Unit 1 – Developing Function Concepts Using Linear Functions: BIG PICTURE

Once a foundational concept is understood, we can develop new knowledge and new ideas as we extend the foundational concept by simply asking "what if ..." or "what about ...." or "how would things change if .... " **Any** topic in math can be CONNECTED to "foundational ideas" & EXTENDED to "new ideas" as new ideas/skills/process get introduced or get applied, so in this unit we will connect & extend our knowledge of linear functions & functions in general.

(1) FORMS & FEATURES: Through a review and an extension of algebraic skills, the equation of a linear function can be converted into **multiple forms**: slope-intercept form: $f(x) = mx + b$, standard form (or implicit form): $Ax + By - C = 0$, point-slope form: $y_1 - y_2 = m(x_1 - x_2)$ & intercept form $(x/a + y/b = 1.)$ What advantages does each form offer? What **key features** become apparent from the form of the equation?

(2) FUNCTION CONCEPTS: We will demonstrate this EXTENSION & CONNECTION in our study of Functions ➔ using linear functions to develop new function concepts like (i) inverse of functions, (ii) composition of functions, (iii) transformation of functions, (iv) restricted domains and ranges of functions

(3) ADVANCED LINEAR related concepts will also now be developed, as we EXTEND our linear ideas from IM2 to new concepts including (i) piecewise functions, (ii) step functions, (iii) absolute value functions, (iv) reciprocal functions, (v) parametric equations and vectors, (vi) lines in 3D, (vii) arithmetic sequences, (viii) tangent and normal lines & local linearity of non-linear functions

Forms and Features

1. Use an algebraic & graphic perspective to review fundamental linear algebra skills & concepts (slope, intercepts, convert) as well as review basic function concepts (notation, evaluate & solve)
2. Generate the graphs of these linear functions on technology (TI-84 & DESMOS)
3. Write linear equations in slope-intercept & point-slope & standard & intercept forms
4. Analyze graphs of linear functions (along with the various forms of the linear equations) to understand the connection between the form of the equation and key features (slope, intercepts) of the graph
5. Work with real world scenarios to write equations that model these scenarios and analyze the scenario as well as apply function basics like domain and range to these scenarios
6. Review algebraic and technology based methods for finding the intersection of two lines and extending these processes to solving systems of equations and thereby extend to **solving inequalities**.

Function Concepts

7. Graph and analyze linear functions with domain/range limitations

8. Use linear functions & associated contexts to present the concept of inverting functions and then invert linear functions (and thus eventually, any type of function) algebraically and graphically
9. Use two linear functions (both in context & as a purely algebraic exercise) and compose the two functions
10. Compose a function and its inverse and see what happens and explain the observation(s).
11. Introduce the concept of function transformations using restricted linear functions or piecewise linear functions with translations and dilations
12. Understand the connection between composition and transformations of functions.

Advanced Linear Related Concepts

13. Extend linear functions to piecewise functions, step functions and the absolute value function, both in real world contexts and as algebraic expressions
14. Graph the function $f(x) = mx + b$ and then use these linear functions to predict the appearances of $y = 1/f(x)$
15. Use the graph of reciprocal of linear functions to introduce key ideas like asymptotes, end behaviours
16. Using the idea that the x and y coordinates of data points can be functions of time, introduce the idea of vector and parametric equations of lines
17. Extend the concept of a line from 2-space to 3-space
18. Introduce the idea of lines that can be tangent & orthogonal/normal to a nonlinear function and then write equations of such lines
19. Introduce the idea that non-linear functions can feature "local linearity" and thus introduce the idea of average and instantaneous rates of change