BIG PICTURE of this UNIT:	• How do we WORK WITH & EXTEND the concept of "functions" • Why are exponential equations written in different forms? • How do we EXTEND our knowledge of exponential functions, beyond the basics of IM2?

This lesson will be based upon a STUDENT DIRECTED DISCUSSION model ….. in your groups, you should be having DISCUSSIONS about how to think and work through and then present the solutions to the following questions. So, discuss & prepare solutions to the following questions. Record the key ideas of your discussions/solutions in your notebook. Then, once you have had your discussions, present your solutions on the board. Solutions do NOT necessarily NEED to be correct – they simply form the basis for DISCUSSIONS !!!! If your group has (i) multiple solutions that lead to the same answers OR (ii) same/different solutions that lead to different answers, present them ANYWAY!!

1. For the following functions, determine {8,9,13}

   a. the equations of their inverses:
   b. the domain and ranges of BOTH the function and its inverse
   c. Include a sketch of the function and its inverse.

(i) $f(x) = 2^x$ (ii) $f(x) = \left(\frac{1}{2}\right)^x$

(iii) $f(x) = 3(2)^x$ (iv) $f(x) = 3 + 2^x$ (v) $f(x) = 2^{x+3}$

2. A sequence of numbers is simply a list of numbers to which there may or not be a given pattern. {22}

   a. Go online and explain what an arithmetic sequence is.
   b. Go online and explain what a geometric sequence is.
   c. Classify the following sequences as either arithmeetic or geometric or neither
      i. 5, 15, 45, 135,….
      ii. 3, 9, 81, 6561,….
      iii. 15, 26, 37, 48, …..
      iv. 6000, 3000, 1500, 750, …..
   d. Determine the 10th term in each sequence.

3. A formula that you can use for "continuous compounding" or "continuously changing" is $A(t) = A_0e^{rt}$. Use this formula to answer these two questions about an investment I made wherein I invested $10,000 in a fund yielding 12% p.a compounded continuously. {4,11,20}

   a. Find the value of the investment after 5 years.
   b. How long does it take for the investment to triple in value?

4. Determine the value after 2 years of a $1000 investment invested at 10% pa under the following compounding conditions: {21}

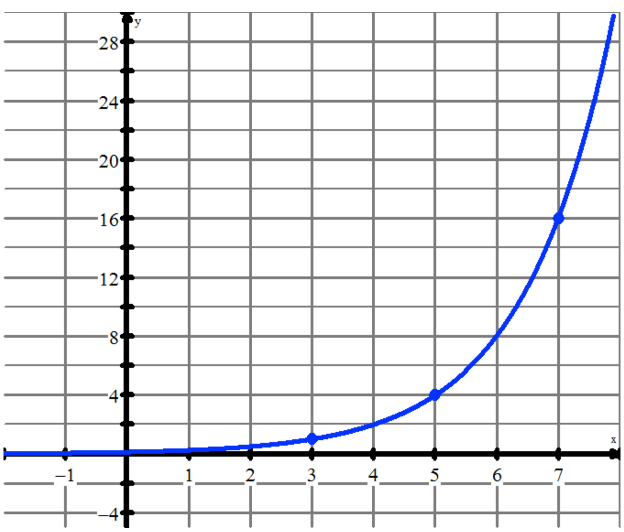
   a. 10% pa compounded quarterly
   b. 10% pa compounded monthly
   c. 10% pa compounded **continuously**
   d. Explain why the three values you calculated are ***not the same***.

5. The population of a town is modeled by the function $P(t) = 15,752(1.045)^t$ where t is time in years since 1990. Answer the questions below using the function given. {11,20}

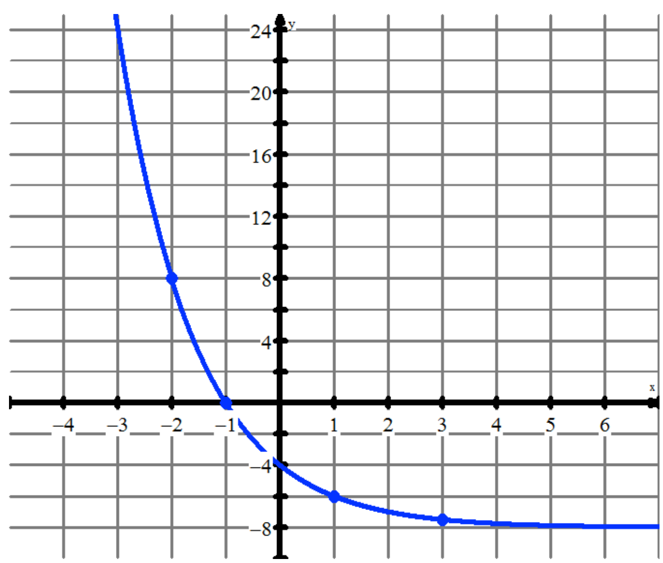
   a. Is the population of the town growing or shrinking… by what percentage?
   b. Find P(5). Interpret.
   c. What was the population of the town in 1990?
   d. When will the population of the town become more that 50,000?
   e. What assumptions are we making in questions (d)?

6. Given the two graphs provided, determine the equation of each function. {6}

   (a) write the equation in the form of $f(x) = ab^x$



   (b) Write the equation in the form of $f(x) = ab^x + c$



7. Solve the following equations, providing both exact and approximate answers. {4,5}

   (a) Solve $2^x = -7$
   (b) Solve $2e^{3x+2} = 14$
   (c) Solve $1 + 2e^{3(x-2)} = 9$
   (d) Solve $\dfrac{16}{2 + 4e^{-x}} = 4$
   (e) Solve $\dfrac{24e^{-2x}}{2 + 4e^{-2x}} = 2$
   (f) Solve $e^{\ln(x-2)} = 2$

8. Convert all log equations to equivalent exponential equations & vice versa (convert exponential equations into equivalent logarithmic equations. {6}

(i) Convert to exponential form

(a) $\log_{16} 256 = 2$

(b) $\log_9 81 = 2$

(c) $\log_2 \frac{1}{8} = -3$

(d) $\log_{25} 5 = \frac{1}{2}$

(e) $\log_{20} 400 = 2$

(f) $\log_{\frac{1}{4}} 64 = -3$

(g) $\log_5 \frac{1}{625} = -4$

(h) $\log_{169} \sqrt{13} = \frac{1}{4}$

(ii) Convert to logarthmic form

(A) $4^{\frac{1}{2}} = 2$

(B) $3^5 = 243$

(C) $14^{-2} = \frac{1}{196}$

(D) $18^2 = 324$

(E) $\left(\frac{1}{3}\right)^3 = \frac{1}{27}$

(F) $6^{-3} = \frac{1}{216}$

(G) $\left(\frac{1}{2}\right)^{-2} = 4$

(H) $32^{-\frac{3}{5}} = \frac{1}{8}$

9. Evaluate & solve the following logarithmic expressions/equations. {5,7}

Evaluate the following logarithmic expressions

$\log_3 \frac{1}{243} =$

$4\log_2 4 =$

$\log_2 64 =$

$\log_{\frac{1}{6}} 36 =$

$2\log_4 2 =$

Solve the following logarithmic equations

$\log_9 x = \frac{1}{2}$

$\log_5 0.04 = x$

$\log_2 \frac{1}{x} = 4$

$\log_x 2 = \frac{1}{3}$

$\log_x 256 = -4$

10. A company is trying to expose a new product to as many people as possible through television advertising in a large city with 2 million potential viewers. A model for the number of people, *N*, in millions, who are aware of the product after *t* days of advertising was found to be $N(t) = 2\left(1 - e^{-0.037t}\right)$. {11,20}

    a. How many viewers are aware of the product after 2 days? After 10 days?
    b. How many days will it take until half of the potential viewers will become aware of the product?

11. Answer the following investment questions: {11,20}

    c. Determine the final value of $12,500 invested at 8% p.a. compounded monthly for 5 years.
    d. How much should I invest now (at 6.25% p.a. compounded **continuously**) so that I have $15,000 in 6 years time?
    e. How many years does it take for an investment of $7,500 @ 3% p.a. compounded quarterly to grow to $9,750?
    f. At what rate should I invest $40,000 so that it grows to $50,000 in 5 years time? (Assume the money is compounded continuously)

12. (**CI**) Evaluate the following expressions: {1,3,6,7}

    (a) $\log_3(81)$ (b) $\log_6\left(\frac{1}{216}\right)$ (c) $16^{\frac{3}{2}} + 16^{-0.5} + 64^{\frac{1}{2}} - 27^{\frac{2}{3}}$

    (d) $81^{\frac{1}{2}} + \sqrt[3]{8} - 32^{\frac{4}{5}} + 16^{\frac{3}{4}}$ (e) $\log_{81}(9)$ (f) $\log_3(\sqrt[3]{9})$

    (g) $\log_{\frac{1}{2}}(16)$ (h) $\sqrt[3]{-27} + \left(\sqrt[3]{-216}\right)^5 + \sqrt[4]{\left(\frac{16}{81}\right)^{-1}}$ (i) $\log_{25}\left(\frac{1}{125}\right)$

13. Given the functions of $f(x) = -\frac{1}{2}(x-4)$ and $g(x) = 9\left(\frac{1}{3}\right)^{x+2}$, determine: {9,10,13,14}

    a. Equations for $(i)\ f \circ g(x) \quad (ii)\ g \circ f(x)$
    b. The domain and range for the functions defined by $(i)\ f \circ g(x) \quad (ii)\ g \circ f(x)$
    c. The intercepts for the functions defined by $(i)\ f \circ g(x) \quad (ii)\ g \circ f(x)$
    d. Equations for the inverses of $f^{-1}(x)$ and $g^{-1}(x)$.
    e. The solution to the inequality $f^{-1}(x) > g(x)$.