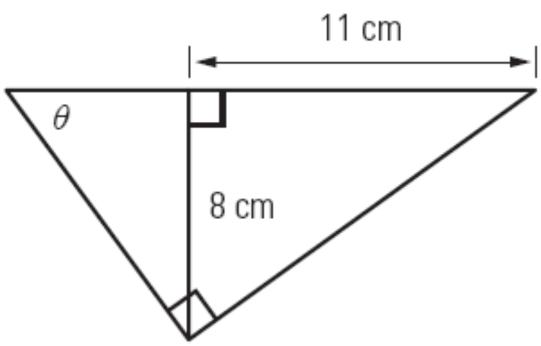
BIG PICTURE of this Unit

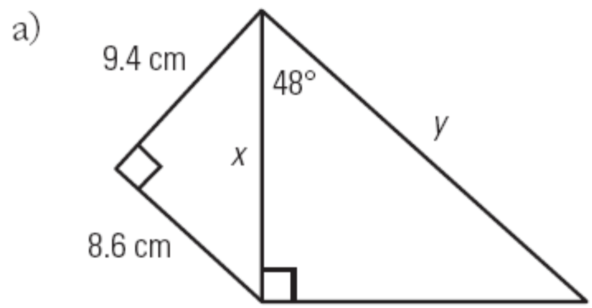
- How can we extend our geometry skills with triangles to go beyond right triangles to (i) obtuse triangles and (ii) circles and Cartesian Planes?
- What do triangles have to do with sinusoidal functions in the first place?
- How can we connect previously learned function concepts and skills to sinusoidal functions?
- How can use the equation of a sinusoidal function be used to analyze for key features of a graph of a sinusoidal curve?
- When and how can triangles and sinusoidal functions be used to model real world scenarios?

1. Determine the value(s) of the following unknowns in these right triangles: {3,4}

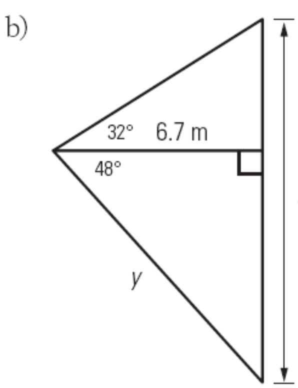
(a) Calculate the measure of angle θ.



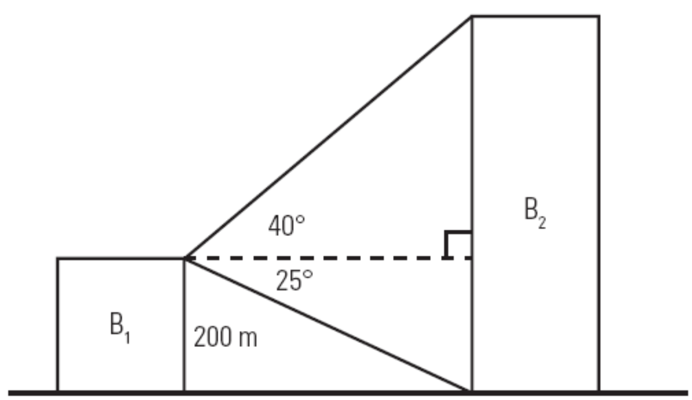
(b) Find *x* and y in the following diagram



(c) Find *x* and y in the following diagram



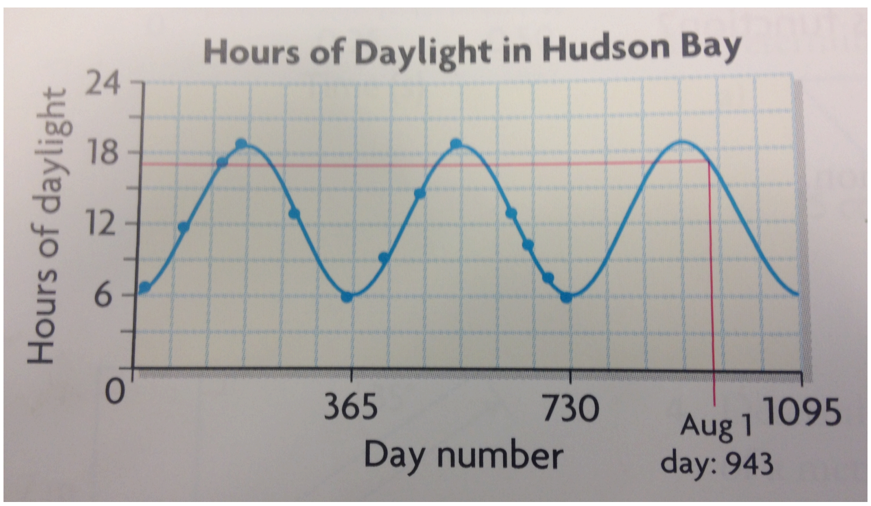
(d) From the top of a 200 m-tall office building, the angle of elevation to the top of another building is 40°. The angle of depression to the bottom of the second building is 25°. ***How tall is the second building?***



2. The number of hours of daylight in any particular location changes with the time of the year. The table shows the average number of hours of daylight for approximately a two year period at Hudson Bay, Nunavut. Day 15 is January 15th 2010. Day 74 is March 15 2010… day 441 is February 15 of 2011…etc. {15}

Day	15	74	135	166	258	349	411	470	531	561	623	653	684	714
Hours	6.7	11.7	17.2	18.8	12.9	5.9	9.2	14.6	18.8	18.1	12.9	10.2	7.5	5.9

This is a scatter plot of the situation. Lets discuss a few key ideas before we move onto the next periodic functions.



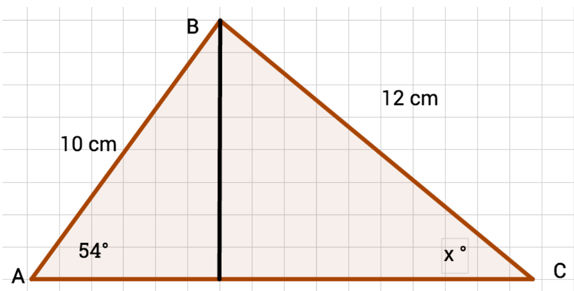
a. Why does it make sense to call a graph of the hours of daylight a **periodic function?** What is a periodic function in the first place?
b. Define the following key terms that relate to periodic functions: (i) period (ii) amplitude, (iii) axis of the curve (also called the equilibrium axis)
c. Use the graph to find the (i) period , (ii) the amplitude and (iii) the axis of the curve for the relationship between day number and hours of daylight.
d. Which points on the graph could help you determine the range (y distance) of the graph?

3. Use your TI-84 (and afterwards, use DESMOS) to graph the following functions: Draw diagrams in your notebooks. (on your TI-84, use zoom trig and make sure you are in DEGREE mode) {16}

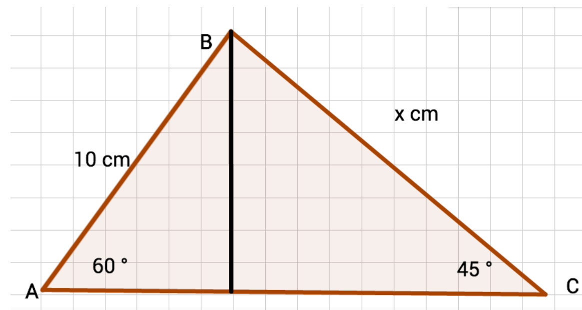
i. $f(x) = \sin(x)$ ii. $f(x) = \cos(x)$

4. Each of the following questions require MULTIPLE steps in order to work out the value of the unknown. FIRST, start by determining the height (or altitude) in each triangle. (NO DIAGRAMS ARE DRAWN TO SCALE!!!!!!!) {2,3,7}

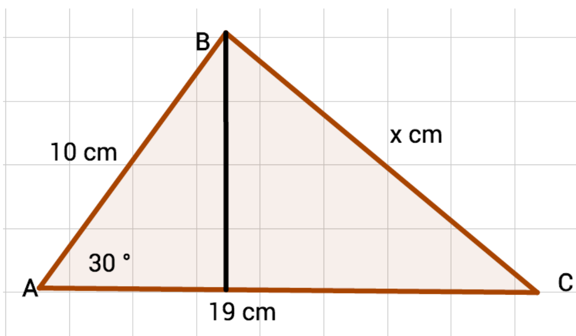
(a)



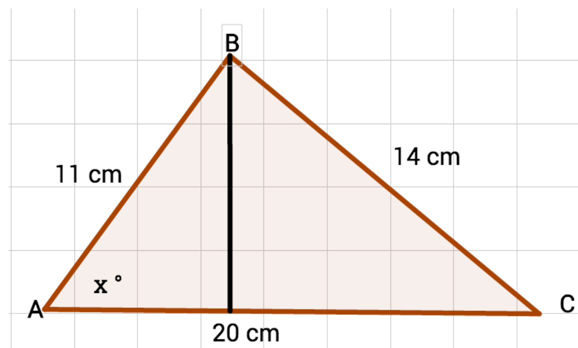
(b)



(c)



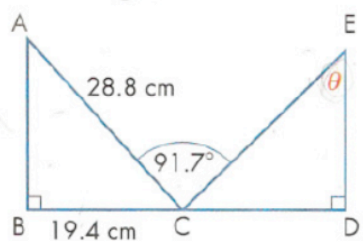
(d)



5. Solve for the unknown angles in the right triangles pictured below: {3}

4. Find the measure of $\angle\theta$, to the nearest tenth of a degree.

a)



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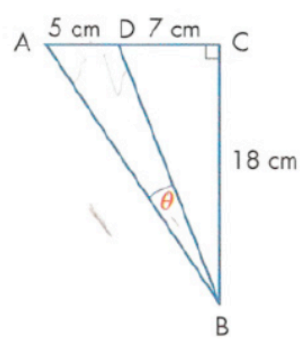
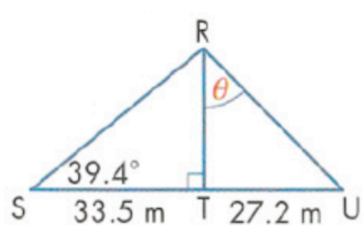


c)



6. **<u>Constructions and Ratios</u>** → CARELFULLY construct an acute triangle and CAREFULLY measure the length of the sides and the corresponding angles. Record your measurements and use these measurements to determine the required ratios: {7,8}

Side a =	Side b =	Side c =	$\frac{a}{\sin(A)}$ =	$\frac{b}{\sin(B)}$ =	$\frac{c}{\sin(C)}$ =
Angle A =	Angle B =	Angle C =			

a. What do you notice about your three ratios?

b. What do you notice about the three ratios of other people at your table?

c. Are your ratios the same as any else at your table?

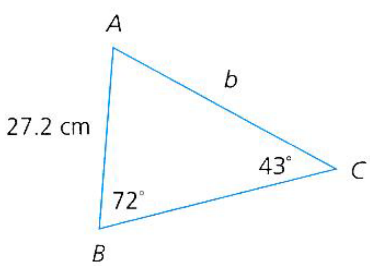
d. See the website https://www.geogebra.org/m/qBctx3Cf and verify your conclusions about these ratios

e. State the Sine Law

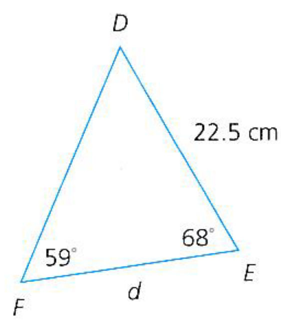
7. Use the Sine Law to answer these questions about the following triangles. {8}
VIDEO HELP: (1) https://youtu.be/LzdDALbFlxY (2) https://youtu.be/cQXSXkBMsXU

1. Find the length of the indicated side, to one decimal place.

(a)

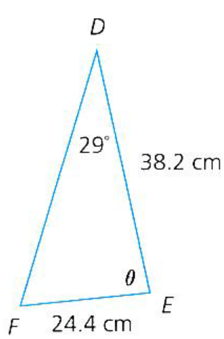


(b)

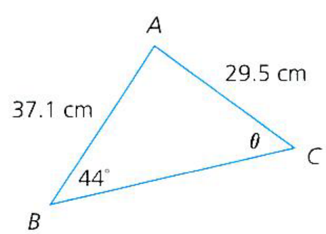


2. Find the measure of angle θ.

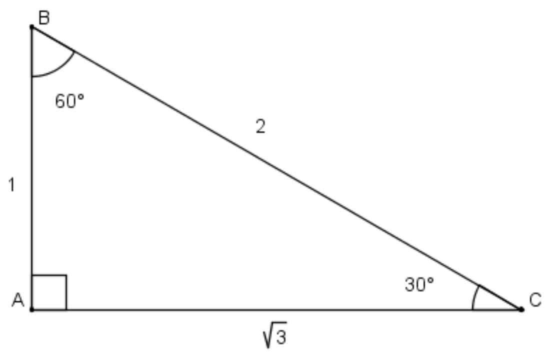
(a)

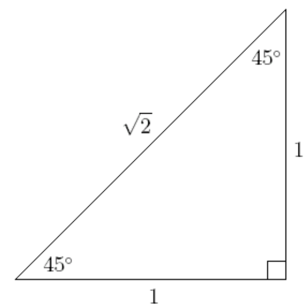


(b)



8. (CI) Below, you will see two special right triangles that will come up repeatedly in your HS Math experiences. Determine the values of the primary trig ratios of these three special angles: 30°, 45°, 60°. {2,6}





(a) sin(30°) =
(b) cos(30°) =
(c) tan(30°) =
(d) sin(60°) =
(e) cos(60°) =
(f) tan(60°) =
(g) sin(45°) =
(h) cos(45°) =
(i) tan(45°) =

9. Use the diagrams above (in Q8) to solve the following for *x*: {2,6}

(a) $x = \sin^{-1}\left(\frac{1}{2}\right)$ (b) $x = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (c) $x = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (d) $x = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (e) $x = \tan^{-1}(1)$

10. Drawing angles: As neatly as possible, draw diagrams of the following **angles in standard position**: {11}

(a) 50° (b) 150° (c) 250° (d) 350° (e) -50° (f) 410°

11. Puzzling questions with trig ratios: use your calculator to evaluate the sine ratios of the following angles: {1,11}

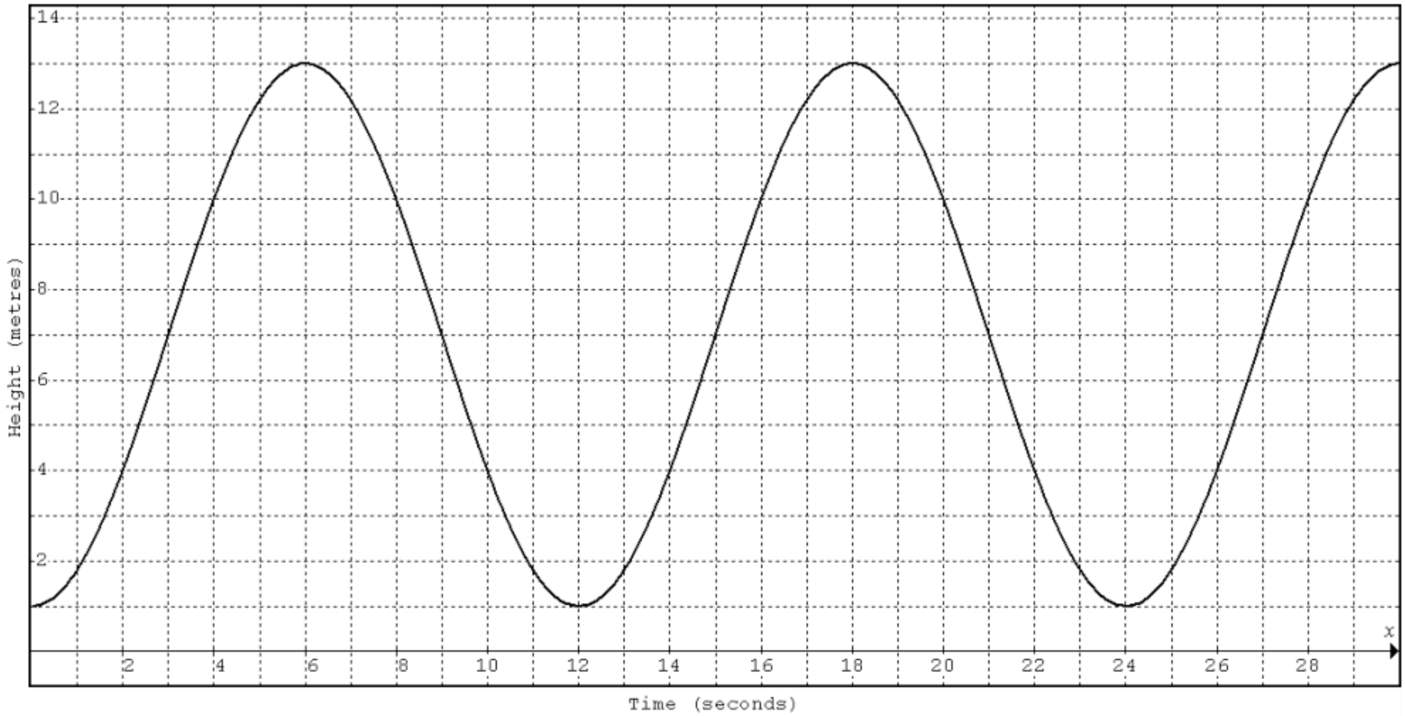
(a) sin(50°) (b) sin(150°) (c) sin(250°) (d) sin(350°) (e) sin(-50°)

(f) sin(130°) (g) sin(410°) (h) sin(-230°) (i) sin(770°) (j) sin(-310°)

a. Describe any patterns you observe. (HINT: It may be VERY helpful to sketch each of these angles in standard position)

b. Hence or otherwise, solve for *x* in the equation $\sin(x) = 0.766$ (or also written as $\sin^{-1}(0.766) = x$)

12. Victoria rode on a Ferris wheel at Cluney Amusements. The graph models Victoria's height above the ground in metres in relation to time in seconds. The data was recorded while the ride was in progress. {15}



a. What is the height of the axle on the Ferris wheel?

b. What is the radius of the Ferris wheel?

c. What is the maximum height of the Ferris wheel?

d. How long does it take for the Ferris wheel to complete one revolution?

e. Victoria boards the Ferris wheel at its lowest point. How high above the ground is this?

f. Within the first 20 seconds, how many times is Victoria at a height of 7 m above the ground?

g. What is Victoria's approximate height above ground at 16 seconds?

h. What is Victoria's approximate height above the ground at 57 seconds?

i. Find the **period** of this function. Label it.

j. Find the **range** of this function. Label it.

k. What is the minimum point… call that the **Trough.** Label it!

l. What is the maximum point… call that the **Peak.** Label it!

m. **Equation of the Axis:** The equation of the horizontal line halfway between the minimum and maximum…Find if for the graph. $y = \dfrac{Max.\,Value + Min.\,Value}{2}$

n. **Amplitude:** Half the distance between the maximum and the minimum. Find it for the graph. Label it.

Higher Level Questions for More Complex Concepts OR an EXTENSION of basic concepts involved with triangle trigonometry and sinusoidal functions.

1. **Working with graphs**: Use your TI-84 as well as DESMOS to graph the following functions. NEATLY sketch these graphs into your notebook. In each case, label the "key points/features" as well as the asymptotes. Each sketch should show at least TWO complete cycles.

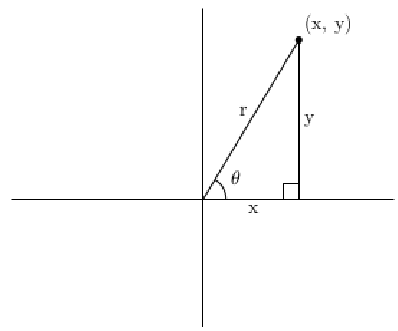
   a. $f(x) = \tan(x)$
   b. $f(x) = \sec(x)$

2. **Working with Identities**: Use online resources to find out what we mean by the term " mathematical identities)

   a. Hence, explain WHY $(x + y)^2 = x^2 + 2xy + y^2$ is an example of a mathematical identity.
   b. Hence, explain WHY $x^2 = x + 6$ is NOT an example of a mathematical identity.

   c. Hence, show why/why not $4(x – 2) = (x – 2)(x + 2) – (x – 2)^2$ is/is not an identity

   Two very common trig identities are (i) the quotient trig identities and then the (ii) the Pythagorean trig identities (see below).



   d. Prove that $\tan x = \dfrac{\sin x}{\cos x}$ and prove that $\sin^2 x + \cos^2 x = 1$. (HINT: Use the diagram to guide your "proof")

3. To prove that a trig equation is also an identity, you will use these two given "facts" (i.e the quotient and the Pythagorean identities) to help develop proofs.

   Two helpful video links ➔ https://youtu.be/Zktxkfr9zJE from Mathispower4u and then https://youtu.be/UpvnqqxmtHk from MathScience Genius

   a. Prove that $\tan x + \dfrac{1}{\tan x} = \dfrac{1}{\sin x \cos x}$ is an identity.
   b. Prove that $\sin^4 x - \cos^4 x = \sin^2 x - \cos^2 x$ is an identity.
   c. Prove that $\dfrac{\sin^2 x}{1 - \cos x} = 1 + \cos x$ is an identity.