

BIG PICTURE of this UNIT:

- How do we WORK WITH & EXTEND the concept of “functions”
- Why are exponential equations written in different forms?
- How do we EXTEND our knowledge of exponential functions, beyond the basics of IM2?

This lesson will be based upon a STUDENT DIRECTED DISCUSSION model in your groups, you should be having DISCUSSIONS about how to think and work through and then present the solutions to the following questions. So, discuss & prepare solutions to the following questions. Record the key ideas of your discussions/solutions in your notebook. Then, once you have had your discussions, present your solutions on the board. Solutions do NOT necessarily NEED to be correct – they simply form the basis for DISCUSSIONS !!!! If your group has (i) multiple solutions that lead to the same answers OR (ii) same/different solutions that lead to different answers, present them ANYWAY!!

- (CI) Given the function $f(x) = 8 - 2^{x+4}$; $\{8,9,13,16,17\}$
 - Determine the domain, range, asymptote(s) and intercept(s) of $f(x)$. Sketch and label key points.
 - The function $f(x) = 8 - 2^{x+4}$ represents a transformation of the “parent function” $y = 2^x$. Describe which transformations must be applied to $f(x) = 8 - 2^{x+4}$ to **get back to** $y = 2^x$.
 - Determine the equation of the inverse of $f^{-1}(x)$. Sketch and label key points.
 - (CA) Use your TI-84 to graph $y = |f(x)|$ (that is $y = |8 - 2^{x+4}|$). Sketch it in your notes. Explain WHY the graph now appears as it does.
 - (HL) Determine the equation of $f^{-1} \circ f(x)$ and $f \circ f^{-1}(x)$. Show the key steps of your work. Explain the significance of the result.
- Questions dealing with half-life can use a “special equation/formula”. Go on line and find this formula. Use this formula to answer the following questions: $\{11,20\}$
 - The half life of caffeine in a child’s system when a child eat or drinks something with caffeine is 1.5 hours. How much caffeine would remain in a child’s body if the child ate a chocolate bar with 30 mg of caffeine 8 hours before?
 - The half-life of Carbon-14 is about 5370 years. What percentage of the original carbon-14 would you expect to find in a sample after 2500 years?
 - Now rework these questions, wherein we now set up the scenario of **continuous changes, hence you must use the special base of e**. So, write new equations using base e to rework your solutions.
- A hard-boiled egg has a temperature of 98 degrees Celsius. If it is put into a sink that maintains a temperature of 18 degrees Celsius, its temperature x minutes later is given by the formula $T(x) = 18 + 80e^{-0.29x}$. Hilda needs her egg to be exactly 30 degrees Celsius for decorating. How long should she leave it in the sink? $\{4,11,20\}$

4. There has been an accident at the local nuclear plant and a new radioactive material (Santogen) has been spilled. This radioactive material begins to decay exponentially. Assume that this decay is **continuous**. There were 1820 Bq (becquerels) of Santogen initially. Eight hours later there were 576 Bq. {11,20}
- What is the decay rate of Santogen?
 - This material becomes non-lethal when there is a max of 20 Bq. When will it be safe for workers to enter the space and clear it out?
 - Will there ever be 0 Bqs left of the Santogen material?
 - What is the half life of Santogen?

5. Use your TI-84 to determine the value of the following logarithms: {6,7}

$\log_2 0$	$\log_2 1$	$\log_2 2$	$\log_2 3$	$\log_2 4$	$\log_2 5$	$\log_2 6$
$\log_2 7$	$\log_2 8$	$\log_2 9$	$\log_2 10$	$\log_2 11$	$\log_2 12$	$\log_2 13$
$\log_2 14$	$\log_2 15$	$\log_2 16$	$\log_2 17$	$\log_2 18$	$\log_2 19$	$\log_2 20$

Look for patterns amongst the numbers & outputs:

- Compare $\log_2 2$ and $\log_2 5$ and $\log_2 10$
 - Compare $\log_2 3$ and $\log_2 4$ and $\log_2 12$
 - Compare $\log_2 3$ and $\log_2 6$ and $\log_2 18$
 - Can you see some patterns that will lead to some GENERALIZATIONS that would then in turn allow us to make PREDICTIONS?
 - So, predict the value of (i) $\log_2 48$, (ii) $\log_2 36$, (iii) $\log_2 75$
 - So, predict the value of (i) $\log_2 \left(\frac{1}{3}\right)$, (ii) $\log_2 7.5$, (iii) $\log_2 \sqrt[3]{12}$
6. Four months ago after it stopped advertising, a manufacturing company noticed that its sales per unit “y” had dropped each month according to the function $y = 100,000e^{-0.05x}$ where “x” is the number of months after the company stopped advertising. {4,11,20}
- Find the projected drop in sales per unit six months after the company stops advertising.
 - How many months until sales per unit had dropped \$50,000?
7. Solve the following logarithmic equations: {6,7}
- SL level (<http://www.mathworksheets4kids.com/logarithms/solve-level2-easy1.pdf>)
 - HL Level (<http://www.mathworksheets4kids.com/logarithms/evaluating-expressions-level2-hard1.pdf>)

8. At any time $t \geq 0$, in days, the number of bacteria present, y , is given by $y = Ce^{kt}$ where k is a constant. The initial population is 1000 and the population triples during the first 5 days. {4,11,20}
- What does “ C ” represent? What is its value?
 - Write an equation for y at any time $t \geq 0$.
 - By what factor will the population have increased in the first 10 days?
 - At what time, t , in days, will the population have increased by a factor of 6?

9. (CI) Evaluate the following: {6,7}

$$\log_4 64 = \quad \log_2 32 = \quad \log_{\frac{1}{5}} 25 = \quad \log_{12} 144 = \quad \log_4 2 = \quad \log_{\frac{2}{3}} \left(\frac{4}{9} \right) =$$

$$\log_{125} 5 = \quad \log_9 3 = \quad \log_8 2 = \quad \log_2 \frac{1}{16} = \quad \log_{243} 27 = \quad \log_8 4 =$$

$$\log_2 \frac{1}{8} = \quad \log_9 \frac{1}{81} = \quad \log_3 \frac{1}{27} = \quad \log_{\frac{3}{5}} \left(\frac{25}{9} \right) = \quad \log_{27} \frac{1}{3} = \quad \log_{128} \frac{1}{2} =$$

10. During a certain epidemic, **the number of people that are infected at any time increases at a rate proportion to the number of people that are infected at that time.** If 1000 people are infected when the epidemic is first discovered and 1200 are infected 7 days later, how many people are infected 12 days after the epidemic is first discovered? {4,11,20}
11. The following data represents the price and quantity supplied in 1997 for IBM personal computers. {4,11,20}

Price (\$/Computer)	2300	2000	1700	1500	1300	1200	1000
Quantity Supplied	180	173	160	150	137	130	113

- Use your calculator draw the scatter plot. Use price as the independent variable. Label your axes.
 - Use your calculator to fit a logarithmic model to this data.
12. Let $P(t)$ represent the number of wolves in a population at time t years, when $t \geq 0$. The population of wolves is given by $P(t) = 800 - Ce^{-kt}$. {4,11,20}
- If $P(0) = 500$, find $P(t)$ in terms of t and k .
 - If $P(2) = 700$, find k .
 - As time increases without bound, what happens to the population of wolves? Support your answer with a graph and a written explanation.