BIG PICTURE of this Unit

- How can we extend our algebra skills to interchange between standard and factored form of polynomial equations? (i.e. synthetic division, factoring)
- Can we use our new polynomial algebra skills in order to find a method for solving EVERY polynomial equation (especially those that don't factor?)
- How can use the equation of a polynomial to analyze for key features of a graph of a polynomial (i.e. end behavior, multiplicity of roots, optimal points, intervals of increase/decrease).
- When and how can polynomial functions be used to model real world scenarios?

This lesson will be based upon a STUDENT DIRECTED DISCUSSION model ….. in your groups, you should be having DISCUSSIONS about how to think and work through and then present the solutions to the following questions. So, discuss & prepare solutions to the following questions. Record the key ideas of your discussions/solutions in your notebook. Then, once you have had your discussions, present your solutions on the board. Solutions do NOT necessarily NEED to be correct – they simply form the basis for DISCUSSIONS!!!! If your group has (i) multiple solutions that lead to the same answers OR (ii) same/different solutions that lead to different answers, present them ANYWAY!!

1. Making an Open-Topped Box {3,17}

   Materials Needed = 2 A4 Pieces of Paper 21.5 x 28 cm, Pair of Scissors, Tape, Ruler measuring cm

   <u>Phase 1: The FIRST "Model" Box</u>

   Step 1: Get a sheet of 21.5 x 28 cm coloured paper.

   Step 2: In one corner, draw a square measuring 2 cm x 2 cm.

   Step 3: Cut this square out.

   Step 4: Go to the other three corners and measure & cut identical 2 cm x 2 cm squares.

   Step 5: You should now have 4 "flaps" that you will fold over (in order to make the sides of a box)

   Step 6: Tape these flaps together to complete the box.

   Step 7: Measure the height, length and width of your box. **<u>Record these values in your notebook</u>**

   Step 8: Calculate the volume of this box. **<u>Record this value in your notebook</u>**

   You now have one ordered pair in data set wherein we will be modelling the volume of a box ➔ (2 cm, 830ish cm^3), where *x* will represent the dimensions of the corner you cut out and *V* will represent the volume of the resultant box.

Phase 2: Creating Class Data

Now we will repeat this box construction, wherein every student chooses/is assigned a different sized square to cut out of each corner.

Step 1: Construct your box, measure the three dimensions & calculate the volume . **Record these values in your notebook and record your data on our google doc**.

Step 2: Construct a graph, showing the relationship between corner size and volume. Prepare the graph electronically (TI-84 or EXCEL) or do it by hand on another sheet of paper. Please label your axes.

Step 3: Looking at your scatterplot, what type of function could we use to model our relationships in this investigation? Justify your choice(s).

Step 4: Use your TI-84 (or EXCEL) to determine an ***equation*** for the curve of best fit. Record this **Volume Equation, V(x), in your notebook**

2. Working with the function, y = V(x):
   a. Use your model from our activity to:
      i. evaluate V(6.25) =   ii. Solve for x when V(x) = 1000 cm^3
   b. Use your model to predict the size of the corner that you should cut out in order to ***optimize the volume*** of the box.
   c. Determine the ***domain and range*** for your model, explaining WHY you've decided upon your domain and range.
   d. EXTENTION: Can you PREDICT what the equation for the model should be, simply given the construction instructions?

3. Use online resources to find the following terms and present there definitions in your notebook: {1}

   a. Leading Coefficient of a polynomial
   b. Degree of a polynomial
   c. Special names of the first ‘five” polynomials
   d. What is meant by a “term” within a polynomial?
   e. Standard form of a polynomial.
   f. Horizontal asymptote
   g. Vertical asymptote

4. Using your definitions from Q3, write the following: {1}

   a. Write an equation of a 4th degree polynomial with a negative leading coefficient, having 3 terms.
   b. Write an equation of a 5th degree polynomial with a positive leading coefficient, having 3 terms.
   c. Write an equation of a 3rd degree polynomial with a negative constant term, having 2 terms.

5. For the following expressions, expand and simplify. Then use Wolframalpha to check your answers. {9}

   a. $(x-2)(x+2)(x-1)$
   b. $-3(x-1)(x+2)(2x+3)$
   c. $(x-5)(x+2)(3-x)$
   d. $(x-2)(x^2+2x-3)$
   e. $(x-2)^3$
   f. $(x-2)(x+1)(x-3)(x-1)$

6. Evaluate the following: {2}

   a. Given the function $P(x) = x^3 - x^2 + 2x - 2$, evaluate $P(2)$, $P(-3)$ and $P(1)$. Explain what your answers mean.

   b. Given the function $P(x) = -3(x-1)(x+2)(2x+3)$, evaluate P(2), $P(-3)$ and $P(1)$. Explain what your answers mean.

   c. Given the function $(x) = -2x^3 + 2x - 5$, evaluate P(-1), P(0) and P(1). What do you notice about your answers. HENCE, **solve** the equation $-5 = -2x^3 + 2x - 5$

7. Evaluate the function $g(x) = \frac{1}{x+2}$ for the following values of *x*:

a. x = 3 b. x = 8 c. x = 28 d. x = 58 e. x = 98

f. What are you noticing happening with both the x-values and the y-values?

g. x = -1 h. x = -1.5 i. x = -1.9 j. x = -1.99 k. x = -1.999

l. What are you noticing happening with both the x-values and the y-values?

m. Graph the function on your TI-84. Explain how the picture of the graph now helps you make sense of you answers to (a) and (c).

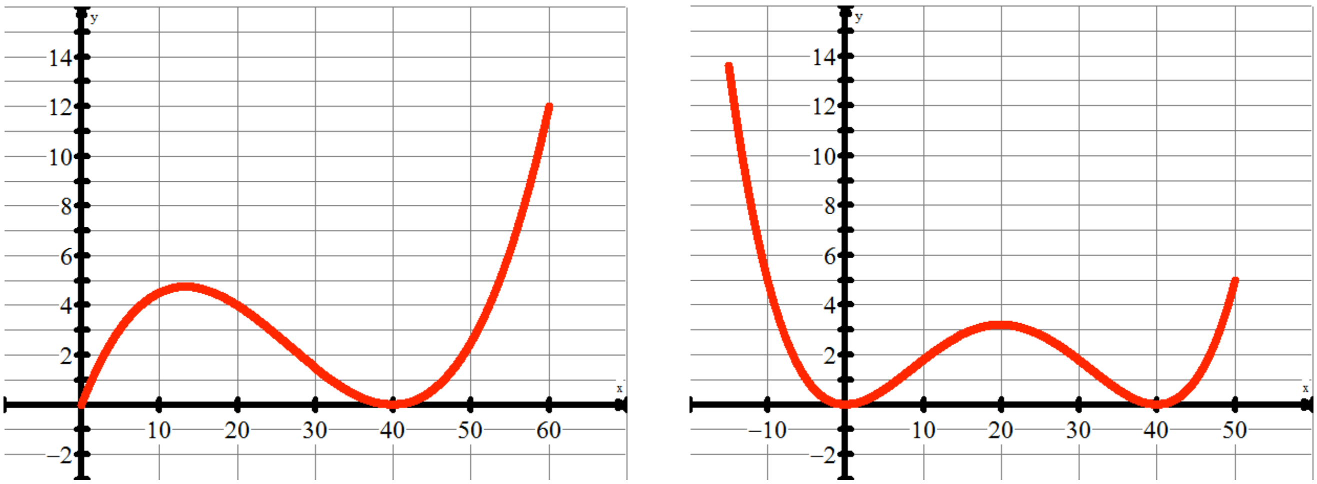
8. Use DESMOS as well as your TI-84 to graph these following "parent" functions. Include a labeled sketch in your notes: (i) $y = x^3$ (ii) $y = x^4$ (iii) $y = \frac{1}{x}$. Label the "important" features/parts of each function. {4}

9. Use DESMOS as well as your TI-84 to graph and then compare (what do all the graphs and equations share in common?) What do you notice about the **way an equation is written and the way a graph looks**? Include labeled sketches in your notes. {16}

a. y = 2(x – 1)
b. y = 2(x – 1)(x + 1)
c. y = 2(x – 1)(x + 1)(x + 2)
d. y = 2(x – 1)(x + 1)(x + 2)(x – 3)

10. Roller Coaster Design {1,4,17}

Your design team has the following "profile" of part of a roller coaster (the x-axis represents horizontal distance and the y-axis represents vertical distance). Your initial task is to find a polynomial model for the profile of the roller coaster.

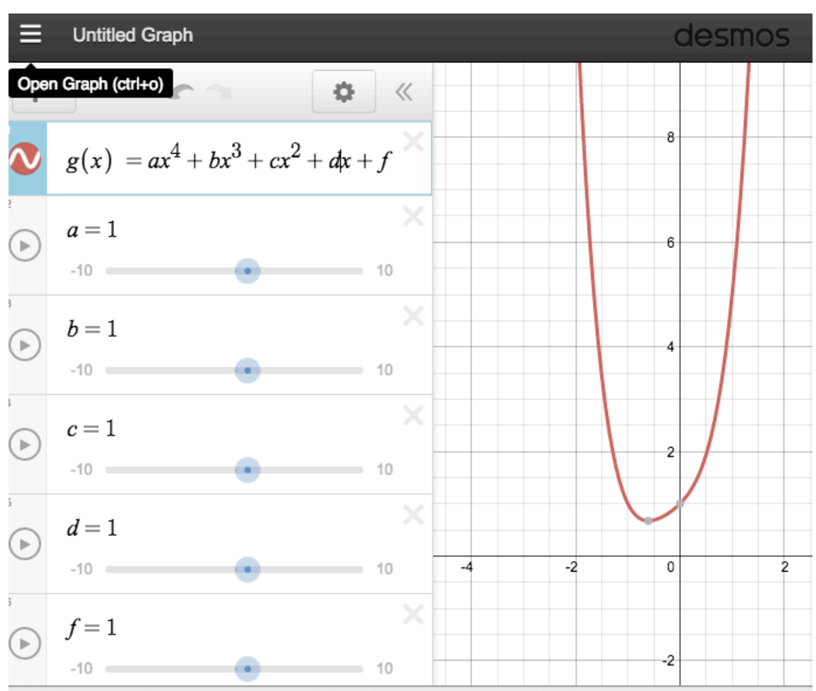


You will carry out this modeling in three ways:

(a) Using DESMOS, program in the standard form of a cubic equation ($y = ax^3 + bx^2 + cx + d$) or quartic ($y = ax^4 + bx^3 + cx^2 + dx + g$) and add sliders for values of a,b,c,d and g. Then adjust the sliders to get an equation that matches this "profile" pictured here. (see picture included for setting up sliders)

(b) Use the graph to read data points from the graph, then your TI-84 to determine the equation (cubicreg/quartreg)

(c) (HL level - NO CALCULATORS – ALTHOUGH SL students should learn how to do this as well) Use algebra & skills you've learned from your Quadratics Unit to determine an equation.



Higher Level Questions for More Complex Concepts OR an EXTENSION of basic concepts involved with Polynomial and Rational Functions.

1. EXTENTION: Boxes with reinforced sides ➔ to construct a box with reinforced sides, use your original box (2 cm corners cut) and make one adjustment on the four side “flaps” ➔ fold this flap TWICE (once at the 2 cm mark and a second time at the 1 cm side), so that your sides are now twice as thick. Again, determine the volume of this box. Then, as before, predict an equation for an equation modeling the relationship between corner size and volume. Hence, what corner size will optimize the volume of the box?

2. Use online resources to find out what the ideas of “even” and “odd” symmetry means. Include sketches.

3. Use online resources to find out how to TEST ALGEBRAICALLY for even and odd symmetry.

Use DESMOS to conduct the following investigation:

4. Graph the following functions: (i) $f(x) = x$ (ii) $f(x) = x^3$ (iii) $f(x) = x^5$ (iv) $f(x) = x^7$.

5. Explain what the function “looks like” when the exponent is odd (i.e. x^{2n+1}, where $n \geq 0$)

6. Graph the following functions: (i) $f(x) = x^2$ (ii) $f(x) = x^4$ (iii) $f(x) = x^6$ (iv) $f(x) = x^8$.

7. Explain what the function “looks like” when the exponent is even (i.e. x^{2n}, where $n \geq 1$)