A. The Idea of an Exploration

I have a link to the IBO's description of the Exploration (which is the Internal Assessment component of the final student grade in an IB Math course). Read it over, familiarize yourself with the idea as well as its assessment (as I have included IB's scoring rubric)

B. The content of This Exploration (comes from Syllabus topic 2.2)

- The graph of a function; its equation $y = f(x)$
- Investigation of key features of graphs, such as maximum and minimum values, intercepts, horizontal and vertical asymptotes and symmetry, and consideration of domain and range.
- The graphs of the functions $y = |f(x)|$ and $f(|x|)$.
- The graph of $y = \dfrac{1}{f(x)}$ as well as $y = f^{-1}(x)$given the graph of $y = f(x)$
- Use of technology to graph a variety of functions.
- TOK Question ➔ Does studying the graph of a function contain the same level of mathematical rigour as studying the function algebraically (analytically)?

C. Working with Absolute Value – Algebraically & Numerically

Do this in a notebook, which you hand in to me in August.

a. Explore the basic idea of taking the absolute value of a number ➔ what does $|-4|$ or $|32.875|$ really mean?

b. So, several key properties of numbers are the commutative, associative & distributive properties. (If you are unfamiliar with these, please look them up). So your initial "guided exploration" is …. Do these properties hold true for absolute value? For example (i) Is $|x|+|y|=|y|+|x|$???? or is $|x|\times|y|=|y|\times|x|$????; (ii) is $|x|+(|y|+|z|)=|x|+(|y|+|z|)$???? or is $|x|\times(|y|\times|z|)=|x|\times(|y|\times|z|)$????; (iii) is $|x|(|y|+|z|)=|x||y|+|x||z|$????.

- **i.** Are these statements (i) never true?, (ii) always true?, (iii) sometimes true?
- **ii.** If they are sometimes true ➔ under what conditions are they "sometimes" true?

c. How about exploring these additional numerical/algebraic ideas ➔ (a) is $|x|+|y|=|x+y|$???? or $|x|-|y|=|x-y|$???? or $|x|\times|y|=|x\times y|$????.

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This second part of the Exploration will be WORD PROCESSED, complete with technology generated graph(s) to reinforce your learning and the presentation of your learning & understandings

D. Working with Absolute Value as a Function

Now, we will switch gears in your "guided exploration." It's one thing to consider simply the behaviour of numbers, but what now happens when we create "relations" from a set of numbers (i.e turn them into ordered pairs & look at them visually as graphs of relations/functions). So, consider some of the following ideas in your exploration:

a. What properties does the graph of $f(x)=|x|$ have? You should explore/study/review properties like domain, range, asymptotes, intercepts, even/odd symmetry, extrema, end behaviour, continuity, boundedness, intervals of increase/decrease, concavities (1 page max in your Exploration)

E. Composing with Absolute Value

Now we get into the real heart of this assignment. What happens when we start composing $f(x)=|x|$ with other functions? What do the graphs look like? Why do they look the way they do? Can we combine our knowledge of algebra with our knowledge of functions to the point where we can PREDICT algebraically what these graphs of composed functions look like? (BIG PICTURE: Can we COMBINE ideas to create NEW IDEAS/KNOWLEDGE?)

a. Start by learning about function composition ➔ what do we mean by the expression $(f \circ g)(x)$? How does it work? (2 page max in your Exploration)

b. Continue your exploration by now looking at linear combinations of $f(x)=|x|$. So, direct your exploration to working with combinations of $g(x)=mx+b$ and $f(x)=|x|$ ➔ so explore graphs of $y=(f \circ g)(x)$ as well as $y=(g \circ f)(x)$. What happens? Describe what you see. Can you come to the point of making any ALGEBRAIC based predictions of what should happen in a graph, especially in the light of not having a graph available!!! In Math, we often look for connections between ideas/topics ➔ so how is what you have just finished exploring CONNECTED to the concept of Transformations of Functions? (3 page max in your Exploration)

c. Now the real work begins in your exploration. In IM2 & IM3, you have been exposed to a variety of functions in addition to the linear you have just finished investigating in part (ii). You have worked with quadratic, exponential, polynomial, rational, trigonometric, logarithmic. So pick several different types of functions and start composing, both $y = (f \circ g)(x)$ as well as $y = (g \circ f)(x)$ where $f(x) = |x|$ and $g(x)$ are the different functions with which you are exploring. What happens? Describe what you see. Can you come to the point of making any ALGEBRAIC based predictions of what should happen in a graph, especially in the light of not having a graph available!!! (6 page max in your Exploration)

F. <u>Connections & Extensions with Absolute Value? (working with Vectors & Complex Numbers)</u>

So, now to finish off, after we make CONNECTIONS within our knowledge, we always look to EXTEND our knowledge into new areas. So let's combine the idea of "absolute value" with vectors & complex numbers (both topics that we will explore in detail in the HL course.) So let's say you have a vector defined by the "equation" $\vec{v} = x\vec{i} + y\vec{j} = \begin{pmatrix} x \\ y \end{pmatrix}$, so what does $|\vec{v}|$ mean? Likewise, if $z = a + bi$ (as a complex number), what does $|z|$ mean? (1 page max in your Exploration)

IMPORTANT NOTE: If you want to see what examples of PROPERLY PREPARED written Math assignments look like, I have links to many EXEMPLARS on the Assignment Page link on my website. (interestingly enough, 3 of the 20 marks – 15% for the mathematically challenged – in an official IB Exploration is awarded on the criteria of "Presentation")