Lesson 9 – Compositions of Functions

IBHL1 Math - Santowski

Opening Example

- Consider the functions $f(x)=\sqrt{x-1}$ and $g(x)=\ln(x)$
- State the domains & ranges of each function
- PREDICT the domain of $y=f\circ g(x)$ and $y=g\circ f(x)$
- Explain/show WHY you picked the domain you did.
- Use your TI-84 to compose the functions and verify your predictions

The **BIG** Picture

- And we are studying this because ....?
- Functions will be a unifying theme throughout the course ➔ so a solid understanding of **what** functions are and **why** they are used and **how** they are used will be very important!
- Sometimes, complicated looking equations can be easier to understand as being combinations of simpler, parent functions

(A) Function Composition

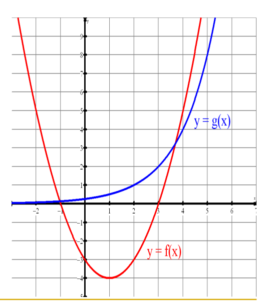
- So we have a way of creating a new function ➔ we can **compose** two functions which is basically a **substitution of one function into another**.
- we have a notation that communicates this idea ➔ if ***f***(***x***) is one functions and ***g***(***x***) is a second function, then the composition notation is ➔ **f o g (x)**

(A) Composition of Functions – Example 1

- We will now define *f* and *g* as follows:
- *f* = {(3,2), (5,1), (7,4), (9,3), (11,5)}
- g = {(1,3), (2,5), (3,7), (4,9), (5,10)}
- We will now work with the composition of these two functions:
  - (i) We will evaluate *fog*(3) (or f(g(3)) and *fog* (1)
  - (ii) evaluate *fog* (5) and see what happens ➔ why?
  - (iii) How does our answer in Q(ii) help explain the idea of "existence"?
  - (iv) evaluate *gof* (9) and *g*(*f*(7)) and *gog* (1)

(A) Composition of Functions – Example 2

- We can define *f* and *g* differently, this time as graphs:
- We will try the following:
- (i) *f*(*g*(3)) or *fog*(3)
- (ii) *gof*(3) or *g*(*f*(3))
- (iii) evaluate *fog*(2) and *fog*(-1)
- (iv) evaluate *gof*(0) and *g*(*f*(1)) and *gog*(2)



(A) Composition of Functions – Example 3

- We can define f and g differently, this time as formulas:
- $f(x) = x^2 - 3$ and $g(x) = 2x + 7$
- We will try the following:
- (i) $f(g(3))$ or $fog(3)$
- (ii) $gof(3)$ or $g(f(3))$
- (ii) $fog(x)$ and $gof(x)$
- (ii) evaluate fog (5)
- (iii) evaluate gof (9) and $g(f(7))$ and gog (1)

(B) "Existence of Composite"

- Use DESMOS to graph the following functions:

$$f(x) = \sqrt{x-1} \text{ and } g(x) = \ln(x)$$

- State the DOMAIN & RANGE of each $f(x)$ and of $g(x)$.
- Determine the equation for fog(x)
- Graph the composite function, fog(x) and determine its DOMAIN
- Q(a)? Does fog(x) exist ➔ Why is the answer NO!?!?!?
- Q(b)? Under what domain conditions of g(x) does fog(x) exist

(C) Composition of Functions – Explaining "Existence of Composite"

- Back to our opening example $f(x) = \sqrt{x-1}$ and $g(x) = \ln(x)$

1. Let's consider a mapping diagram. Complete the mapping diagram for the function:

   g(x) = ln(x), where the Domain = {0.1, 0.5, 1, 2, e, 5, 8} (ideally {xE***R***|x ≥ 0}

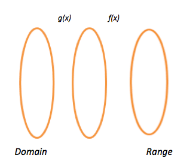
   What is the Range of ***g(x)*** (use calculator) ____________________ ? (ideally {yE***R***}

(C) Composition of Functions – Explaining "Existence of Composite"

2. Sometimes functions undergo <u>more than one</u> mapping or transformation ➔ we will do $f \circ g(x)$

Let's consider these two functions: $f(x) = \sqrt{x-1}$ and $g(x) = \ln(x)$, what would this mapping look like below.

Let the Domain = {0.1, 0.5, 1, 2, e, 5, 8} , as before



Determine the Range ____________________________

If our data is mapped via g(x) and then mapped by f(x), the question then remains:

(1) When does this doubling mapping work & when doesn't it?

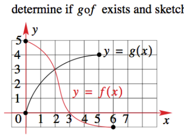
(2) What is the new equation of the twice mapped data that will allow us to get the same result in 1 step?"

Composition, Existance & Domains of Existance

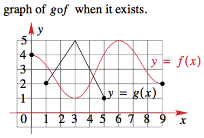
15. Given the functions $f(x) = \sqrt{x^2 - 9}, x \in \mathbf{S}$ and $g(x) = |x| - 3, x \in \mathbf{T}$, find the largest positive subsets of $\mathbb{R}$ so that (a) gof exists (b) fog exists.

16. For each of the following functions
(a) determine if fog exists and sketch the graph of fog when it exists.
(b) determine if gof exists and sketch the graph of gof when it exists.

i.



ii.



Composition, Existance & Domains of Existance

18. The functions f and g are given by $f(x) = \begin{cases} \sqrt{x-1} & \text{if } x \geq 1 \\ x - 1 & \text{if } 0 < x < 1 \end{cases}$ and $g(x) = x^2 + 1$.

(a) Show that fog is defined. (b) Find $(fog)(x)$ and determine its range.

19. Let $f : \mathbb{R}^+ \mapsto \mathbb{R}^+$ where $f(x) = \begin{cases} \frac{1}{x^2}, & 0 < x \leq 1 \\ \frac{1}{\sqrt{x}}, & x > 1 \end{cases}$.

(a) Sketch the graph of f.
(b) Define the composition fof, justifying its existence.
(c) Sketch the graph of fof, giving its range.

Composition, Existance & Domains of Existance

3. All of the following functions are mappings of $\mathbb{R} \mapsto \mathbb{R}$ unless otherwise stated.

(a) Determine the composite functions $(f \circ g)(x)$ and $(g \circ f)(x)$, if they exist.

(b) For the composite functions in (a) that do exist, find their range.

i. $f(x) = x + 1, g(x) = x^3$

ii. $f(x) = x^2 + 1, g(x) = \sqrt{x}, x \geq 0$

iii. $f(x) = (x+2)^2, g(x) = x - 2$

iv. $f(x) = \frac{1}{x}, x \neq 0, g(x) = \frac{1}{x}, x \neq 0$,

v. $f(x) = x^2, g(x) = \sqrt{x}, x \geq 0$

vi. $f(x) = x^2 - 1, g(x) = \frac{1}{x}, x \neq 0$

vii. $f(x) = \frac{1}{x}, x \neq 0, g(x) = \frac{1}{x^2}, x \neq 0$

viii. $f(x) = x - 4, g(x) = |x|$

ix. $f(x) = x^3 - 2, g(x) = |x+2|$

x. $f(x) = \sqrt{4-x}, x \leq 4, g(x) = x^2$

xi. $f(x) = \frac{x}{x+1}, x \neq -1, g(x) = x^2$

xii. $f(x) = x^2 + x + 1, g(x) = |x|$

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(F) Composition of Functions – Example

- For the following pairs of functions
- (a) Determine fog (x)
- (b) Determine gof (x)
- (c) Graph the original two functions in a square view window & make observations about the graph → then relate these observations back to the composition result

$(a)\ f(x) = 3x - 6 \quad \text{and} \quad g(x) = \frac{1}{3}x + 2$

$(b)\ f(x) = \frac{1}{x+3} \quad \text{and} \quad g(x) = \frac{1-3x}{x}$

$(c)\ f(x) = 3 - (x+2)^2 \quad \text{where } x \geq -2$

$\quad \text{and} \quad g(x) = \sqrt{3-x} - 2$

$(d)\ f(x) = e^{2x+1} \quad \text{and} \quad g(x) = \frac{1}{2}(\ln(x) - 1)$

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