Unit Objectives | Applying Function Concepts Using Exponential & Logarithmic Functions (2016)

***Unit 2 – Applying Function Concepts Using Exponential & Logarithmic Functions: BIG PICTURE*:**

Once a foundational concept is understood, we can develop new knowledge and new ideas as we extend the foundational concept by simply asking "what if ..." or "what about ...." or "how would things change if .... " **Any** topic in math can be CONNECTED to "foundational ideas" & EXTENDED to "new ideas" as new ideas/skills/process get introduced or get applied, so in this unit we will demonstrate this connection & extension of our knowledge of functions in general, but also specifically exponential & logarithmic Functions.

(1) FORMS, FEATURES AND EXPONENTIAL ALGEBRA: Fundamental algebraic skills related to working with exponents will be reviewed and extended (fractional exponents, roots and powers); multiple forms of exponential equations will be explored (all variations of $y = ab^x$ though); conversion between exponential and logarithmic forms; contextual applications of exponential models

(2) FUNCTION CONCEPTS: We will demonstrate this EXTENSION & CONNECTION in our study of Functions ➔ using exponential functions to develop new function concepts like (i) inverse of functions (and hence introduce the logarithmic function), (ii) composition of functions, (iii) transformation of functions

i.e. use the idea that "solving" for a variable is simply the idea of "undoing" operations and thereby reinforce the idea of undoing as being "inversing" the operations and now asking the extending question ➔ how do I "undo" (or inverse) an exponent, be it a (i) known exponent (for example $4 = x^7$); or (ii) be it an unknown exponent (for example $4 = 7^x$)

(3) ADVANCED EXPONENTIAL/LOGARITHMIC related concepts, as we EXTEND our exponential ideas from IM2 to new concepts including (i) the natural base, ***e***, (ii) continuous change (like compounding); (iii) the logistic function, (iv) catenary functions like $f(x) = e^x + e^{-x}$, (v) the normal distribution curve, $f(x) = e^{-x\hat{}2}$, (vi) arithmetic and geometric sequences

i.e. use the idea of compounding interest on smaller and smaller time intervals (or more and more times per year) to develop the concept of "continuous" compounding and thus extend our knowledge to the "natural base", e.

FORMS, FEATURES AND EXPONENTIAL ALGEBRA

1. Evaluate expressions involving negative, zero & rational exponents
2. Understand the terminology of powers, exponents & bases
3. Determine (with and without TI-84) roots of common powers (i.e. 5^{th} root of 32 or 7^{th} root of 100)
4. Solve exponential equations in various forms of $y = ab^x$ and solve for any of y,a,or b
5. Solve exponential equations specically for the exponent using graphs or logarithms, both with and without the TI-84 wherein the base can be any number, including the natural base (i.e. solve $5 = 2^{2x+3}$ or $5 = e^{2x}$)
6. Convert exponential equations into logarithmic equations
7. Solve simple logarithmic equations using conversion to exponential form strategy, with & without the TI-84

8. Generate the graphs of exponential and logarithmic functions on technology, both with and without restricted domains (TI-84 & DESMOS)
9. Analyze graphs of exponential functions and identify asymptotes, & intercepts, with and without technology
10. Solve systems of equations and inequalities involving exponential – exponential systems as well as exponential-linear systems

11. Write exponential equations (in multiple variations of $f(x) = ab^x$) from word problems and then be able to solve for f(x), a, b or x (including the use of logs) as well as the appropriate use of the natural base, e, in word problems (i.e. $A(t) = Pe^{rt}$)

FUNCTION CONCEPTS:

12. Use an algebraic & graphic perspective to review basic function concepts (notation, evaluate & solve)
13. Use exponential functions to reinforce the concept of inverting functions and then invert exponential functions algebraically and graphically to introduce the concept of a logarithm
14. Compose the exponential and linear functions
15. Understand the connection between composition and transformations of functions.
16. Compose a function and its inverse and see what happens and explain the observation(s).
17. Transform the parent functions of $f(x) = 2^x$ and $f(x) = e^x$ & work with the new function of $f(x) = Ae^{k(x-c)} + D$
18. Apply various **reflections** to exponential functions (i.e. $y = -(2^{-x})$ or $y = -(0.5^{-x})$)
19. Write an equation of a transformed exponential function from its graph & vice versa (graphing/sketching from an equation)
20. Write equation & make sketches of exponential functions from a data set & from a contextual description

Advanced Exponential/Logarithmic Related Concepts

21. Understand how continuous change (like compounding) leads to the "natural" base, e.
22. Work with data set and look for number patterns and thereby introduce the idea of arithmetic and geometric sequences and series
23. Use technology to graph and analyze and apply new exponential based functions like: the logistic function, catenary functions like $f(x) = e^x + e^{-x}$ and the normal distribution curve, $f(x) = e^{-x\hat{}2}$,