axis of the curve (or equilibrium axis)
 - (d) phase shift or phase angle

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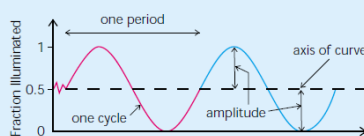
3

(A) Key Terms

Key Ideas

- Repeating data forms a periodic function.
- A periodic function has a self-repeating graph.
- The cycle of a graph is the smallest complete repeating pattern of the graph.
- The length of one cycle is called the **period**.
- The horizontal line that is halfway between the maximum and minimum values of a periodic curve is called the **axis of the curve**.
- The equation of the axis of the curve is

$$y = \frac{\text{maximum value} + \text{minimum value}}{2}$$



- The magnitude of the vertical distance from the axis of the curve to either the maximum or minimum value is called the **amplitude** of the function. The amplitude, a , is calculated as

$$a = \frac{\text{maximum value} - \text{minimum value}}{2}$$

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4

The General Sinusoidal Equation

- In the equation $f(x) = a\sin(k(x+c)) + d$, explain what:
 - a represents?
 - k represents?
 - c represents?
 - d represents?

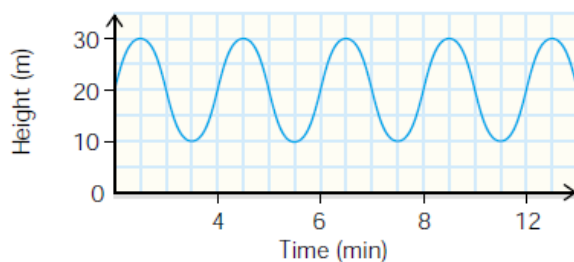
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5

Modeling Periodic Phenomenon

- The graph shows John's height above the ground as a function of time as he rides a Ferris wheel.
 - (a) State the maximum and minimum height of the ride.
 - (b) How long does the Ferris wheel take to make one complete revolution?
 - (c) What is the amplitude of the curve? How does this relate to the Ferris wheel?
 - (d) Determine the equation of the axis of the curve.



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6

Modeling Periodic Phenomenon

- A spring bounces up and down according to the model $d(t) = 0.5 \cos 2t$, where d is the displacement in centimetres from the rest position and t is the time in seconds. The model does not consider the effects of gravity.
 - (a) Draw the graph.
 - (b) Explain why the function models periodic behaviour.
 - (c) What is the period and what does it represent in the context of this question?
 - (d) What is the amplitude and what does it represent in the context of this question?

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7

Modeling Periodic Phenomenon

- A small windmill has its center 6 m above the ground and the blades are 2 m in length. In a steady wind, one blade makes a rotation in 12 sec. Use the point P as a reference point on a blade that started at the highest point above the ground.
 - (a) Determine an equation of the function that relates the height of a tip of a blade, h in meters, above the ground at a time t .
 - (b) What is the height of the point P at the tip of a blade at 5s? 40s?
 - (c) At what time is the point P exactly 7 m above the ground?

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8

Analysis of Sinusoidal Graphs

- (a) Determine the maximums and zeroes of the function $f(x) = -2\sin(2x-60^\circ)+1$
- (b) Determine the equation of the asymptotes and zeroes of the function $g(\theta) = \tan\left(\frac{\theta}{4} + \pi^2\right)$
- (c) Determine the period, min value and the equation of the axis of the curve of $y = 5 - \frac{\sqrt{2}}{\pi} \cos\left(\frac{3}{\sqrt{\pi}}x\right)$

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9

Analysis of Systems of Trig Functions

- (a) Sketch $y = \sin(2x-\pi)$, given the domain of $[0, 2\pi]$
- (b) Sketch $y = \cos(x)$, given the same domain
- (c) Provide a solution to the equation $\sin(2x-\pi) = \cos(x)$

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10

Advanced Sinusoidals

- (a) Use your GDC to graph $f(x) = |\sin(x)|$ on $[-2\pi, 4\pi]$
- (b) State the 6 maximums of $y = f(x)$
- (c) Now graph $g(x) = e^{-0.1x}f(x)$
- (d) Determine the 6 maximums of $y = g(x)$
- (e) Now graph $y = e^{-0.1x}$
- (f) Explain what $y = e^{-0.1x}$ does to the sinusoidal function.
- (g) Explain what dampened simple harmonic motion is

Advanced Sinusoidals

- Predict the domains of the following composite functions

$$(a) f(x) = \sqrt{\ln(\cos(x))}$$

$$(b) g(x) = \tan(\ln(x))$$

$$(c) h(x) = \frac{\tan(x)}{1 - 2\sin^2(x)}$$

Sinusoidals: $f(x) = A\sin(x) + B\cos(x)$

- (a) Graph the function $f(x) = \sqrt{3}\sin x + \cos x$
- (b) This function can be rewritten in the form $g(x) = K\sin(x + \alpha)$. Determine the values of K and α .
- (c) Determine the value of $\sin(\alpha)$ and $\cos(\alpha)$

11/19/16

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13

Sinusoidals: $f(x) = A\sin(x) + B\cos(x)$

- (a) Graph the function $f(x) = 2\sin x - 2\cos x$
- (b) This function can be rewritten in the form $g(x) = K\sin(x + \alpha)$. Determine the values of K and α .
- (c) Determine the value of $\sin(\alpha)$ and $\cos(\alpha)$

11/19/16

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14

Sinusoidals: $f(x) = A\sin(x) + B\cos(x)$

The function $f(x) = A\sin x + B\cos x$ can be rewritten in the form $g(x) = K\sin(x + \alpha)$

Predict the values of K and α , based upon the connections you established in your previous work on the previous 2 slides

Test your conjecture by rewriting
 $f(x) = 5\sin x + 12\cos x$ as $g(x) = A\sin(x + \alpha)$