Lesson 19 – The Logarithmic Function – An Inverse Perspective

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FAST FIVE

- Graph the exponential function $f(x) = 2^x$ by making a table of values ➔ USE A MAPPING DIAGRAM
- What does the input/domain of the function represent?
- What does the output/range of the function represent?
- What are the key graphical features of the function $f(x) = 2^x$?

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Lesson Objectives

- Introduce the function that represents the inverse relation of an exponential function from a graphic and numeric perspective
- Introduce logarithms from an algebraic perspective
- Apply the basic algebraic equivalence of exponential and logarithmic equations to evaluating/simplifying/solving simple logarithmic equations/expressions.

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Terminology (to clarify … I hope)

- In algebra, the terms EXPONENT and POWER unfortunately are used interchangeably, leading to confusion.
- We will exclusively refer to the number that the base is raised to AS THE EXPONENT and NOT THE POWER.
- For the statement that $2^3 = 8$,
  - a) the base is 2: the base is the number that is repeatedly multiplied by itself.
  - b) the exponent is 3: the exponent is the number of times that the base is multiplied by itself.
  - c) the power is 8: the power is the ANSWER of the base raised to an exponent, or the product of repeatedly multiplying the base by itself an integral exponent number of times.

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(A) Graph of Exponential Functions

- Graph the exponential function $f(x) = 2^x$ by making a table of values
- What does the input/domain of the function represent?
- What does the output/range of the function represent?
- What are the key graphical features of the function $f(x) = 2^x$?

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(A) Table of Values for Exponential Functions

x	y
-5.00000	0.03125
-4.00000	0.06250
-3.00000	0.12500
-2.00000	0.25000
-1.00000	0.50000
0.00000	1.00000
1.00000	2.00000
2.00000	4.00000
3.00000	8.00000
4.00000	16.00000
5.00000	32.00000
6.00000	64.00000
7.00000	128.00000

- What does $f(-2) = ¼$ mean ➔ $2^{-2} = ¼$
- Domain/input ➔ the value of the exponent to which the base is raised
- Range/output ➔ the result of raising the base to the specific exponent (i.e. the power)
- Graphical features ➔ y-intercept (0,1); asymptote; curve always increases

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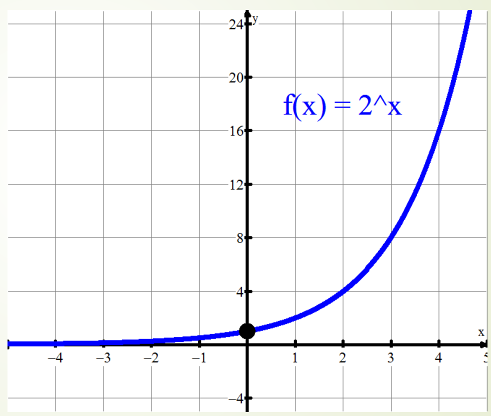
(A) Graph of Exponential Functions

- x y
- -5.00000 0.03125
- -4.00000 0.06250
- -3.00000 0.12500
- -2.00000 0.25000
- -1.00000 0.50000
- 0.00000 1.00000
- 1.00000 2.00000
- 2.00000 4.00000
- 3.00000 8.00000
- 4.00000 16.00000
- 5.00000 32.00000
- 6.00000 64.00000
- 7.00000 128.00000



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(B) Inverse of Exponential Functions

- List the ordered pairs of the inverse function and then graph the inverse
- Let's call the inverse I(x) for now ➔ so what does I(¼) mean and equal ➔ I(¼) = -2
- of course I(x) = $f^{-1}(x)$, so what am I asking if I write $f^{-1}(¼)$ = ????
- After seeing the graph, we can analyze the features of the graph of the logarithmic function

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(B) Table of Values for the Inverse Function

x	y
0.03125	-5.00000
0.06250	-4.00000
0.12500	-3.00000
0.25000	-2.00000
0.50000	-1.00000
1.00000	0.00000
2.00000	1.00000
4.00000	2.00000
8.00000	3.00000
16.00000	4.00000
32.00000	5.00000
64.00000	6.00000
128.0000	7.00000

- What does $f^{-1}(¼) = -2$ mean
  - ➔ $¼ = 2^{-2}$
- Domain/input ➔ the power
- Range/output ➔ the value of the exponent to which the base is raised that produced the power
- Graphical features ➔ x-intercept (1,0); asymptote; curve always increases

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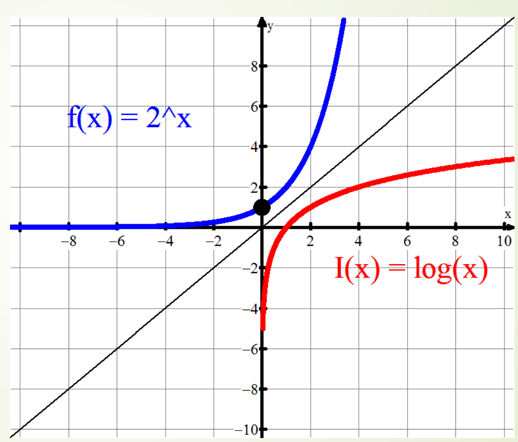
(B) Table of Values & Graphs for the Inverse Function

x	y
0.03125	-5.00000
0.06250	-4.00000
0.12500	-3.00000
0.25000	-2.00000
0.50000	-1.00000
1.00000	0.00000
2.00000	1.00000
4.00000	2.00000
8.00000	3.00000
16.00000	4.00000
32.00000	5.00000
64.00000	6.00000
128.00000	7.00000



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(C) TheLogarithmic Function

- If the inverse is $f^{-1}(x)$ so what I am really asking for if I write $f^{-1}(1/32) = -5$
- The equation of the inverse can be written as $x = 2^y$.
- But we would like to write the equation EXPLICITLY (having the y isolated ➔ having the exponent isolated)
- This inverse is called a logarithm and is written as $y = \log_2(x)$

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(C) Terminology (to clarify … I hope)

- So what's the deal with the terminology ???
- Given that $f(x) = 2^x$, I can write $f(5) = 2^5 = 32$ and what is meant is that the **EXPONENT 5** is "applied" to the **BASE 2**, resulting in 2 multiplied by itself 5 times (2x2x2x2x2) giving us the result of the **POWER of 32**
- The inverse of the exponential is now called a logarithm and is written as $y = \log_2(x)$ or in our case $5 = \log_2(32)$ and what is meant now is that I am taking the **logarithm of the POWER 32** (while working in **BASE 2**) and I get an **EXPONENT of 5** as a result!

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(C) Terminology (to clarify … I hope)

- So the conclusion ➔ To PRODUCE the POWER, I take a BASE and multiply the base a given number of times (ie. The EXPONENT) ➔ this is the idea of an exponential function
- With a logarithmic function ➔ I start with a given POWER and determine the number of times the BASE was exponentiated (i.e. the EXPONENT)
- In math, we use shortcut notations ALL THE TIME. The mathematical shorthand for "What is the exponent on b (the base) that yields p (the power)?" is read as $\log_b p$.

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(D) Playing With Numbers

- Write the following equations in logarithmic form:
- a) $2^3 = 8$ b) $\sqrt{9} = 3$
- c) $125^{1/3} = 5$ d) $4^{-1/2} = ½$
- Write the following equations in exponential form:
- a) $\log_5 25 = 2$ b) $\log_2(1/16) = -4$
- c) $\log_4 2 = 0.5$ d) $\log_7 1 = 0$
- Evaluate the following:
- a) $\log_3(1/27)$ b) $\log_4 8$ c) $\log_{½} 4$
- d) $\log_3 27 + \log_3 81$

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(D) Common Logarithms

- A common logarithm is a logarithm that uses base 10. You can ignore writing the base in this case: $\log_{10}p = \log p$.
- Interpret and evaluate the following:
- a) log10 b) log100 c) log1000
- d) log1 e) log (1/10000) f) log $1/\sqrt{10}$
- Evaluate the following with your calculator and write the value to three decimal places).
- a) log9 b) log10 c) log11 d) log π

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(D) Working With Logarithms

- Solve: (HINT: switch to exponential form)
- (a) $\log_x 27 = 3$ (b) $\log_x \sqrt[3]{25} = 5$
- (c) $\log_x 8 = ¾$ (d) $\log_x 25 = 2/3$
- (e) $\log_4 \sqrt{2} = x$ (f) $\log_2 2^7 = x$
- (g) $5\log_3 9 = x$ (h) $\log_4 x = -3$
- (i) $\log_9 x = -1.5$ (j) $\log_2(x + 4) = 5$
- (k) $\log_3(x – 3) = 3$ (l) $\log_2(x^2 – x) = \log_2 12$
- (m) $\log_3 \sqrt[5]{9} = x$ (n) $\log_{1/3} 9\sqrt{27} = x$
- (o) $\log_x 81 = -4$ (p) $\log_2 \sqrt{0.125} = x$

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Further Examples

- http://www.kutasoftware.com/FreeWorksheets/Alg2Worksheets/Meaning%20of%20Logarithms.pdf
- http://www.kutasoftware.com/FreeWorksheets/Alg2Worksheets/Graphing%20Logarithms.pdf

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(D) Working With Logarithms

- Evaluate $\log_3 9$ and $\log_9 3$
- Evaluate $\log_5 125$ and $\log_{125} 5$
- What relationship exists between the values of $\log_a b$ and $\log_b a$
- Solve $\log_7(\log_4 x) = 0$
- Solve $\log_5(\log_2(\log_3 x)) = 0$

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Extra Practice

- Follow this link to access a Practice Sheet from last year's PreCal class:
- http://mrsantowski.tripod.com/2013PreCalculus/Homework/Exp_and_Log_Assignment.pdf
- Work through Q1aef,2,3,6-10,11a,12a,14a
- http://mrsantowski.tripod.com/2014MathHL/EXAM%20PREP/Exp_and_Log_Graphs.pdf

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(E) Transformed Logarithmic Functions

- As will be seen in the next exercises, the graph maintains the same "shape" or characteristics when transformed
- Depending on the transformations, the various key features (domain, range, intercepts, asymptotes) will change

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(E) Transformed Logarithmic Functions

- So we can now do a complete graphic analysis of this graph
- (i) no y-intercept and the x-intercept is 1
- (ii) the y axis is an asymptote
- (iii) range {yER}
- (iv) domain {x > 0}
- (v) it increases over its domain
- (vi) it has no max/min or turning points

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(E) Graphing Log Functions

- Without using graphing technology, graph the following functions (it may help to recall your knowledge of function transformations)
- (1) $f(x) = \log_2(x + 2)$
- (2) $f(x) = -3\log_2(x - 4)$
- (3) $f(x) = \log_5(4x – 4) + 5$
- Examples and discussions on how to make these graphs is found at the following website:
- Graphs of Logarithmic Functions from AnalyzeMath