Lesson 7 – Combinations of Functions

IBHL1 Math - Santowski

Warm Up Exercises – Fast Five

- Show supporting evidence as you determine the domain of:
$$f(x) = \sqrt{\frac{2}{(x-1)^2} - 8}$$

- Show supporting evidence as you determine the range of:
$$g(x) = \frac{|x|}{x}$$
$$h(x) = x^2 - 4x + 2$$

Lesson Objectives

- Perform a variety of operations with functions including evaluating functions; add, subtract, multiply, and divide and then analyze the resultant functions in terms of domains and ranges, end behaviours, asymptotes,

- Review & practice with the composition of functions

The **BIG** Picture

- And we are studying this because ....?

- Functions will be a unifying theme throughout the course ➔ so a solid understanding of **what** functions are and **why** they are used and **how** they are used will be very important!

- Sometimes, complicated looking equations can be easier to understand as being combinations of simpler, parent functions
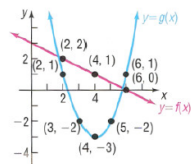
(C) Operations with Functions – Graphic Analysis

The graph of two functions, f and g, is illustrated. Use the graph to answer parts (a)–(f).

(a) $(f + g)(2)$ (b) $(f + g)(4)$
(c) $(f - g)(6)$ (d) $(g - f)(6)$
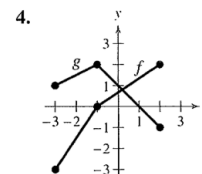(e) $(f \cdot g)(2)$ (f) $\left(\dfrac{f}{g}\right)(4)$

(C) Operations with Functions – Graphic Analysis

- Given the graphs of f(x) and g(x), draw the graph of the following combinations:

4.

- (a) f + g
- (b) f - | g |

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(C) Operations with Functions – Function Analysis

- We will work through this pair of equations together. Write an equation for each new function. Discuss the domain, range, and intercepts of the newly formed functions

- So f(x) = x + 2 & g(x) = x^2 – x – 6

- (i) State D & R of f and g
- (ii) Evaluate f(2) and g(4)
- (iii) f(x) + g(x) = (f + g)(x)
- (iv) f(x) - g(x) = (f – g)(x)
- (v) g(x) - f(x) = (g – f)(x)
- (vi) | f(x) – g(x)| (absolute value)

(C) Operations with Functions – Function Analysis

- We will work through this pair of equations together. Write an equation for each new function. Discuss the domain, range, and intercepts of the newly formed functions

- So f(x) = x + 2 & g(x) = x^2 – x – 6

- (i) (fg)(x)
- (ii) (gf)(x)
- (iii) (f÷g)(x) = (f/g)(x)
- (iv) (g÷f)(x) = (g/f)(x)
- (v) 1/f(x)
- (vi) 1/g(x)

(C) Operations with Functions – Function Analysis

Let f(x) =

- (a) f(x) = x + 2
- (b) f(x) = $x^2 + 5$
- (c) f(x) = $\sqrt{x^2 - 4}$
- (d) f(x) = $\dfrac{1}{x}$
- (e) f(x) = $\dfrac{x}{x+1}$
- (f) f(x) = 2x - 5

Let g(x) =

- (a) g(x) = x^2 - x - 6
- (b) g(x) = $\sqrt{1 - x}$
- (c) g(x) = $\dfrac{x^2}{x^2 - 1}$
- (d) g(x) = x^3
- (e) g(x) = $\dfrac{1}{x^2}$
- (f) g(x) = $\sqrt[3]{1 - x}$

(C) Operations with Functions – Function Analysis

- Write an equation for each new function. Discuss the domain, range, and intercepts of the newly formed functions

- (i) f(x) + g(x) = (f + g)(x)
- (ii) f(x) - g(x) = (f – g)(x)
- (iii) g(x) - f(x) = (g – f)(x)
- (iv) | f(x) – g(x)| (absolute value)

- Graph the following functions on the TI-84 or graphing software to verify

(C) Operations with Functions – Function Analysis

- Write an equation for each new function. Discuss the domain, range, and intercepts of the newly formed functions

- (i) (fg)(x)
- (ii) (gf)(x)
- (iii) (f÷g)(x) = (f/g)(x)
- (iv) (g÷f)(x) = (g/f)(x)

- Graph the following functions on the TI-84 or graphing software to verify.

(D) Operations with Functions - Reciprocals

- Write an equation for each new function. Discuss the domain, range, asymptotes, and intercepts of the newly formed functions

- (i) 1/f(x)
- (ii) 1/g(x)

- Graph the following functions on the TI-84 or graphing software to verify.

(E) Operations with Functions - Function Composition

- So we have a way of creating a new function ➔ we can **compose** two functions which is basically a **substitution of one function into another**.

- we have a notation that communicates this idea ➔ if $f(x)$ is one functions and $g(x)$ is a second function, then the composition notation is ➔ **f o g (x)**

(G) Composition of Functions – Example

- We will now define f and g as follows:
- f = {(3,2), (5,1), (7,4), (9,3), (11,5)}
- g = {(1,3), (2,5), (3,7), (4,9), (5,10)}

- We will now work with the composition of these two functions:
  - (i) We will evaluate fog(3) (or f(g(3))
  - (ii) evaluate fog (1)
  - (iii) evaluate fog (5) and see what happens ➔ why?
  - (iv) evaluate gof (9) and g(f(7) and gog (1)

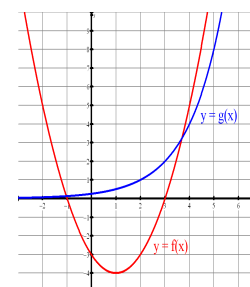## (G) Composition of Functions – Example

- We can define f and g differently, this time as formulas:

- $f(x) = x^2 - 3$ and $g(x) = 2x + 7$

- We will try the following:

- (i) $f(g$(3)) or fog(3)
- (ii) gof(3) or $g(f$(3))
- (ii) fog(x) and gof(x)
- (ii) evaluate fog (5)
- (iii) evaluate gof (9) and g(f(7)) and gog (1)

(G) Composition of Functions – Example

- We can define f and g differently, this time as graphs:

- We will try the following:

- (i) $f(g$(3)) or fog(3)
- (ii) gof(3) or $g(f$(3))
- (iii) evaluate fog(2) and fog(-1)
- (iv) evaluate gof(0) and g(f(1)) and gog(2)

(G) Composition of Functions – Example

- We can define f and g differently, this time as formulas:

- Repeat if f(x) = $-2|x^2 + x - 1|$ and g(x) = 2^x

- We will try the following:

- (i) $f(g$(3)) or fog(3)
- (ii) gof(3) or $g(f$(3))
- (ii) fog(x) and gof(x)
- (ii) evaluate fog (5)
- (iii) evaluate gof (9) and g(f(7)) and gog (1)

(G) Composition of Functions – Example

- For the following pairs of functions

- (a) Determine fog (x)
- (b) Determine gof (x)

- (c) Graph the original two functions in a square view window & make observations about the graph ➔ then relate these observations back to the composition result

$(a)\ f(x) = 3x - 6$ and $g(x) = \dfrac{1}{3}x + 2$

$(b)\ f(x) = \dfrac{1}{x+3}$ and $g(x) = \dfrac{1-3x}{x}$

$(c)\ f(x) = 3 - (x+2)^2$ where $x \geq -2$
and $g(x) = \sqrt{3-x} - 2$