Lesson 4 – Function Analysis – Algebraic Perspective

HL1 Math - Santowski

Lesson Objectives

- Consolidate our understanding of the key features of the parent functions studied in Lesson 3
- Extend our knowledge of these key features by now considering ALGEBRAIC COMBINATIONS of these parent functions
- Finally, consider various algebraic strategies for analyzing the key features of these functions. We will algebraically investigate:
- (i) domain & range
- (ii) symmetries (even & odd)
- (iii) end behavior
- (iv) asymptotic behavior
- (v) intercepts

BIG PICTURE

- Each type of function that we will be studying in this course will have some **features common** with other types of functions BUT will also have some features **unique** to itself
- How can we efficiently use our knowledge of these **key features** to make the algebraic anaysis of a myriad of functions that much easier?

(A) Domain and Range

- Quick recap ➔ which of our parent functions have domain restrictions? Explain why.
- Quick recap ➔ which of our parent functions have range restrictions? Explain why.

(A) Domain & Range

- HINT: when looking at the Domain of "complex" functions, rather than asking yourself what the domain IS, ask yourself rather, what the domain ISN'T!!
- For example:

If $f(x)=\sqrt{x-3}$ and $g(x)=\dfrac{1}{2-x}$

(a) State the domain and range of $y=f(x)$ and $y=g(x)$

(b) PREDICT the domain of $y=f\circ g(x)$

(c) PREDICT the domain of $y=g\circ f(x)$

(A) Domain & Range

- Predict the domains & ranges of the following functions. Include a justification for your chosen D & R
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$$f(x)=1-\sqrt{5-2x}$$

$$g(x)=\frac{|x-3|}{1-2x}$$

(A) Domain & Range

- HINT: when looking at the Domain of "complex" functions, rather than asking yourself what the domain IS, ask yourself rather, what the domain ISN'T!!
- For example, state the domain and range of the following functions:

$$(a)\ f(x) = 2 - |3 - x|$$
$$(b)\ g(x) = \frac{2x+5}{x+2}$$
$$(c)\ h(x) = -3x^2 + 6x - 1$$
$$(d)\ k(x) = \frac{1}{\sqrt{|x-4|}}$$

(A) Domain & Range

7. Determine the domain of each function.

a) $y = 3 - 5x$

b) $y = x^2 - 4$

c) $y = -2x^2 - 8x$

d) $y = \sqrt{-x}$

e) $y = \sqrt{5x - \frac{\sqrt{3}}{7}}$

f) $y = \sqrt{x^2 - 4}$

g) $y = \frac{x^2 - 9}{x - 3}$

h) $y = \frac{x - 3}{x^2 - 9}$

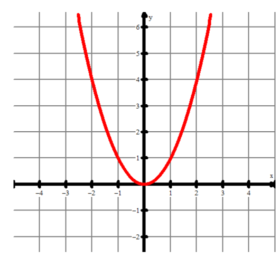
i) $y = \sqrt[4]{\frac{x^2 + 3x + 2}{4 - x^2}}$

(B) Symmetries

- The two most common symmetries that we will consider for functions will be:
- (a) symmetrical about the y-axis (called EVEN symmetry)
- (b) symmetrical about about the origin (called ODD symmetry – two fold rotational symmetry))
- Q ➔ How do we ALGEBRAICALLY determine this??

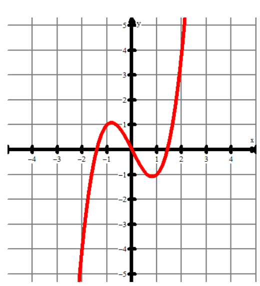
(B) Symmetries

- Consider $y = x^2$
- Notice what happens when you evaluate f(2) and f(-2) ➔
- So EXPLAIN why we make the statement that to test for EVEN symmetry, we state that f(x) = f(-x) for all values of x.



(B) Symmetries

- Consider $y = x^3 – 2x$
- Notice what happens when you evaluate f(2) and f(-2) ➔
- So EXPLAIN why we make the statement that to test for ODD symmetry, we state that f(x) = -f(-x) for all values of x.



(B) Symmetries

- Test the following functions for having either even, odd or neither symmetry:

$$(a)\ f(x) = |x - 2|$$
$$(b)\ f(x) = |x| - 2$$
$$(c)\ f(x) = x^2 - x - 4$$
$$(d)\ f(x) = \sqrt{x^2 + 2}$$
$$(e)\ f(x) = \frac{x}{x-2}$$
$$(f)\ f(x) = 2x^3 - 4x$$
$$(g)\ f(x) = -\frac{3}{2x}$$

(B) Symmetries

6. Determine the symmetry of the following. Use proper "proof" format.

a) $f(x) = 4x^3 + \pi x - \sqrt[5]{2x}$

b) $f^{-1}(x) = 3x^{100} - \frac{5}{4} + x^{-2}$

c) $f(x) = \frac{\sqrt{|x|} + \sqrt[3]{x^2}}{x}$

d) $f(x) = \frac{1}{x^2 - x}$

e) $f(x) = \frac{3x}{5 - x^2}$

f) $h(x) = \frac{x^{-1} - x^{\frac{1}{3}}}{x^2 - x^{-2}} - 2$

(C) End Behaviour

- Here we shall simply ask ourselves the question ➔ what happens at the "positive" and "negative" ends of a function (we could be looking for Horizontal Asymptotes here as well)
- So in symbols, as we've described, as x➔+∞ and as x ➔ -∞ (as our domain elements get infinitely larger, both negatively so & positively so
- A couple of key algebraic ideas ➔ what do these expressions equal?

$$(\infty)^x \qquad (\infty)^{-x} \qquad (2)^{-\infty} \qquad (0.5)^{-\infty} \qquad \infty + 2$$

(C) End Behaviour

- Predict the "end behaviours" of the following functions:

$$\text{(a)}\, g(x) = x^3 - x^2$$
$$\text{(b)}\, g(x) = \frac{2}{x-3}$$
$$\text{(c)}\, g(x) = \frac{4x-1}{2x}$$
$$\text{(d)}\, g(x) = 4 - 2^{x+1}$$
$$\text{(e)}\, g(x) = x^4 - \sqrt{2x}$$
$$\text{(f)}\, g(x) = \frac{-2}{x^2}$$

(D) Vertical Asymptotes

- Again, we will narrow down our analysis, because, for now, some of our parent functions have vertical asymptotes (tan(x), cot(x), sec(x), csc(x), ln(x) & 1/x and 1/x^2)
- Question is WHY do they have VAs?
- And then, how can I algebraically predict WHERE the VA's are?

(D) Vertical Asymptotes

- Since we have an idea as to WHY VA's occur, let's predict algebraically where they are in the following functions:

$$\text{(a)}\, f(x) = \frac{150}{2x-6}$$
$$\text{(b)}\, f(x) = \frac{\sec(x)}{3x+6}$$
$$\text{(c)}\, f(x) = \ln(4+x)$$
$$\text{(d)}\, f(x) = \frac{x+1}{\sqrt{3-x}}$$
$$\text{(e)}\, f(x) = \frac{2}{x^2-4}$$