BIG PICTURE of this Unit

- How can we extend our geometry skills with triangles to go beyond right triangles to (i) obtuse triangles and (ii) circles and Cartesian Planes?
- What do triangles have to do with sinusoidal functions in the first place?
- How can we connect previously learned function concepts and skills to sinusoidal functions?
- How can use the equation of a sinusoidal function be used to analyze for key features of a graph of a sinusoidal curve?
- When and how can triangles and sinusoidal functions be used to model real world scenarios?

1. (CA) Believe it or not, Mr. S. is a superhero in his spare time (when he is not busy writing lessons for his beloved "other favorite" class of course). So one night (it was a Thursday I recall), I was standing on top of a building (as is my superhero duty - watching over the city of course), when I happen to notice the evil Dr. MathNoLikius on top of a building, close to the one I was on. So I quickly used my InfraRed Supervision and I quickly determined that the angle of elevation of my line of sight to Dr. MathNoLikius was 12°. I also quickly determined that the angle of depression to the base of the building upon which Dr. MathNoLikius was standing happened to be 34°. Amazingly enough, I also knew that the two buildings where 150 meters apart (Wow, imagine that!!) {2,3,4}

   a. So being a superhero, I was able to use my trig knowledge to determine the height of the building that the evil Dr. MathNoLikius was standing upon to be 356.6 m. Was I correct? Correct me if I was wrong (HAHAHAHAHAHA)

   b. But I also needed to know exactly the direct distance between me and the evil Dr. M. (as of course I would FLY there – or at least jump in a single bound – well, maybe attempt to anyway). Anyway, once again, I used my super trig powers to calculate that distance to be 600 meters. Was I right???

2. Using the domain of $\{x \in R \mid -360° \le x \le 360°\}$, graph the following two "parent functions": $f(x) = \sin(x)$ and $g(x) = \cos(x)$ in your notebooks. {16,21}

   a. Label the five key points within each cycle.

   b. State the period and amplitude of each function.

   c. Use the graphs to solve the following equations for *x*, given the domain of $\{x \in R \mid -360° \le x \le 360°\}$

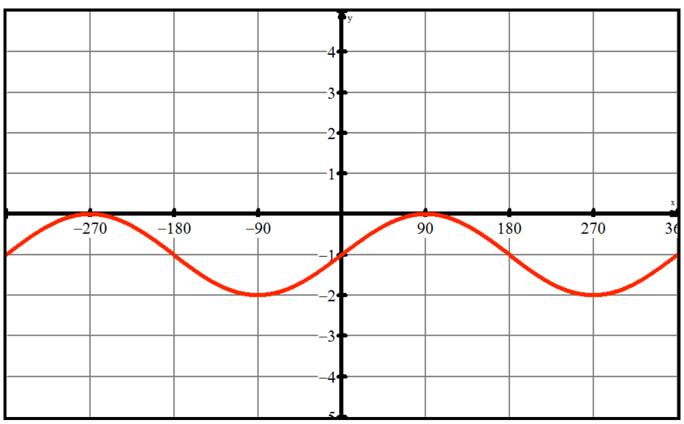
      i. $\sin(x) = 1$ ii. $1 + \cos(x) = 0$ iii. $\cos(x)(\cos(x) - 1) = 0$ (HINT: Let B = cos(x))

3. Given the following 4 equations of cosine functions, determine: (i) the amplitude, (ii) the period, (iii) the equation of the equilibrium axis and hence, make a detailed, labeled sketch of each curve. {17,18}

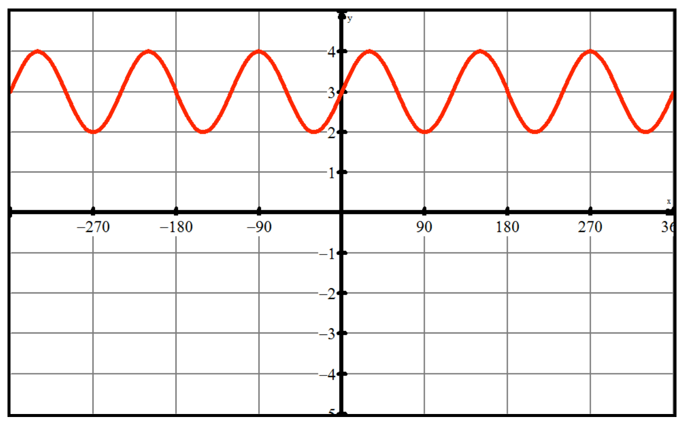
   a. $f(x) = 2\cos(3x)$ b. $f(x) = 10\cos(x) + 5$ c. $f(x) = -6\cos(x + 90°)$ d. $f(x) = \cos(3(x - 45°))$

4. Given the following 4 graphs of sine curves, determine: (i) the amplitude, (ii) the period, (iii) the equation of the equilibrium axis and hence, determine the equation of each curve. {17,18}

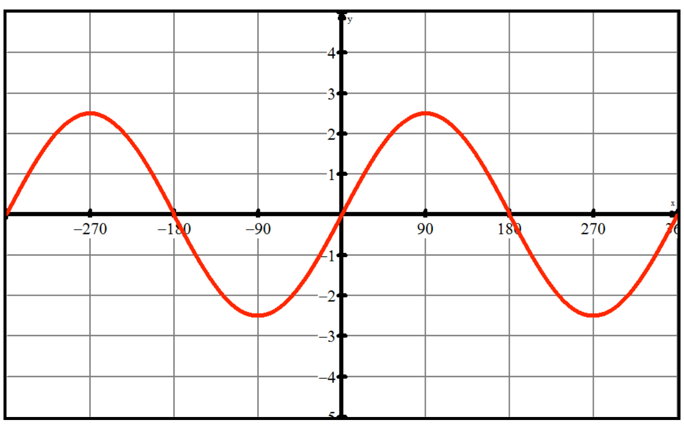
(i)



(ii)



(iii)



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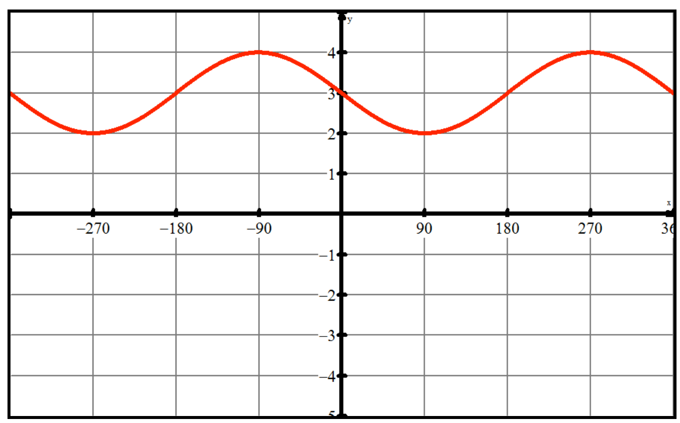
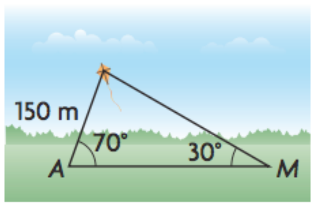


5. A triangle has a “base” side of AB = 12 cm and a second side, AC of 10 cm. The total area of ΔABC is 36 cm^2. Determine the measure(s) of ∠BAC. Now, actually draw and cut out this/these triangle(s) and show me. {8,10}

6. Answer the following two word problems. {8,9,10}

6. Allison is flying a kite. She has released the entire 150 m ball of kite string. She notices that the string forms a 70° angle with the ground. Marc is on the other side of the kite and sights the kite at an angle of elevation of 30°. How far is Marc from Allison? 	A bicycle race follows a triangular course. The three legs of the race are, in order, 23 km, 59 km and 62 km. Find the angle between the starting leg and finishing leg to the nearest degree.

7. The function $D(t) = 4\sin\left[\frac{360}{365}(t-80)\right]^{\circ} + 12$ is a model of the number of hours of daylight, *D*, on a specific day, *t*, on the 50° of north latitude. {15,17,19}

a. Explain why a trigonometric function is a reasonable model for predicting the number of hours of daylight.

b. How many hours of daylight do March 21 and September 21 have? What is the significance of each of these days?

c. What is the significance of the number 80 in the model?

d. How many hours of daylight do June 21 and December 21 have? What is the significance of each of these days?

e. Explain what the number 12 represents in the model.

f. Graph the model.

g. What are the maximum hours of daylight? the minimum hours of daylight? On what days do these values occur?

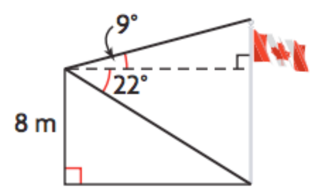
h. Use the graph to determine *t* when $D(t) = 15$. What dates correspond to *t* ?

i. Evaluate $D(246)$ and explain the solution in the context of the problem.

8. (CI ideally) You are given the following angles: 210°, 330°, -150°, -30°, 570°, 690°. {6,11,21}

   a. To start with, draw the 30-60-90 right triangle (as per PS 6.2/Q8), labeling each side and angle.
   b. Draw these given angles (210°, 330°, -150°, -30°, 570°, 690°) using the idea of "standard position"
   c. Determine the sine ratio of each angle given in the list above. Explain why this happens.
   d. Now, solve the equation $2\sin(x) + 1 = 0$ on the domain of $-360° < x < 720°$ WITHOUT graphing and without a calculator.
   e. Now solve the equation $2\sin(x) + 1 = 0$ using a calculator, but WITHOUT GRAPHING. Is/Are your answer(s) the same as in Qd? Why/why not?

9. Solve the following word problems. {8,9,10}

A triangular lot sits at the corner of two streets that intersect at an angle of 58°. One street side of the lot is 32 meters long and the other street side is 40 meters long. (a) This triangular field is to be fenced. Determine the total perimeter of the field. (b) Determine the total area of the field.	**11.** From the top of an 8 m house, the angle of elevation to the top of a flagpole across the street is 9°. The angle of depression is 22° to the base of the flagpole. How tall is the flagpole? 

10. The maximum height of a Ferris wheel is 35 m. The wheel takes 2 min to make one revolution. Passengers board the Ferris wheel 2 m above the ground at the bottom of its rotation. {17,19}

   a. Write an equation to represent the position of a passenger at any time, t, in seconds.
   b. How high is the passenger after 45 s?
   c. The ride lasts for 4 min. At what time(s) will the passenger be at the maximum height during this ride?

Higher Level Questions for More Complex Concepts OR an EXTENSION of basic concepts involved with triangle trigonometry and sinusoidal functions.

1. Ambiguous Case of the Sine Law. Investigate the "ambiguous case" of the sine law and explain WHY it happens. Then, answer the questions below:

   Solve each triangle. Begin by sketching and labelling a diagram. Account for all possible solutions. Express each angle to the nearest degree and each length to the nearest tenth of a unit.

   (a) ΔABC, $\angle A = 68°$, $a = 11.9$ cm, $b = 10.1$ cm

   (b) ΔDEF, $\angle D = 52°$, $d = 7.2$ cm, $e = 9.6$ cm

   (c) ΔHIF, $\angle H = 35°$, $h = 9.3$ cm, $i = 12.5$ cm

   (d) ΔDEF, $\angle E = 45°$, $e = 81$ cm, $f = 12.2$ cm

   (e) ΔXYZ, $\angle Y = 38°$, $y = 11.3$ cm, $x = 15.2$ cm

2. The angles of a triangle are 120°, 40°, and 20°. The longest side is 10 cm longer than the shortest side. Find the perimeter of the triangle to the nearest hundredth of a centimetre.

3. In quadrilateral QRST, QR 3 cm, RS 4 cm, ST 5 cm, and TQ 6 cm. Also, diagonal RT is 7 cm. How long is the other diagonal to the nearest tenth of a centimeter?

4. A regular pentagon has all sides equal and all central angles equal. Calculate, to the nearest tenth, the area of the pentagon shown.