BIG PICTURE of this Unit

- How can we extend our geometry skills with triangles to go beyond right triangles to (i) obtuse triangles and (ii) circles and Cartesian Planes?
- What do triangles have to do with sinusoidal functions in the first place?
- How can we connect previously learned function concepts and skills to sinusoidal functions?
- How can use the equation of a sinusoidal function be used to analyze for key features of a graph of a sinusoidal curve?
- When and how can triangles and sinusoidal functions be used to model real world scenarios?

1. (CI - Ideally) The average monthly temperature in a region of Australia is modelled by the function $T(t) = 9 + 23\cos(30t)$, where *T* is the temperature in degrees Celsius and *t* is the month of the year. For *t* = 0, the month is January. {6,15,17,20}

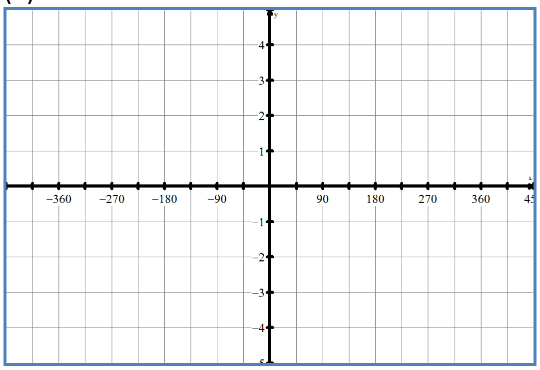
   a. Explain how to use the axis of the curve and the amplitude to determine the maximum and minimum values of the function.
   b. Determine the period of the function from the graph. Verify your answer algebraically.
   c. Prepare a table for $0 \le t \le 13$. (Ideally, no calculator, but you may use one if needed ➔ HINT: 30-60-90 right triangle??)
   d. Prepare a scatter plot of the data (WITHOUT the TI-84).
   e. Verify the graph in Q(d) by using a graphing calculator.
   f. Explain how to sketch a similar graph using transformations of *y* = cos(t).

2. Graph the point A(-6,-12). The terminal arm of an angle goes through this point. {1,11,12}

   a. Draw the angle described in this question. Label the principle angle and the related acute angle.
   b. Determine the sine and cosine and tangent ratios for this angle. (Use EXACT values for the hypotenuse)
   c. Explain WHY the sine and cosine ratios are negative.
   d. Determine the measure of the principle angle and the related acute angles.
   e. Determine the value of the following: (i) $\sin^{-1}\left(-\frac{12}{\sqrt{180}}\right)$, (ii) $\cos^{-1}\left(-\frac{6}{\sqrt{180}}\right)$, (iii) $\tan^{-1}(-2)$
   f. Explain WHY your answers for Qd and Qe are NOT the same? Should they be?
   g. Sketch two cycles of y = sin(x) as well as y = cos(x) and use this graph to explain the reason(s) behind what is happening in Qd, Qe, Qf

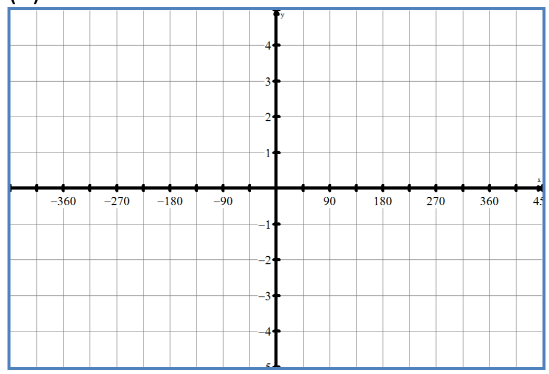
3. (CI) Graph the following data sets and then determine the equation of the sinusoidal function that best matches the data set. {17,19,20}

(a)



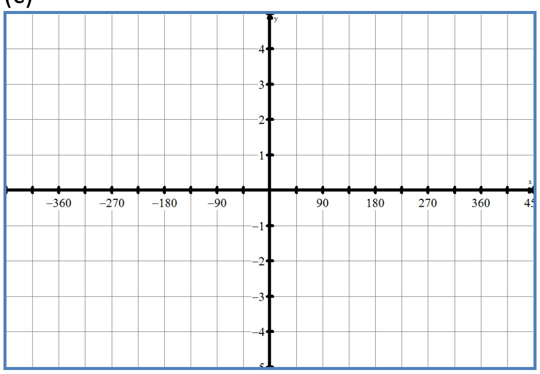
X°	0	90	180	270	360
y	-3	0	3	0	-3

(b)



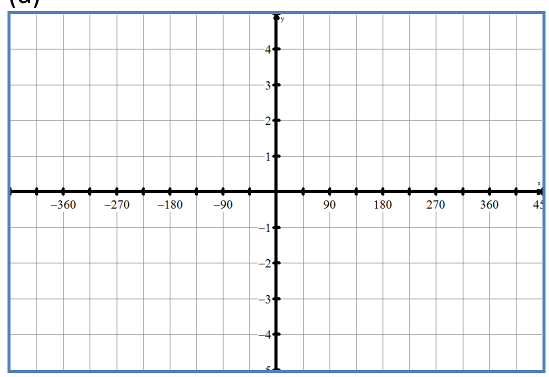
X°	45	135	225	315	405
y	3.5	2	0.5	2	3.5

(c)



X°	0	45	90	135	180
y	0	3	0	-3	0

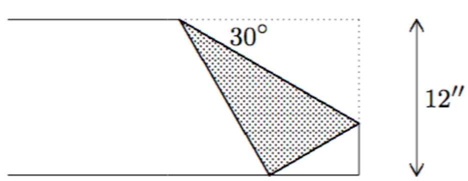
(d)



X°	-90	-30	30	90	150
y	0	3.5	5	3.5	0

4. (CI – Ideally) {3,4,5}

4. The figure at right shows a long rectangular strip of paper, one corner of which has been folded over to meet the opposite edge, thereby creating a 30-degree angle. Given that the width of the strip is 12 inches, find the length of the crease.

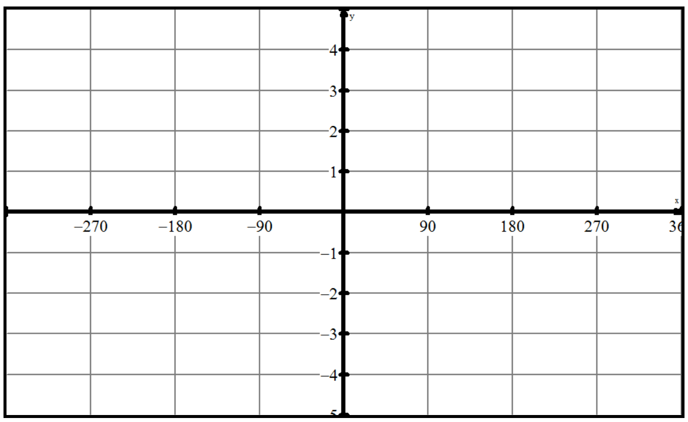


5. (CA) Two forest fire towers, A and B, are 20.3 km apart. The **bearing** from A to B is N70°E. The ranger in each tower observes a fire and reports the fire's **bearing** from the tower. The **bearing** from tower A to the fire is N25°E. From tower B, the **bearing** to the fire is N15°W. How far from is the fire from each tower? {8,9,10}

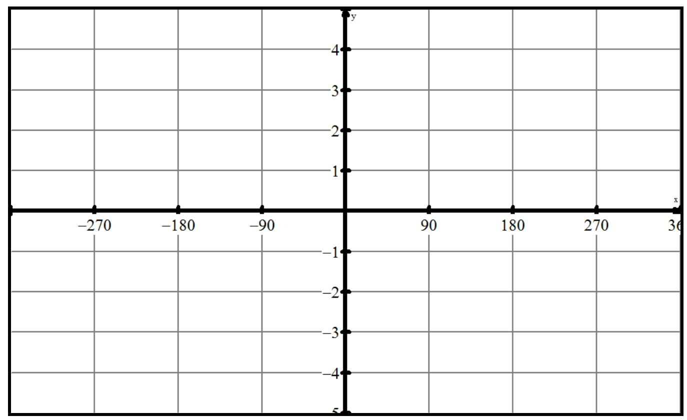
6. (CA) A surveyor needs to estimate the length of a swampy area. She starts at one end of the swamp and walks in a straight line a distance of 450 paces and then turns 60° towards the swamp. She then walks in another straight line a distance of 380 paces before arriving at the end of the swamp. One pace is about 75 cm. Estimate the length of the swamp, in meters. {8,9,10}

7. (CI) Prepare sketches of the following functions. Label the five key points in each cycle. {17,19,20}

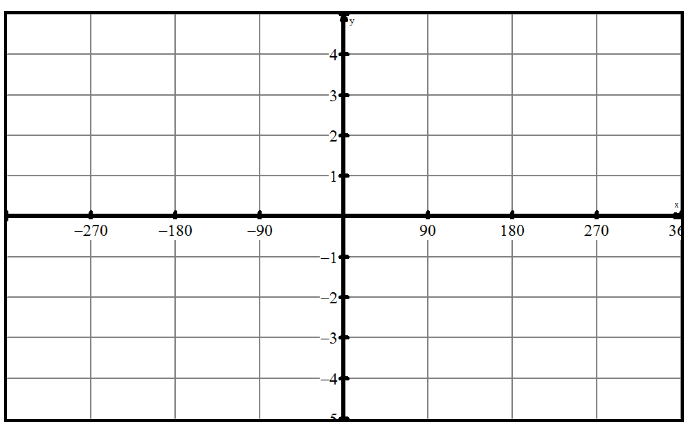
(a) $f(x) = -2\cos(x°) + 2$



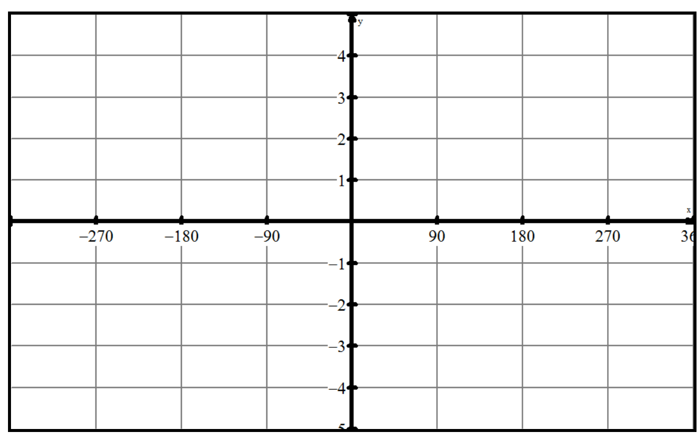
(b) $g(x) = 2\sin(3x°)$



(c) $f(x) = \cos(x - 60°) + 3$



(d) $g(x) = 2\cos 2(x - 45°)$

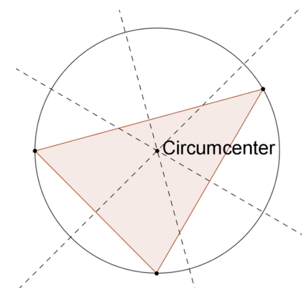


8. (CI) In Canada's Wonderland (an amusement park), there is a roller coaster that is a continuous series of identical hills that are 18m high from the ground. The platform to get on the ride is on top of the first hill. It takes 3 seconds for the coaster to reach the bottom of the hill 2m off the ground. {15,17,19}

   a. Sketch a graph below which expresses the path of the roller coaster.
   b. What is the sinusoidal equation (sine and cosine) that best reflects this roller coaster's motion?

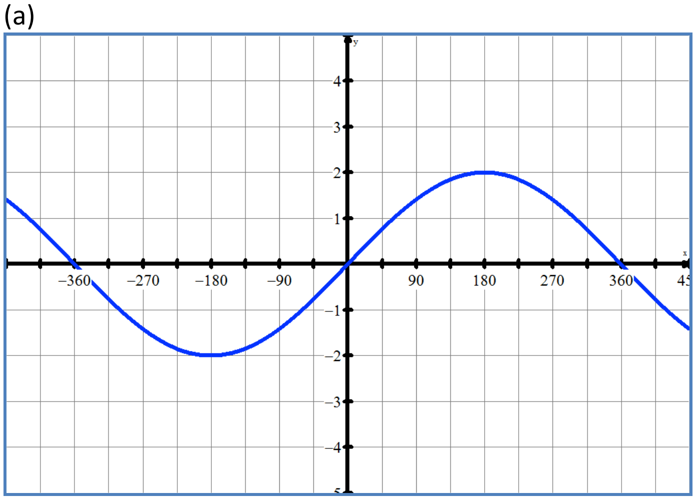
9. (CA) A triangle has side lengths of 30 cm, 42 cm and 55 cm. {5,8,9,10}

   a. True or false? This is a right triangle. Show/explain your reasoning.
   b. Determine the area of this triangle.
   c. HL ONLY: This triangle is **circumscribed** by a circle. The area **between** the circle and the triangle is to be coloured in with blue. Determine the area of this blue region. (see diagram)

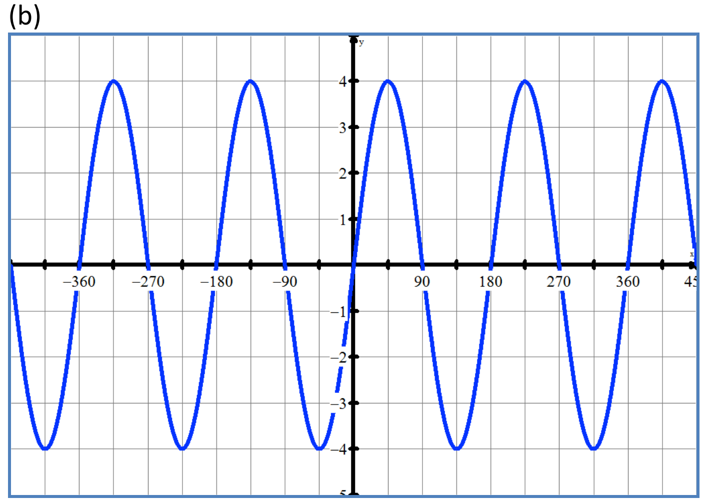


10. (CI) Determine the equation of each graph shown below. {18}

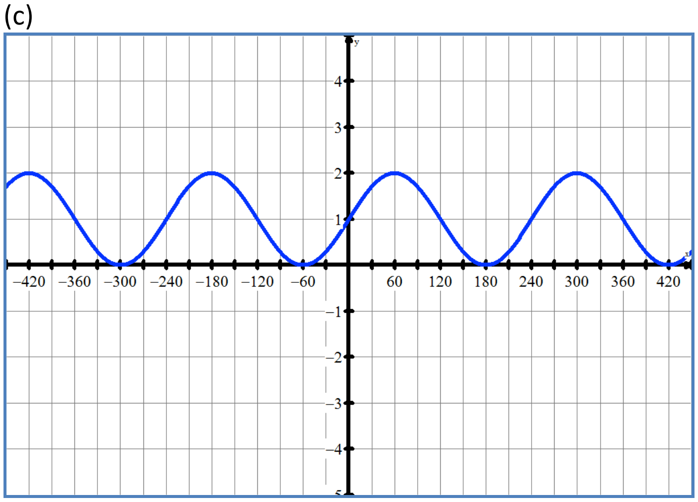
(a)



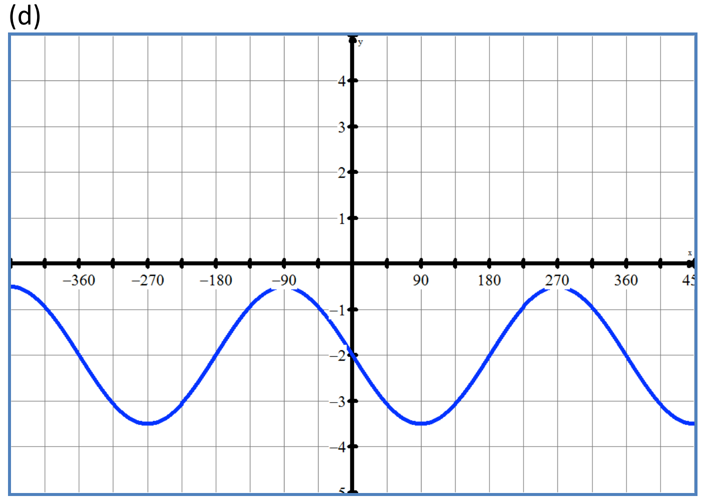
(b)



(c)

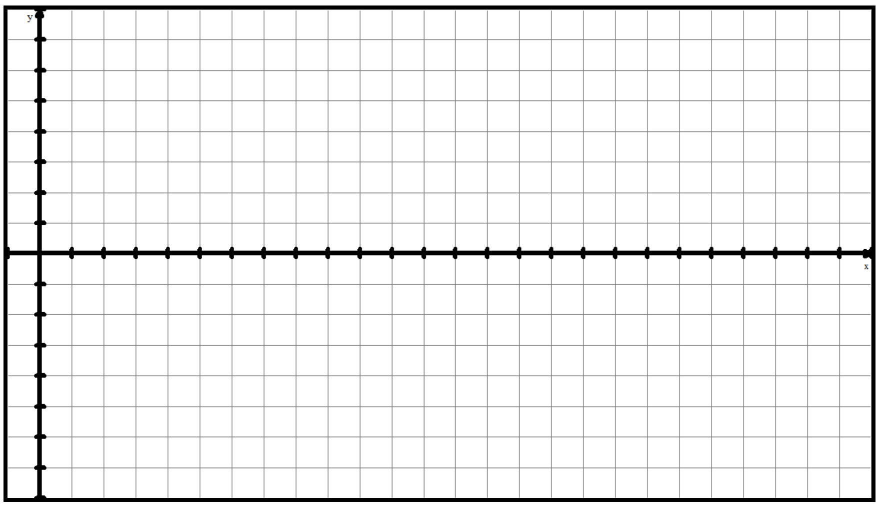


(d)



11. (CA) The depth of water in a harbour on the Bay of Fundy changes each hour, as shown on the table below: {19,20}

Time (h)	00:00	01:00	02:00	03:00	04:00	05:00	06:00	07:00	08:00	09:00	10:00	11:00	12:00
Depth (m)	5.5	6.3	8.5	11.5	14.5	16.7	17.5	16.7	14.5	11.5	8.5	6.3	5.5



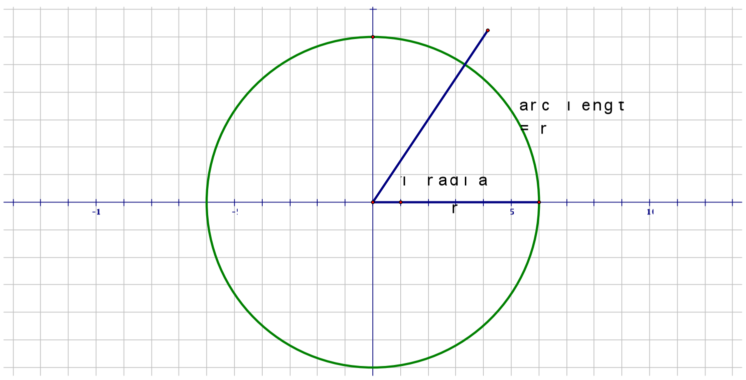
a. Complete a scatter plot (by hand & on GDC & DESMOS)
b. Determine equation using (i) your knowledge of the key features of a sinusoidal function and the data given and then (ii) using sinreg on your TI-84. Are the equations the same? Why/why not?
c. Use the equation to determine the depth of water at 10:30. Verify your answer using the graph.
d. When is the water 7 m deep?

Higher Level Questions for More Complex Concepts OR an EXTENSION of basic concepts involved with triangle trigonometry and sinusoidal functions.

See worksheet below

Worksheet on Radians

When a central angle intercepts an arc that has the same length as a radius of the circle, the measure of this angle is defined to be one **radian**.



The circumference of a circle is 2π r, where r is the length of a radius. There are 2π radians in one complete revolution about a point and one complete revolution equals 360°.

2π radians = 360° π radians = 180° 1 radian $\approx 57.3^{\circ}$

Convert each degree measure to radian measure.

1. 120°

2. -245°

Convert each radian measure to degree measure.

3. $\frac{\pi}{3}$ radians

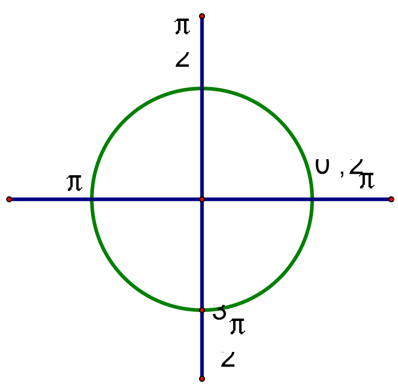
4. $-\frac{3\pi}{4}$ radians

Quadrant I if $0 < \theta < \frac{\pi}{2}$

Quadrant II if $\frac{\pi}{2} < \theta < \pi$

Quadrant III if $\pi < \theta < \frac{3\pi}{2}$

Quadrant IV if $\frac{3\pi}{2} < \theta < 2\pi$



In which quadrant or on which axis does the terminal side of the angle lie?

5. $\frac{4\pi}{3}$

6. $-\frac{5\pi}{4}$

7. $\frac{9\pi}{2}$

In which quadrant, or on which axis, does the terminal side of the each angle lie?

8. 150°
9. 210°
10. -60°
11. 180°
12. -240°
13. 540°
14. 2π
15. $\frac{\pi}{3}$
16. $\frac{3\pi}{4}$
17. $\frac{7\pi}{3}$
18. $\frac{5\pi}{4}$
19. $\frac{10\pi}{3}$

Convert each degree measure to radian measure.

20. 150°
21. 210°
22. 45°
23. 240°

Each radian measure to degree measure.

24. $\frac{\pi}{6}$
25. $\frac{\pi}{4}$
26. $\frac{5\pi}{6}$
27. $\frac{7\pi}{6}$

In exercises 28 to 31, find the measure of the central angle in both degrees and radians with the given radius and arc length.

28) r = 4 cm, s = 16 cm

29) r = 5 m, s = 4 m

30) r = 3.2 in, s = 7.8 in

31) The minute hand on the clock at the City Hall clock in Stratford measures 2.2 metres from the tip to the axle.

a) Through what angle does the minute hand pass between 7:07 A.M. and 7:43 A.M.?

b) What distance does the tip of the minute hand travel during this period?

32) You are at the stern of a paddle boat and observe that the paddle wheel has a radius of 10ft and makes 15 revolutions per minute. What is the speed of the paddle boat with respect to the water?

33) A car with a 12 inch radius wheel is moving with a velocity of 65 mph. Find the angular velocity of the tire in revolutions per second.