BIG PICTURE of this Unit

- How can we extend our algebra skills to interchange between standard and factored form of polynomial equations? (i.e. synthetic division, factoring)
- Can we use our new polynomial algebra skills in order to find a method for solving EVERY polynomial equation (especially those that don't factor?)
- How can use the equation of a polynomial to analyze for key features of a graph of a polynomial (i.e. end behavior, multiplicity of roots, optimal points, intervals of increase/decrease).
- When and how can polynomial functions be used to model real world scenarios?

1. (CI) Now that you understand the importance of (i) factors & roots, (ii) multiplicities of zeroes & (iii) end behaviour, prepare sketches of the following polynomials. In your sketches, pay attention to the correct end behvaviour and the appearance of each graph as it crosses the x-axis. {6}

   a. $f(x) = (x-8)(x-2)(x+4)^2$

   b. $f(x) = -2(x+2)^2(x-2)^2$

   c. $f(x) = (x-3)^2(x-1)(x+5)^2$

   d. $f(x) = (x+3)(x-6)^3(x+7)(x-1)(x+4)$

2. (CI) Working with the parent function $y = \frac{1}{x}$, you will apply the following transformations and determine: {7}

   i. Ex #1 ➔ the parent function has been moved left by 5, down by 3.

   ii. Ex #2 ➔ the parent function has been moved right by 2, up by 4 and vertically stretched by a factor of 3.

   iii. Ex #3 ➔ the parent function has been reflected across the x-axis and then moved up 3.

   a. the equation of the asymptotes in each of i, ii, iii

   b. the "new" location for the "original" points of (1,1) and (-1,-1) in each of i, ii, iii

   c. The new equation, given the transformations. Write the equation in "transformational form" and in "linear/linear" form in each of i, ii, iii

   d. the x- and y-intercepts of the transformed function in each of i, ii, iii

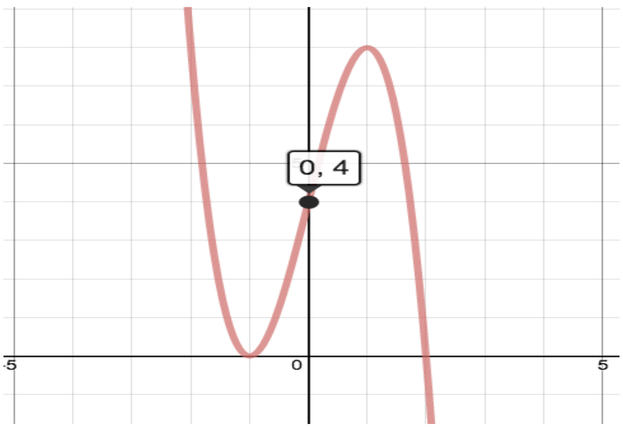
   e. sketch the new, transformed function in each of i, ii, iii

3. (CI) From the following graphs, {5,15}

   a. determine the equation of the polynomial, $p(x)$, in both factored and standard form.

   b. Solve the inequality $p(x) < 0$.

a.



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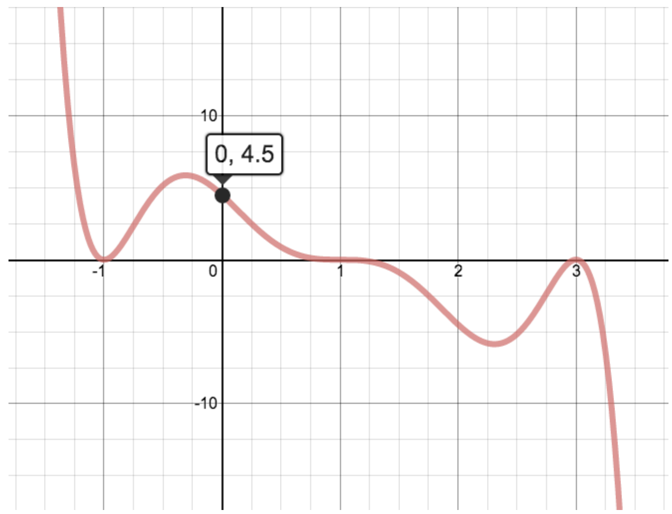


4. Completely factor each of the following polynomials, given some information about the polynomial. {11,13}

   a. Factor $p(x) = 5x^3 + 9x^2 - 26x - 24$, given that $(x+3)$ is one factor of $p(x)$

   b. Factor $p(x) = 6x^3 + 7x^2 - 1$, given that $p(-0.5) = 0$.

   c. Factor $p(x) = 5x^3 + 4x^2 - 20x - 16$, given that x = 2 is one of the zeroes.

   d. Factor $p(x) = 2x^4 - 11x^3 + 20x^2 - 12x$, given that $x = 2$ is a double root.

5. Given this data set (number of students at Juan Fine High School), calculate the second and the third differences. {3,4,8}

Year	1998	1999	2000	2001	2002	2003	2004	2005
# of children	3204	3165	3187	3268	3391	3527	3654	3744

   a. HENCE, decide whether the data set is best modelled by a quadratic function or a cubic function.

   b. Use your TI-84 to determine the quadreg and cubicreg equations for the data set. Record the equations and the coefficient of determination (r^2) from your calculator (turn stat diagnostics on).

   c. According to your calculator, is a quadratic model or cubic model more appropriate? How do you know?

6. Given the following graph: {1,5,6}

   a. Estimate domain interval in which the function is increasing

   b. Estimate the domain interval in which the function is decreasing.

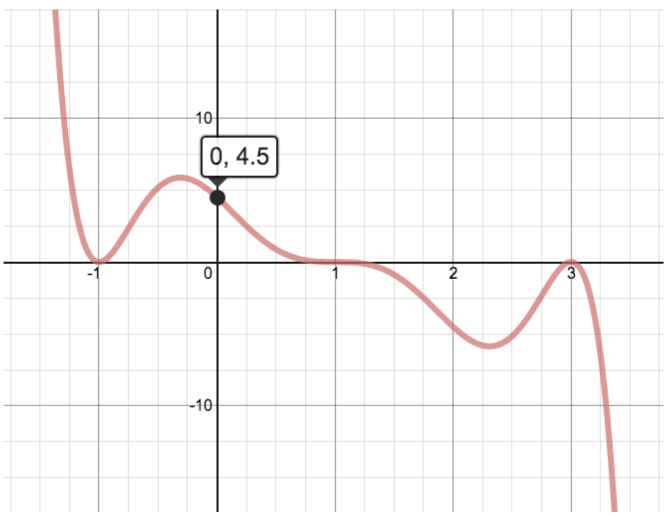
   c. State the end behaviours

   d. Determine the degree of the polynomial

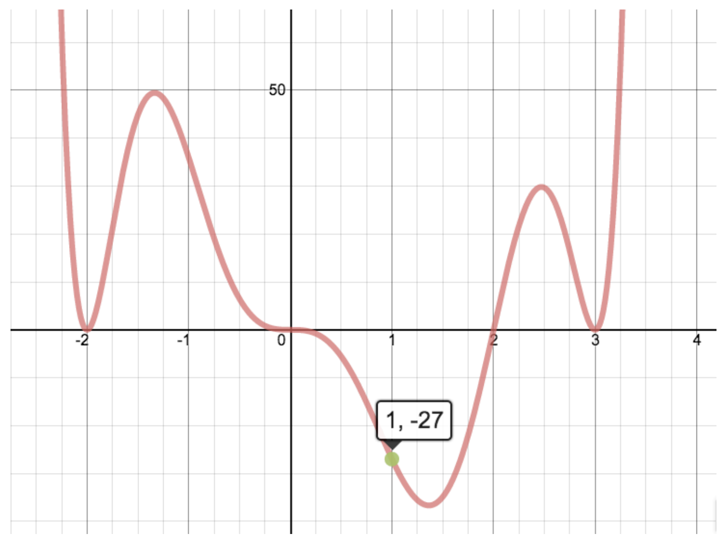
   e. Determine the sign of the leading coefficient

   f. Write the equation in factored form

a.



b.



7. The number of students, $N(t)$, at CAC who have had absences due to illnesses t days after Feb 1, can be modeled by the formula $N(t) = 500 - \dfrac{450}{1+0.3t}$ for $t \geq 0$. {4,8}

   a. Find and interpret N(0).

   b. How long will it take until 300 students will have had the flu?

   c. Determine the behavior of N as t → ∞. Interpret this result graphically and within the context of the problem.

   d. Include a sketch of the graph in your solution.

8. (CA) The cost *C* in dollars to remove *p*% of the invasive species of IttyBitty fishy from Sasquatch Pond is given by $C(p) = \dfrac{1770p}{100 - p}$, where $0 \le p < 100$. {4,8}

   a. Find and interpret C(25) and C(95).

   b. What does the vertical asymptote at x = 100 mean within the context of the problem?

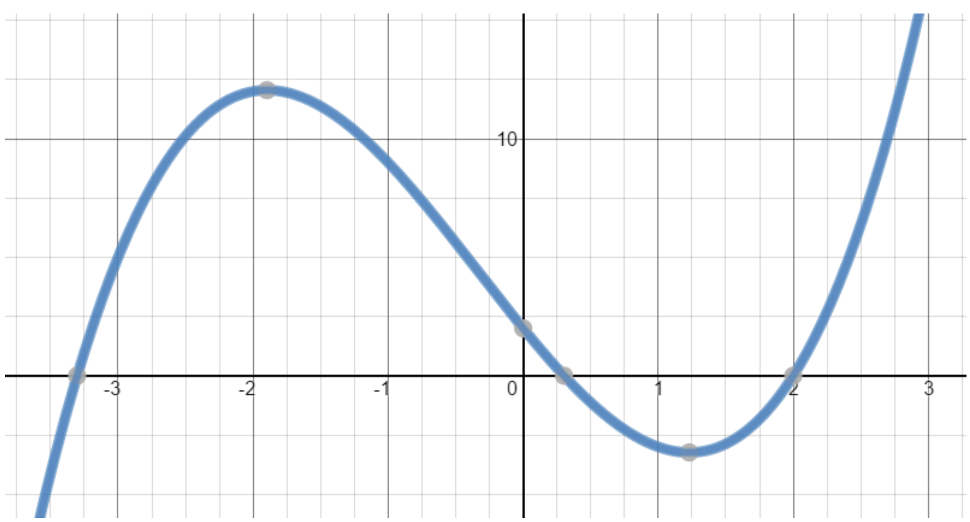
   c. What percentage of the IttyBitty fishy can you remove for $40,000?

   d. How much does it cost to remove the **first** 10% of the fish? How much does it cost to remove the **last** 10% of the fish? Why would/should these costs be different?

   e. How much would it cost to so that only 25% of the fish **remain**?

   f. Include a sketch of this cost function in your notes.

9. Determine the exact value of all roots of the polynomial $A(x) = x^3 + x^2 - 7x + 2$, given the following graph of A(x) {5,6}



10. Determine the equation of the **inverse** of the following rational functions:

   a. $y = \dfrac{2x - 3}{x + 1}$ b. $y = \dfrac{x + 4}{2x - 1}$

11. The function $v(t) = 0.05t^3 - 1.35t^2 + 7.6t + 49$ describes the value (in $US per gram) of a precious metal over an 18 month period. {4,8}

    a. What does the y-intercept mean?

    b. During which month did the metal achieve its greatest value?

    c. During which months has the value of the metal been decreasing?

    d. Determine the lowest value since then.

    e. Describe the value of the metal over the last ten months.

    f. If the function continues to model the value of the precious metal, when will the value first exceed its previous greatest value?

Higher Level Questions for More Complex Concepts OR an EXTENSION of basic concepts involved with Polynomial and Rational Functions.

1. (CI) Solve the following equations for $x \in C$.

    a. $0 = x^3 - 3x^2 + x + 5$

    b. $0 = x^4 - x^3 - 3x^2 - 9x - 108$

    c. $1 - x = 4x^3 - 4x^2$

    d. $-4x - 13 = x^4 + 4x^3 + 14x^2$, knowing that $x = i$ is one of the solutions.

2. (CI) Since you now understand the "derived function" and where it comes from, you will work with the function $y = 2x^3 - \frac{3}{2}x^2 - 3x - 4$:

    a. Determine the equation of the line that is tangent to the curve at x = -1.

    b. Determine the other intersection point(s) of the tangent line with the cubic.

3. (CI) A cubic function has two of its zeroes at -6 and $\sqrt{3}i$ and its y-intercept is (0, 6). Determine the vertical distance between its local extrema.

4. (CI) Given the following info about the roots of a polynomial, write the equation of the polynomial in standard form.
   a. A cubic equation has a double root of 3 and has a zero at x = -6. If the y-intercept is -27, determine the equation of the cubic.
   b. Two roots of a cubic polynomial are 2 and $\frac{1}{2}(1-\sqrt{5})$. Determine the general equation of the family of cubics with these roots.
   c. Two roots of a cubic polynomial are 2 and $1-2i$ and it passes through (5, -180)
   d. Two roots of a fourth degree polynomial are $1+i$ and $3-2i$. Determine the general equation of the family of quartics with these roots.