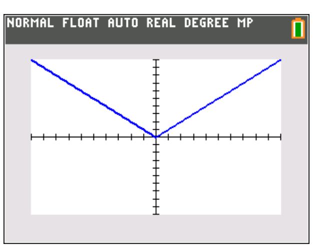
IM3 L1.7 – Function Concepts & Linear (HL) | Unit 1 – Function Concepts & Linear

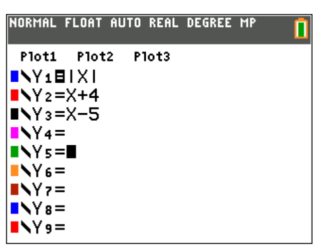
BIG PICTURE of this UNIT:	• How do we WORK WITH & EXTEND the concept of "functions" • Why are linear equations written in different forms? • How do we EXTEND our knowledge of LINEAR functions, beyond the basics of IM2?

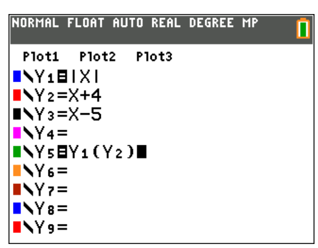
In your group, discuss & prepare solutions to the following **CONCEPT EXTENSION** questions.

1. Consider these three linear functions ➔ $f(x) = 7 - 2x$ and $3x - 5y - 30 = 0$ and $h(x) - 2 = -\frac{2}{3}(x + 6)$.
{3,6,8,9,10}
   a. **Show that** $3x - 5y - 30 = 0$ can be written as $g(x) = \frac{3}{5}x - 6$. {3}
   b. Solve the equation $f(x) = g(x)$ and **hence** solve the inequality $f(x) < g(x)$. Verify graphically. {6}
   c. Find the equations of $f^{-1}(x)$ and $g^{-1}(x)$. {8}
   d. **Hence**, perform the following compositions: $f \circ f^{-1}(x),\ f^{-1} \circ f(x),\ g \circ g^{-1}(x),\ g^{-1} \circ g(x)$. What do you notice? {9,10}
   e. Find the equations of the following compositions: {9}
   $\text{(i)}\, f \circ g(x)\quad \text{(ii)}\, f \circ h(x)\quad \text{(iii)}\, f^{-1} \circ g(x)\quad \text{(iv)}\, h \circ g^{-1}(x)\quad \text{(v)}\, f \circ g \circ h(x)$
   f. Write equations of the following combinations of functions and PREDICT what the functions should look like. Once you have made your predictions, use DESMOS to verify your predictions.
   $\text{(i)}\, f(x) + g(x)\quad \text{(ii)}\, g(x) - f(x)\quad \text{(iii)}\, f(x) \times g(x)\quad \text{(iv)}\, f(x) \times g(x) \times h(x)\quad \text{(v)}\, \frac{f(x)}{g(x)}$

2. Use your TI-84 for the following graphical investigation. Start by graphing $f(x) = |x|$ in a standard view window. Then program in $g(x) = x + 4$ and $h(x) = x - 5$, inactivate these equations. You should have the following in your equation editor on the TI-84: {9,11,12,13}
   a. Find the equations for (i) $f \circ g(x)$ (ii) $f \circ h(x)$ (iii) $g \circ f(x)$ (iv) $h \circ f(x)$
   b. Graph the composition $f \circ g(x)$ in your calculator as: $f \circ g(x)$ as $Y_5 = Y_1(Y_2)$. Describe what does and doesn't change in the appearance of the graph of $f(x) = |x|$.
   c. Graph the composition $f \circ h(x)$ in your calculator as: $f \circ g(x)$ as $Y_6 = Y_1(Y_3)$. Describe what does and doesn't change in the appearance of the graph of $f(x) = |x|$.
   d. Repeat for $g \circ f(x)$ & $h \circ f(x)$.
   e. Summarize your observations and thus make some generalizations about the effect of composing any function with a linear function.







3. The following example will illustrate one way of understanding the composition given a practical context. Let's say my son Andrew is a carpenter & earns a **daily** wage of $20/h plus $15 for travel expenses. {5,9}

   a. Write this information as an equation. Use W(h) in writing your equation. Why?
   b. Complete the Daily Wages column in the table of values below.

   However, Andrew belongs to a Carpenter's Union and must pay union fees at 2.5% of his daily wages.

   c. Write an equation, relating his union fees, F, to his wages, W. So use F(W) in writing your equation. Why?
   d. Complete the Union Fees column in the table of values below.
   e. Write an equation relating Union Fees to hours worked. Use F(h) in your equation. Use the data table to test/verify your equation.
   f. Write a composite function equation $F \circ W(h)$. Use the data table to test/verify your equation.
   g. What do you notice about the two equations you have generated in part (e) and (f)

Hours worked	Daily Wages	Union Fees
2		
4		
6		
8		
9		
10		
12		

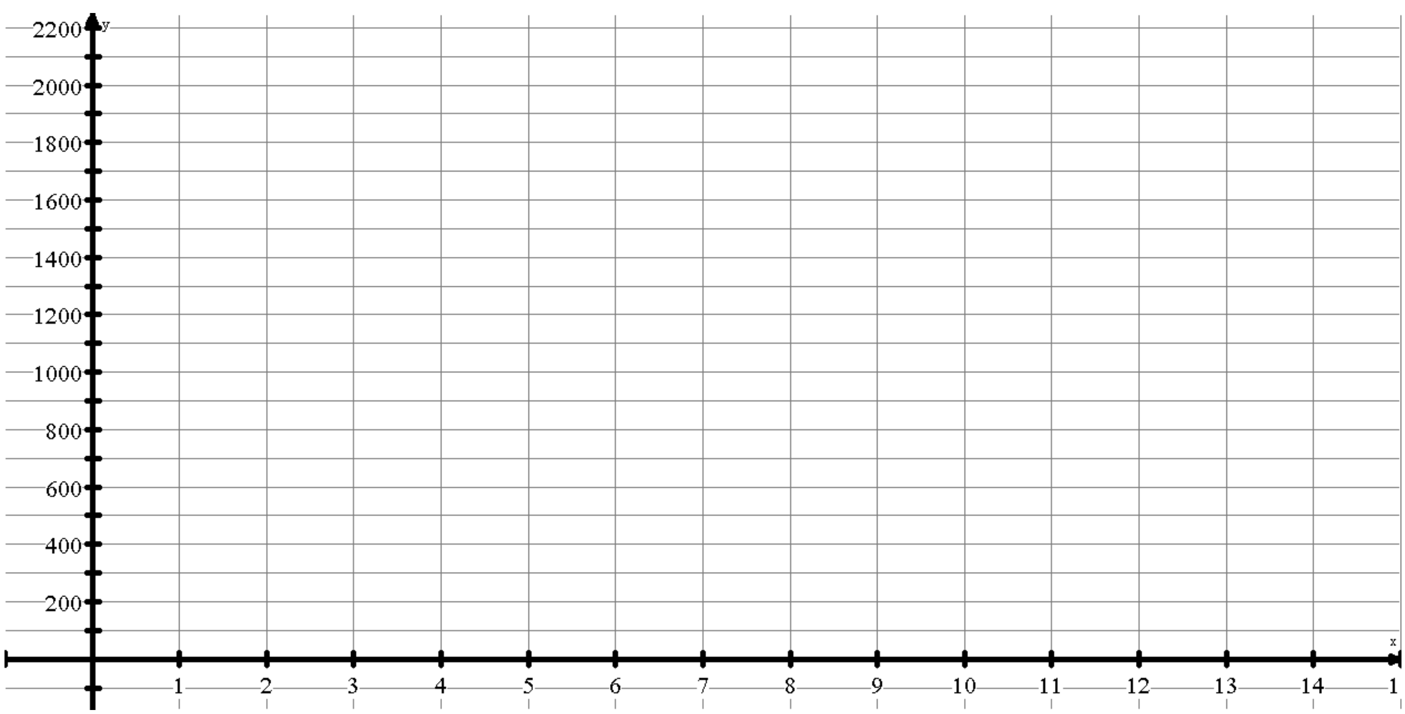
4. Given the functions $f(x) = \frac{1}{x}$ and $g(x) = 2x - 6$, {3,9,15,19}

   a. Use your TI-84 to graph $y = f \circ g(x)$ and label the asymptotes. Write the equation for $f \circ g(x)$.
   b. Give one reason why Mr S prefers $g(x) = 2x - 6$ to be written as $g(x) = 2(x - 3)$
   c. Determine the average rate of change of $y = f \circ g(x)$ between the x-values of x = 4 and x = 5.
   d. Determine the average rate of change of $y = f \circ g(x)$ between the x-values of x = 4 and x = 4.1.
   e. Determine the average rate of change of $y = f \circ g(x)$ between the x-values of x = 4 and x = 4.001.
   f. PREDICT the INSTANTANEOUS rate of change of $y = f \circ g(x)$ at x= 4.
   g. Use the TI-84 to draw the tangent line at x = 4 to verify your prediction.

5. A hotel has the following rates that apply to groups who rent their ballroom. They charge \$400 for any time of 2 hours or less. If the rental time exceeds 2 hours, then an additional rate of \$200 per hour are charged. However, if the total rental time is more than 8 hours, they only charge an hourly rate of \$100. All rentals are not allowed to exceed 12 hours. {5,13}

   a. What is the independent variable (input)? What would the domain be?
   b. What is the dependent variable (output)? What would the range be?
   c. Would you expect this relation to be a function? Why/why not?
   d. Evaluate C(7) as well as C(11).
   e. Evaluate \$1150 = C(t) and interpret.
   f. To help draw a graph, complete the following table of values. Then graph this relation.

Time									
Cost (\$)									



Now, how would you write an equation for this relation?

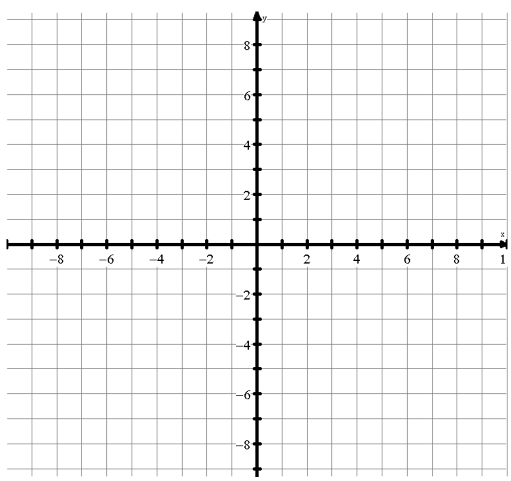
6. You now extend your working knowledge of absolute value. {13}

   a. Explain what the Absolute Value "function" does to an input, for example the numbers -3 and +5
   b. Evaluate $|-2+5+7-13\times 2|$ and evaluate $(-2+5+7-13\times 2)$ and explain WHY the answers are different.
   c. The function $f(x)=|2x+5|$ can be understood as a piecewise function. What are the two "pieces" and in which restricted domain does each piece apply?
   d. Solve $|2x+5|=4$ GRAPHICALLY on DESMOS and explain WHY there are two solutions.
   e. Explain HOW to solve the equation $|2x+5|=4$ ALGEBRAICALLY.
   f. Solve $|2x+5|=x+4$ GRAPHICALLY and explain WHY there are two solutions.
   g. Explain HOW to solve the equation $|2x+5|=x+4$ ALGEBRAICALLY.

7. Review your working knowledge of piecewise functions: {13}

(a) Graph

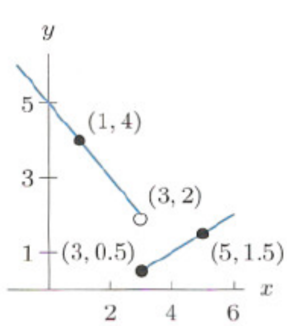
$$a(x)=\begin{cases}1-x & \text{if } -6\le x<-2\\ 3 & \text{if } x=-2\\ x-2 & \text{if } -2<x<4\end{cases}$$



(b) Now, using the same grid, graph $y=f\circ a(x)$ where $f(x)=-x+2$

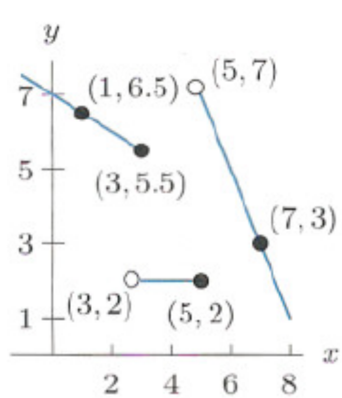
(c) Write an equation for the function

7.



(d) Write an equation for the function

8.



(e) Write an equation for the function

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