BIG PICTURE of this Unit

- How can we extend our algebra skills to interchange between standard and factored form of polynomial equations? (i.e. synthetic division, factoring)
- Can we use our new polynomial algebra skills in order to find a method for solving EVERY polynomial equation (especially those that don't factor?)
- How can use the equation of a polynomial to analyze for key features of a graph of a polynomial (i.e. end behavior, multiplicity of roots, optimal points, intervals of increase/decrease).
- When and how can polynomial functions be used to model real world scenarios?

1. With this question, you will (hopefully) see the connection between (i) evaluating a polynomial and (ii) the remainder that results from a division. {11}

   a. Example #1
      i. Evaluate *P*(2) if $P(x) = x^2 - 6x - 7$
      ii. Divide $P(x) = x^2 - 6x - 7$ by $x - 2$ using synthetic division.
      iii. What is the remainder after the division?

   b. Example #2
      i. Evaluate *P*(2) if $P(x) = 4x^3 - 2x^2 - 6x - 1$
      ii. Divide $P(x) = 4x^3 - 2x^2 - 6x - 1$ by $x - 2$ using synthetic division.
      iii. What is the remainder after the division?

   c. Example #3
      i. Evaluate *P*(-3) if $P(x) = 2x^3 - x^2 - 7x + 6$
      ii. Divide $P(x) = 2x^3 - x^2 - 7x + 6$ by $x + 3$ using synthetic division.
      iii. What is the remainder after the division?

2. The following exercise is based on the EXTREMA and ZEROS of a polynomial function. From the description of each polynomial function, sketch a possible graph of each function clearly showing the correct number of extrema or zeros. DO NOT repeat any graphs! If it is not possible to sketch a function fitting the criteria, EXPLAIN why. {1}
   a. 5^{th} degree polynomial with 4 *x*-intercepts
   b. 5^{th} degree polynomial with 2 extrema
   c. 4^{th} degree polynomial with 2 *x*-intercepts
   d. 6^{th} degree polynomial with 3 extrema

3. Use DESMOS and your TI-84 to graph the parent function $f(x)=\frac{1}{x}$ and sketch it in your notes. Then, as a second function, graph the rational function $g(x)=\frac{1}{x+3}+1$ and sketch it in your notes. Explain the form of the second equation can be called "transformational form." {4,7}

4. Convert the following rational expression from transformation form to linear/linear form. (i.e. Write the following as ONE fraction (think adding fractions)): {4,7}

   a. $\frac{2}{x}+3$ b. $4-\frac{3}{x}$ c. $\frac{1}{x+3}+1$ d. $\frac{1}{x-2}-4$

5. You are required to perform the following algebraic manipulations with polynomial functions: {6,9}

   (i) $f(x)=-2(2x-1)^2(x+4)(x-3)$ and **(ii)** $f(x)=-3x(2x-3)^3$

   a. expand the polynomial
   b. detemine the leading coefficient, the degree, and the constant term
   c. State the leading term and predict the end behaviour of these quartic polynomials
   d. from FACTORED FORM find the y-intercept and f(-2)
   e. from STANDARD FORM find the y-intercept and f(-2)
   f. Sketch what you think the polynomial should look like
   g. Graph it using your TI-84 as well as DESMOS (but only after answering questions a – f).

6. The volume of a box is modeled by $V(x)=x^3-15x^2+66x-80$. Graph in a standard view window. {5,8}

   a. Re-adjust the window to a more reasonable setting. Justify your settings.
   b. Factor the expression $x^3-15x^2+66x-80$ and HENCE determine expressions for the dimensions of the box. (Use either the graph or use wolframalpha)
   c. Evaluate and interpret $V(3.5)$.
   d. Substitute x = 3.5 into the **factored equation** and explain why x = 3.5 is NOT valid in our context.
   e. Explain why x = 7 is inadmissible in the context of this question?
   f. Suggest a domain for this model and explain/justify your domain.
   g. Is there an optimal volume for this box?

7. Complete the following reflections, based upon your current understanding of polynomial functions. {1,6}

   a. Reflect upon and write about any connections between the factored form of the equation and the graph. Include a picture or sketch to support your thinking.
   b. Reflect upon and write about any connections between the degree of the polynomial and the shape of the graph. Include a picture or sketch to support your thinking

8. Use synthetic division to divide the following. At the end of the synthetic division state the quotient and the reminder. {10}

   a. Divide $x^2+3x-28$ by $x+5$
   b. Divide $2x^2-5x-3$ by $x-4$
   c. Divide x^3-7x+8 by $x-2$
   d. Divide $12x^3+2x^2+11x+16$ by $3x+2$ ➔ what's different and how do we deal with it?

9. Use synthetic division to help answer the following questions that deal with factoring polynomials. {10,11}

   a. Example #1
      i. Is $(x-2)$ a factor of $2x^2+3x-2$?
      ii. Is $(x+2)$ a factor of $2x^2+3x-2$?
      iii. How did you use SD to help answer the question?

   b. Example #2
      i. Is $(x-1)$ a factor of $x^3+2x^2-11x-12$?
      ii. Is $(x+1)$ a factor of $x^3+2x^2-11x-12$?
      iii. How did you use SD to help answer the question?

   c. Example #3
      i. Is $x-1$ a factor of $3x^3+x^2-22x-24$?
      ii. Is $x+1$ a factor?
      iii. Is $x-2$ a factor?
      iv. Is $x+2$ a factor?

10. For the following polynomials, use DESMOS and your TI-84 to generate the graphs. We will focus our attention on the *x*-intercept at *x* = 3 and simply draw the sketches and record how the function behaves at the *x*-intercept. {6,16}

    a. $f(x) = (x-3)$
    b. $f(x) = (x-3)(x-1)$
    c. $f(x) = (x-3)^2$
    d. $f(x) = (x-3)^2(x-1)$
    e. $f(x) = (x-3)^3$
    f. $f(x) = (x-3)^3(x-1)$
    g. $f(x) = (x-3)^4$
    h. $f(x) = (x-3)^5$

    i. Finally, go on line and research the term "multiplicities of zeroes" and explain what this concept means.

Higher Level Questions for More Complex Concepts OR an EXTENSION of basic concepts involved with Polynomial and Rational Functions.

1. Go online and research the "Remainder Theorem". Then, practice finding the remainder in the following divisions.

   a. Find the remainder when $x^2 + 6x - 17$ is divided by $x - 1$.
   b. Find the remainder when $4x^2 - x + 3$ is divided by $x + 2$.
   c. Find the remainder when $2x^3 - x^2 - x - 1$ is divided by $x - 1$.
   d. Find the remainder when $x^4 - x^3 - x^2 - x - 1$ is divided by $x - 3$.
   e. Find the remainder when $x^4 - 5x^3 + x^2 - 10x - 5$ is divided by $2x + 3$.
   f. Solve for k: When $x^3 + kx + 1$ is divided by $x - 2$, the remainder is -3.
   g. Solve for k: When $2x^4 + kx^2 - 3x + 5$ is divided by $x - 2$, the remainder is 3.
   h. When $x^3 + kx^2 - 2x - 7$ is divided by $x + 1$, the remainder is 5. What is the remainder when it is divided by $x - 1$?
   i. When the polynomial $x^n + x - 8$ is divided by $x - 2$ the remainder is 10. What is the value of n ?

2. Research the Rational Root Theorem (link to PurpleMath ➔ http://www.purplemath.com/modules/rtnlroot.htm) and here are two video from patrickjmt (VIDEO #1 ➔ https://www.youtube.com/watch?v=GTx-rXTQg1o and VIDEO #2 ➔ https://www.youtube.com/watch?v=rP-__zFngio)

3. Predict the rational roots of the following polynomials and hence, factor the polynomials.

   a. $p(x) = 3x^3 + x^2 - 22x - 24$
   b. $p(x) = 6x^3 + 5x^2 - 21x + 10$
   c. $p(x) = 2x^4 - x^3 - 26x^2 - 11x + 12$

4. General Conclusions – Parent Functions and End Behaviour and Multiplicity and Zeroes and Extrema:

   a. What is the general appearance of an even degree polynomial function? An odd degree polynomial function?
   b. What are the generalized end behaviours of even degree polynomials?
   c. What are the generalized end behaviours of odd degree polynomials?
   d. How can you predict the maximum number of zeroes of a polynomial?
   e. How do you predict the appearance of a function near the x-axis if the multiplicity of its zeroes is even?
   f. How do you predict the appearance of a function near the x-axis if the multiplicity of its zeroes is odd?
   g. How can you predict the maximum number of extrema in a polynomial?