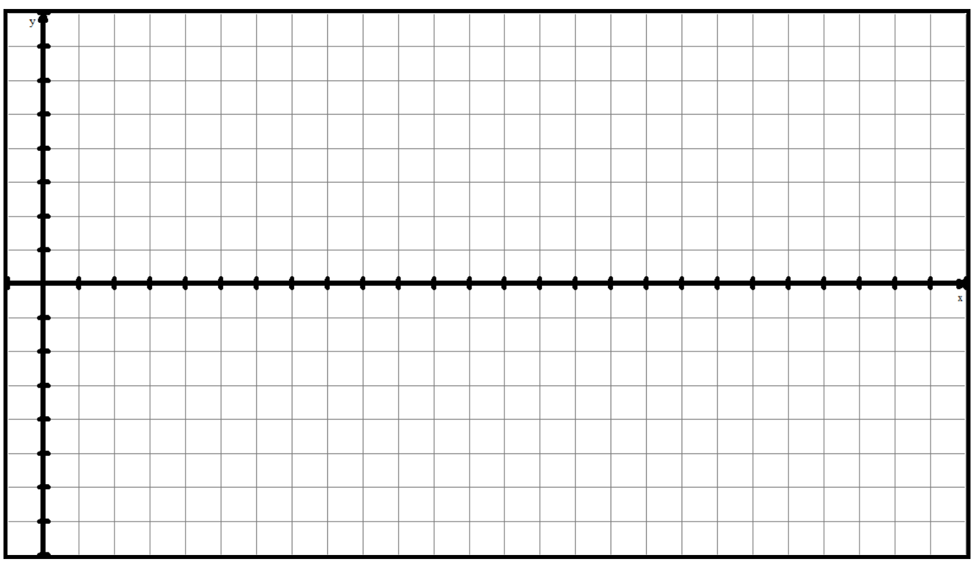
BIG PICTURE of this Unit

- How can we extend our geometry skills with triangles to go beyond right triangles to (i) obtuse triangles and (ii) circles and Cartesian Planes?
- What do triangles have to do with sinusoidal functions in the first place?
- How can we connect previously learned function concepts and skills to sinusoidal functions?
- How can use the equation of a sinusoidal function be used to analyze for key features of a graph of a sinusoidal curve?
- When and how can triangles and sinusoidal functions be used to model real world scenarios?

1. (CA) The table shows the average monthly high temperature for one year in Kapuskasing (town in Ontario, near Mr S' home town) {19,20}

Time (months)	J	F	M	A	M	J	J	A	S	O	N	D
Temperature (°C)	−18.6	−16.3	−9.1	0.4	8.5	13.8	17.0	15.4	10.3	4.4	−4.3	−14.8



a. Complete a scatter plot (by hand & on GDC & DESMOS)
b. Determine equation using (i) your knowledge of the key features of a sinusoidal function and the data given and then (ii) using sinreg on your TI-84. Are the equations the same? Why/why not?.
c. What is the average monthly temperature for the 38th month?

2. (CA) A bush pilot delivers supplies to a remote camp by flying 255 km in the direction N52°E. While at the camp, the pilot receives a radio message to pick up a passenger at a village. The village is 85 km S21°E from the camp. What is the total distance that the pilot will have flown by the time he returns to his starting point? {8,9,10}

3. (CA) Solve ΔABC, if: {5,8,9,10}

   a. $\angle A = 75°$, $\angle B = 50°$, and the side between these angles is 8.0 cm.
   b. $\angle B = 31°$, $b = 22$ cm, and $c = 12$ cm.
   c. Find the area of each triangle.

4. (CA) The population, *R*, of rabbits and the population, *F*, of foxes in a given region are modelled by the functions $R(t) = 10{,}000 + 5{,}000\cos(15t)$ and $F(t) = 1{,}000 + 500\sin(15t)$ where *t* is the time in months. Explain, referring to each graph, how the number of rabbits and the number of foxes are related by answering the following questions: {15,19}

   a. When does each population reach their maximum populations? Their minimum populations?
   b. When does the rabbit population increase? How do you know? What is happening to the fox population in the same time interval?
   c. Evaluate $R(5)$ as well as $F(5)$.
   d. Unfortunately, a pathogenic bacteria gets introduced to the ecosystem. The rabbit population halves and the population now fluctuates by only 2000 rabbits per period.
      i. Write a new equation to represent this situation.
      ii. What would happen to the fox population? Write a new equation to present a realistic model for the fox population. Explain your equation.

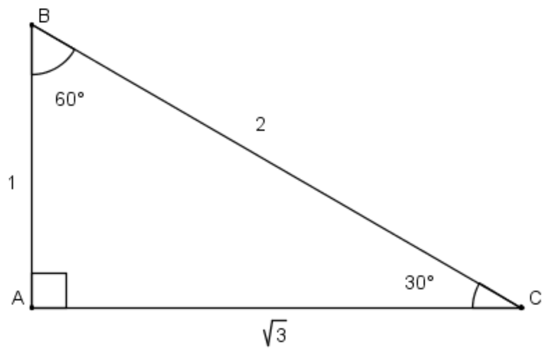
5. Open the following three geogebra files and then explain the connection between the (i) angles in standard position, (ii) triangles, and (iii) the graphs of the trig functions: {14}

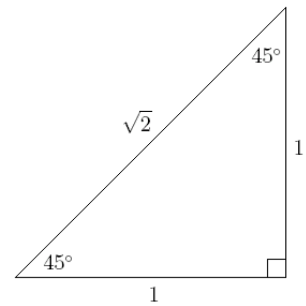
   a. Sine Function ➔ https://www.geogebra.org/m/S2gMrkbD
   b. Cosine Function ➔ https://www.geogebra.org/m/MjFgAfBv
   c. Tangent Function ➔ https://www.geogebra.org/m/cf6KYJeb

6. (CI) Mr. Smith, disguised as MathMan, a costumed crime fighter, is swinging back and forth in front of the window to Mr. Dunham's math class. At t = 3s, he is at one end of his swing and 4m from the window. At t = 7s, he is at the other end of his swing and 20m from the window. {15,17,19}

   a. Sketch the curve. Use the distance from the window on the vertical axis and the time in seconds along the horizontal axis.
   b. What is the equation (in terms of sine and cosine), which represents Mathman's motion?

7. (CI) Below, you will see two special right triangles that will come up repeatedly in your HS Math experiences. Determine the values of the primary trig ratios of these three special angles: 30°, 45°, 60°. {2,6,21}





Use the diagrams above to help solve the following problems:

a. Solve the equation $2a^2 - 1 = 0$.
b. Solve for $\{x \in R \mid -360° < x < 360°\}$ if $0 = (\sqrt{2}\sin x - 1)(\sqrt{2}\sin x + 1)$.
c. Solve for $\{x \in R \mid -360° < x < 360°\}$ if $0 = 2\cos^2 x - 1$.
d. Solve for $\{x \in R \mid -360° < x < 360°\}$ if $0 = (1 - \tan x)(2\sin x - \sqrt{3})$.

8. (CI) Use the diagrams of the special triangles above to help solve the following problems. Helpful suggestion is to draw each angle in standard position and determine the measure of the related acute angle. {2,6,11}

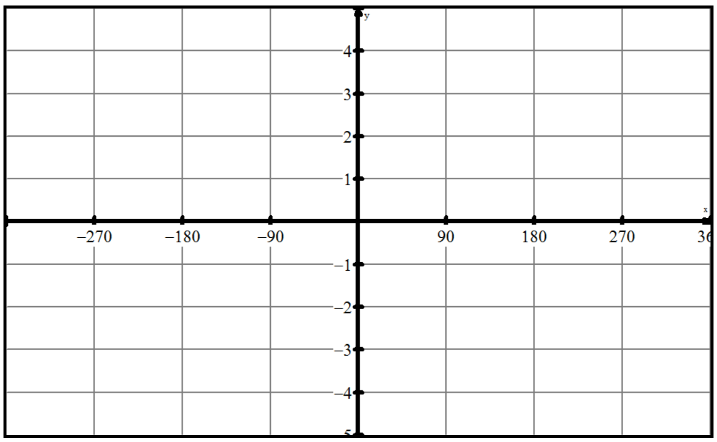
(a) sin(210°)	(b) cos(300°)	(c) tan(150°)
(d) sin(300°)	(e) cos(-135°)	(f) tan(405°)
(g) sin(135°)	(h) cos(150°)	(i) tan(-60°)

9. (CA) John is floating on a tube in a wave tank. At t = 1 s, John reaches a maximum height of 14 m above the bottom of the pool. At t = 9 s, John reaches a minimum height of 2 m above the bottom of the pool. {17,19}

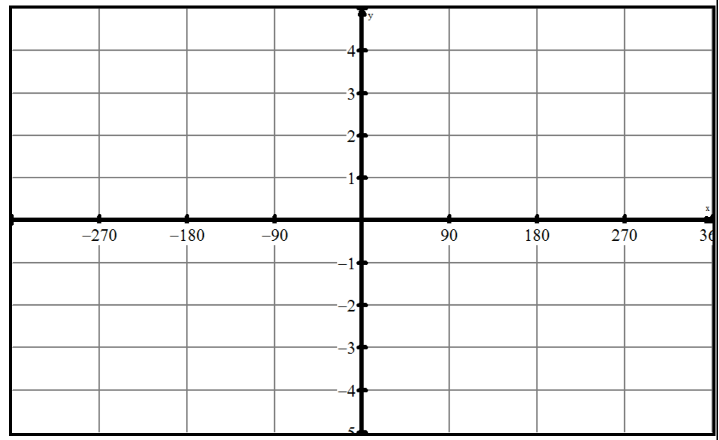
a. Sketch a graph below which expresses John's height from the bottom of the pool with respect to time.
b. What is the equation (in terms of sine and cosine), which represents John's motion?
c. What is John's height from the bottom of the pool at 21 seconds?

10. (CI) Prepare sketches of the following functions. Label the five key points in each cycle. {17,19,20}

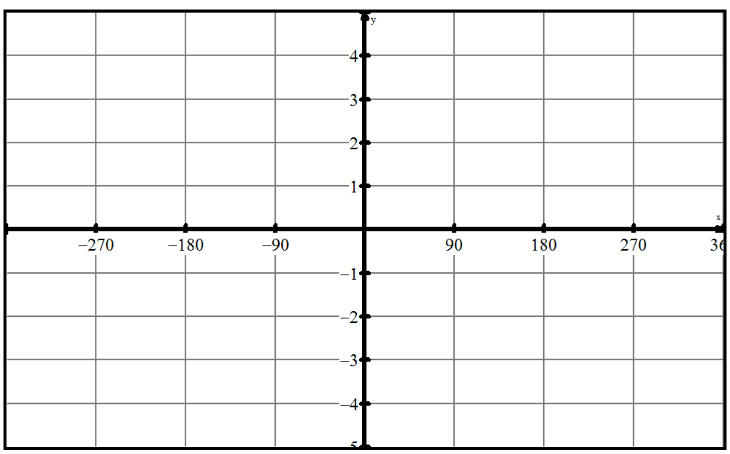
(a) $f(x) = 4\sin(x^\circ) - 1$



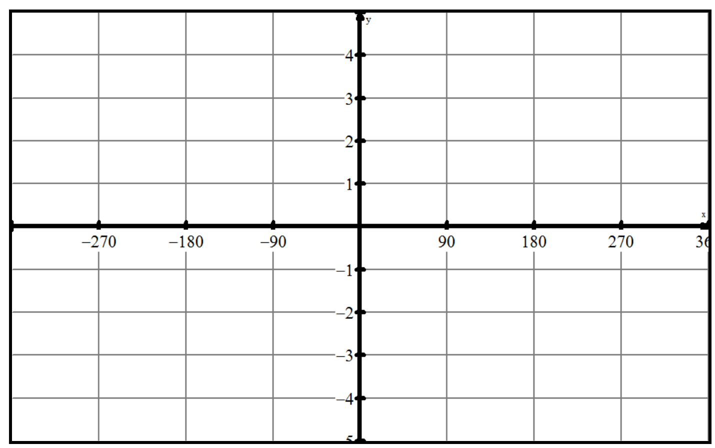
(b) $g(x) = 3\cos(2x^\circ)$



(c) $f(x) = \sin(x + 45^\circ) - 2$



(d) $g(x) = \frac{1}{2}\cos(x - 90^\circ)$



Higher Level Questions for More Complex Concepts OR an EXTENSION of basic concepts involved with triangle trigonometry and sinusoidal functions.

1. Graph the function f(x) = sin(x) on DESMOS. Sketch the result and once again, label the 5 key points in each cycle. Use degrees as the unit of angle measure.

2. Use DESMOS to then graph the inverse as x = sin(y). Sketch. Is the inverse of y = sin(x) a function or not? Explain.

3. Using both graphs, y = sin(x) as well as x = sin(y), try to **restrict the domain** of the original function (y = sin(x)) so that now the inverse is a function. Explain the reasoning behind your domain restriction.

4. Now, use DESMOS to graph $y = \sin^{-1}(x)$. Label the key points and then explain the domain and range $y = \sin^{-1}(x)$ as well as the **restrictions** on the original function, y = sin(x), to ensure that the inverse of y = sin(x) is a function.

5. Repeat the previous 4 steps with both y = cos(x) and y = tan(x)