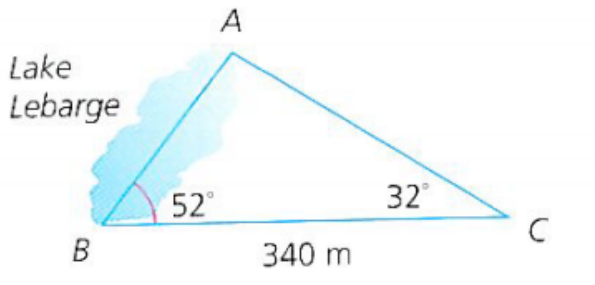
BIG PICTURE of this Unit

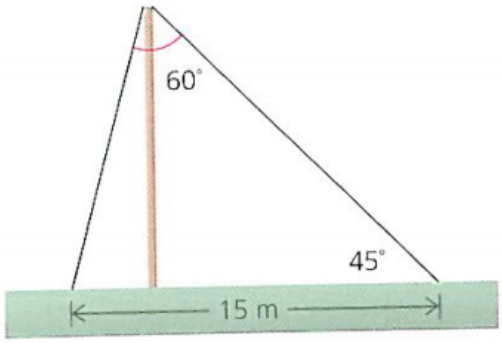
- How can we extend our geometry skills with triangles to go beyond right triangles to (i) obtuse triangles and (ii) circles and Cartesian Planes?
- What do triangles have to do with sinusoidal functions in the first place?
- How can we connect previously learned function concepts and skills to sinusoidal functions?
- How can use the equation of a sinusoidal function be used to analyze for key features of a graph of a sinusoidal curve?
- When and how can triangles and sinusoidal functions be used to model real world scenarios?

1. Louise is a naturalist studying the effect of acid rain on the fish population in lakes. As part of her research, she needs to know the length of Lake Labarge. Louise makes the measurements shown below. How long is the lake? {8}

1.



2.



2. A radio wire is supported by two wires on opposite sides as shown above. The wires form an angle of 60° at the top of the tower. On the ground, the ends of the wires are 15.0 m apart and one wire is at a 45° angle to the ground. How long will the wires be? {8}

3. Use online resources to find out what the term "**angles in standard position**" means. Illustrate your understanding by drawing an example of an angle in standard position. {11}

   a. Use your diagram (or additional diagrams) & your understanding to explain/illustrate the following terms:
      i. Terminal arm
      ii. Initial arm
      iii. Coterminal angles
      iv. Negative angles
      v. Principal angle
      vi. Related acute angle (also called the reference angle)

   b. Now, for the following angles (-225°, 315°, 120°, 510°), draw the angle in standard position and illustrate and calculate the:
      i. Principal angle and the related acute angle
      ii. The next two positive coterminal angles as well as the next two negative coterminal angles

4. In the following **right triangle trig** problems, **you must first create a diagram to visualize the problem**. Then, use the diagram to help you solve the problem. {2,4}

   a. A plane is flying at an elevation of 35, 000 feet within sight of the Giza Pyramids. The pilot would like to estimate her distance from the pyramids. She finds that the **angle of depression** to a point on the ground beside the pyramid is 22°.
      i. What is the direct distance between the plane and the pyramid?
      ii. What is the distance between a point on the ground directly below the plane and the pyramid? (along the ground)

   b. From the top of a 200 foot lighthouse, the **angle of depression** to a ship on the ocean is 23°. How far is the ship from the base of the lighthouse?

   c. A 96 foot tree casts a shadow that is 120 feet long. What is the **angle of elevation** of the sun?

   d. A man who is 6 feet tall is on the beach, flying a kite. He holds the end of the kite string and estimates the **angle of elevation** of the kite to be 50°. If the string is 45 feet long, how high is the kite above the ground?

   e. The altitude of an equilateral triangle is 5 cm. What is the length of a side of the triangle?

   f. Find the altitude of an isosceles triangle with base 4.24 feet. The vertex angle of the triangle measures 85°.

5. To investigate the relationship between linear functions and angles: {2,4}

   a. Graph the line $f(x) = 2x$
   b. Label the following points on the line you graphed: A(0,0); B(3,0); C(3,6)
   c. Draw right triangle ABC
   d. To determine the measure of angle BAC, which trigonometric ratio could you use?
   e. Hence, determine the measure of angle BAC
   f. OBSERVATION: Determine the slope of the line, $f(x) = 2x$.
   g. OBSERVATION: State the value of the tangent ratio of angle BAC.
   h. CONCLUSION: What do you notice about your answers from Q(f) & Q(g)?

6. Solve the following triangles using the Sine Law (you will find the measure of ALL 3 sides and ALL 3 angles) OR explain why it may not be possible to solve these triangles using the Sine Law. For those that are NOT possible, solve them using right triangle trigonometry based strategies. {8,9,10}

   a. In ΔABC, ∠B = 79° and ∠C = 66° and a = 36 cm

   b. In ΔABC, ∠A = 56° and a = 6.7 cm and b = 7.2 cm

   c. In ΔDEF, ∠F = 63° and e = 9.0 cm and d = 9.5 cm

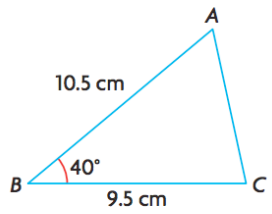
7. Go to this GEOGEBRA animation to investigate the cosine law ➔ https://www.geogebra.org/m/CPeCKmBu. Hence:

   a. Explain WHEN the cosine law needs to be used {9}

   b. Solve for the required unknowns in the following diagrams {9}

3. Determine each unknown side length.

a)



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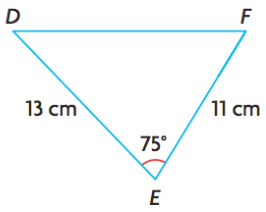
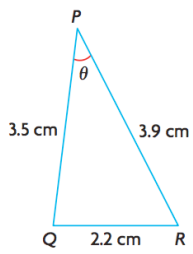


4. Determine the measure of each indicated angle to the nearest degree.

a)



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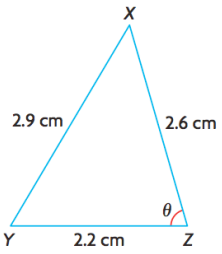


8. The point *P* (-3, 4) is on the terminal arm of an angle, θ, in standard position. {11,12}

   a. Sketch the principal angle, θ and show the related acute/reference angle.

   b. Determine the values of the sine, cosine and tangent ratios of θ.

   c. Determine the value of the related acute angle to the nearest degree.

   d. What is the measure of θ to the nearest degree?

9. Omar rides on a Ferris wheel. The vertical distance, in meters, of a rider **with respect to the horizontal diameter** is modelled by $h(t) = 5 \cos (18t)°$, where t is the number of seconds.

   a. To one decimal place, what is the riders verical distance with respect to the horizontal diameter of the wheel when t = 8 s? t = 16 s? t = 30 s?

   b. When is the rider first at $h(t)$ = 4.5 m?

   c. When is the third time the rider is at -2.5 m?

10. **INVESTIGATION** of $f(x) = A \sin\left(k\left(x + C\right)\right) + D$. Complete the following steps & record all observations {16,17}

a. Graph y = Asin(x) (use slider on DESMOS) and explain what happens when:

i. A > 1

ii. 0 < A < 1

iii. A < 0

iv. Which features PREDICTABLY change when the value of A change? (Circle choices: domain, range, period, amplitude, axis of the curve). Explain HOW the selected features change

b. Graph y = sin(kx) (use slider on DESMOS) and explain what happens when:

i. k > 1

ii. 0 < k < 1

iii. k < 0

iv. Which features PREDICTABLY change when the value of k change? (Circle choices: domain, range, period, amplitude, axis of the curve). Explain HOW the selected features change

c. Graph y = sin(x) + D (use slider on DESMOS) and explain what happens when:

i. D > 0

ii. D < 0

iii. Which features PREDICTABLY change when the value of D change? (Circle choices: domain, range, period, amplitude, axis of the curve.) Explain HOW the selected features change.

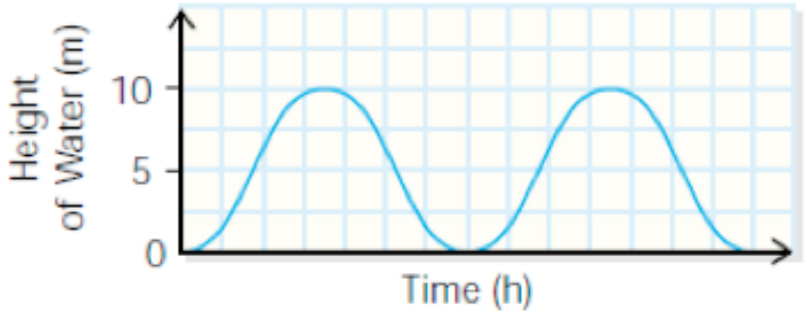
d. Graph y = sin(x – C) (use slider on DESMOS) and explain what happens when:

i. C > 0

ii. C < 0

iii. Which features PREDICTABLY change when the value of c change? (Circle choices: domain, range, period, amplitude, axis of the curve). Explain HOW the selected features change

11. The Bay of Fundy, which is between New Brunswick and Nova Scotia (in Canada), has the highest tides in the world. At low tide, there is no water on the beach, while at high tide, the water covers the beach. Below, is a graph showing the relationship between the height of the water, (in meters) as a function of the time (in hours) {15,17,19}



a. Why can you use a periodic function to model tides?
b. What is the change in the depth of the water from low tide to high tide?
c. Determine the equation of the axis of the curve.
d. What is the amplitude of the curve?
e. What is the period of the curve.
f. Given your work in Q10, predict an equation for the function.

12. Sketch periodic graphs to satisfy the given properties: {15}

Shape	Period	Amplitude	Equation of Axis	Number of Cycles
	4	6	$y = 2$	2
	3	4	$y = 1$	3
	$\frac{1}{2}$	5	$y = -3$	2

Higher Level Questions for More Complex Concepts OR an EXTENSION of basic concepts involved with triangle trigonometry and sinusoidal functions.

1. You are given a triangle enclosed by the following lines: Line #1: $y = x + 10$ and then Line #2: $y = -2x + 6$ and then Line #3: $x + 2y = 10$.

    a. Draw the lines, outline the triangle and then determine the measure of all three interior angles.
    b. Determine the area of the triangle using Heron's formula

2. Word problems

Problem 1.

18. Thinking, Inquiry, Problem Solving: A given pyramid has a regular hexagonal base. Each side of the base is 12.5 cm and the vertical height of the pyramid is 20.0 cm. Determine

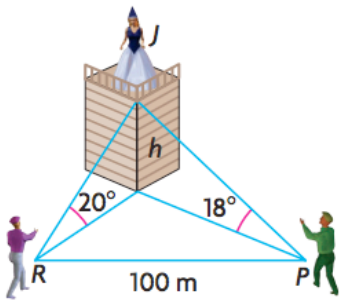
(a) the measure of the apex angle of each face

(b) the surface area of the pyramid

(c) the volume of the pyramid

Problem 2.

7. Suppose Romeo is serenading Juliet while she is on her balcony. Romeo is facing north and sees the balcony at an angle of elevation of 20°. Paris, Juliet's other suitor, is observing the situation and is facing west. Paris sees the balcony at an angle of elevation of 18°. Romeo and Paris are 100 m apart as shown. Determine the height of Juliet's balcony above the ground, to the nearest metre.

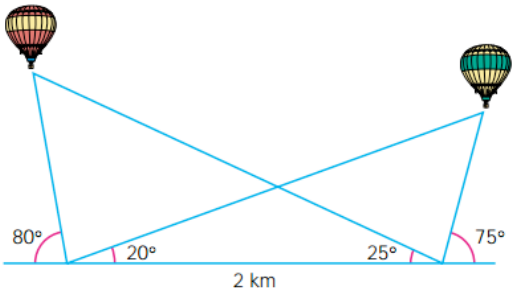


Problem 3.

20. Two hot air balloons are moored directly over a level road. The diagram shows the angle of elevation of the balloons from two observers 2 km apart.

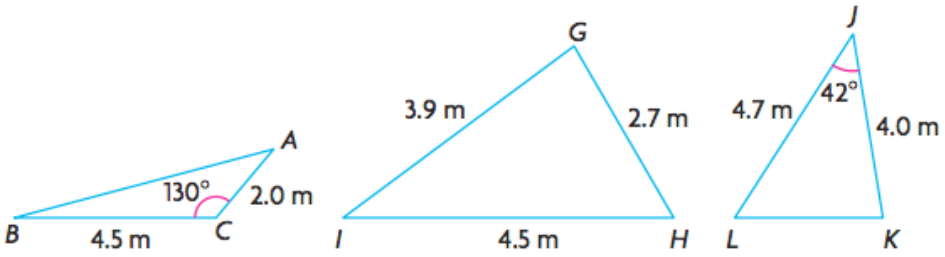
(a) To the nearest tenth of a kilometre, how far apart are the balloons?

(b) Which balloon is higher, and by how many metres?



Problem 4.

10. In setting up for an outdoor concert, a stage platform has been dismantled into three triangular pieces as shown.



There are three vehicles available to transport the pieces. In order to prevent damaging the platform, each piece must fit exactly inside the vehicle. Explain how you would match each piece of the platform to the best-suited vehicle. Justify your reasoning with calculations.