

BIG PICTURE of this UNIT:

- How do we WORK WITH & EXTEND the concept of “functions”
- Why are linear equations written in different forms?
- How do we EXTEND our knowledge of LINEAR functions, beyond the basics of IM2?

This lesson will be based upon a STUDENT DIRECTED DISCUSSION model in your groups, you should be having DISCUSSIONS about how to think and work through and then present the solutions to the following questions. The questions will involve basic ideas from IM2 including (i) functions, (ii) linear functions, (iii) exponential functions, and (iv) quadratic functions. EVERY LESSON this semester will involve **spiralling through** these 4 major concepts as you will be given the opportunity to deepen and extend your conceptual knowledge & skill set on these 4 major themes as you see them multiple times in our lessons.

So, in your group, discuss & prepare solutions to the following questions. Record the key ideas of your discussions/solutions in your notebook. Then, once you have had your discussions, present your solutions on the board. Solutions do NOT necessarily NEED to be correct – they simply form the basis for DISCUSSIONS !!!! If your group has (i) multiple solutions that lead to the same answers OR (ii) same/different solutions that lead to different answers, present them ANYWAY!!

- For $g(x) = 4 - 5x$, determine the: {1,4}
 - input for x when the output for $g(x)$ is (i) -6 (ii) 2 (iii) 0.6
 - the slope of the line
 - the x - and y -intercepts. HENCE, write the linear equation in the form $\frac{x}{a} + \frac{y}{b} = 1$
- Consider the function $g(t) = 3t + 5$. {1,4}
 - Create a table of values and prepare a graph of $y = g(t)$.
 - Determine each of the following values:
 - $g(0)$
 - $g(3)$
 - $g(1) - g(0)$
 - $g(2) - g(1)$
 - $g(1001) - g(1000)$
 - $g(a+1) - g(a)$
- A freight delivery company charges \$4.00/kg for any order less than 100 kg and \$3.50/kg for any order of at least 100 kg. {5}
 - Why must this relation be a function?
 - What is the domain of this function? What is its range?
 - Graph the function.
 - What suggestions can you offer to the company for a better pricing structure? Support your answer.

- Use DESMOS to graph the function defined by $f(x) = \begin{cases} 2 - x & \text{if } x < 4 \\ 5 & \text{if } 4 \leq x \leq 6 \\ \frac{1}{2}x + 2 & \text{if } x > 6 \end{cases}$. Sketch it in your notebook. {14}

- Functions can be described as being **continuous** and/or **discontinuous**. Given the graph of $y = f(x)$, would you describe the function as being continuous or discontinuous? Explain why.
- On what interval are the function values of $f(x)$ **increasing**?

5. Have your group in unison clap out a rhythm given the following characteristics: 5 claps, two of which are “short” and three of which are “long”. How many such rhythms can be composed? Explain/show the reasoning that leads to your answer. {P1}
6. A piecewise function, $y = h(x)$ has the following characteristics: {6,13}
- The domain is $\{x \mid -2 \leq x \leq 6\}$,
 - The value of $h(-2)$ is 5 and $h(6) = 17$,
 - The function decreases for the first half of its domain and then increases for the second half of the domain
 - CHALLENGE OPTION: (The function decreases for the first half of its domain and then increases for the second half of the domain at a rate that is twice as great as first piece’s rate-of decrease).**
- Sketch the function and determine it’s equation.
 - Determine the domain and range of $y = h(x)$
7. Let $f(x) = 3x + 1$ and let $g(x) = 2 - x$. {1,6,9}
- Determine a value of K such that $f(K) = g(K)$. Explain the significance of K .
 - Determine a value for M such that $f(M^2) = g(2M)$.
 - Evaluate: (i) $f(3)$ (ii) $g(-1)$ (iii) $f(g(-1))$ (iv) $g(f(-2))$ (v) $f(f(3))$ (vi) $f(f(-2))$
8. Have every student at the table move one seat to their left. Explain whether this is or isn’t a “new” seating arrangement. Would your answer change if your group was seated at a round table? {P1}
9. Use DESMOS to graph the following functions and then use the graph to determine the domain and range of each function:
- $f(x) = -|x - 4|$ (this is a variation of the “absolute value” function) {13}
 - $g(x) = 5 + \sqrt{x + 3}$ (this is a variation of a “root” function) {8}
 - $h(x) = \frac{1}{x + 2} - 3$ (this is a variation of a “reciprocal” function) {15}

10. Each of the functions above represent “transformations” of “parent” functions. The “parent” functions were $f(x) = |x|$, $g(x) = \sqrt{x}$, $h(x) = \frac{1}{x}$. Graph the parent functions as well and then state how the parent functions had been “transformed” to produce the graphs described by the function equations in Q9. {11}
11. The highest and lowest marks awarded on an examination were 285 and 75. All the marks must be reduced so that the highest and lowest marks become 100 and 45. {5}
- Determine a linear function that will convert 285 to 100 and 75 to 45.
 - Use your function to determine the new marks that correspond to original marks of 95, 175, 215 and 255.
12. Equation writing: {5}
4. For each statement, define two variables and write a linear equation that models the sentence.
- The sum of two numbers is 12.
 - The sum of the width and length of a rectangle is 36 m.
 - The total value of nickels and dimes is 75¢.
 - Fives times some number less three times another number is ten.
 - A sum of money invested at 5% per year and another sum at 8% per year earned a total of \$150 in the first year.
 - The cost of the rental is \$50 plus \$5/h.
 - A rectangle is 2 m longer than it is wide.
 - The total value of \$5 bills and \$10 bills is \$135.
13. An investor has two investments. One investment, Investment A, earns 7% p.a. and the other investment, Investment B, earns 12% p.a. Last year, the original investment of \$14,000 earned \$1230. You are going to work toward determining how much money was invested in EACH of the two investments, A and B. {5,6}
- Define your variables and write an equation modeling this scenario.
 - Determine the x- and y-intercepts and explain their meaning.
 - Determine the slope and explain its meaning.
 - How much did the investor put into Investment B?
 - Would the investor earn more or less if she switched the amounts?
 - State the domain and range of the function(s) in this model. Explain.

HOMEWORK – House Values {5,13}

Mr Santowski has a summer cottage for which he paid \$120,000. Each year, the value of the house increases by \$8,000. For this question, you will model how the value of the cottage is related to the years during which Mr. S owns the cottage.

Questions:

- (a) Write an equation to help analyze this situation.
- (b) Determine the slope & state its meaning.
- (c) Determine the y-intercept and state its meaning.
- (d) When will my cottage double in value?
- (e) What will be the value of my cottage in 5 years time?
- (f) At what rate is the value of the house changing from year to year?
- (g) Explain the meaning of statement $V(10) = 200,000$
- (h) What is the x-intercept and what does it mean?
- (i) What would be a reasonable domain for this scenario? Why

CONCEPT EXTENSION Questions:

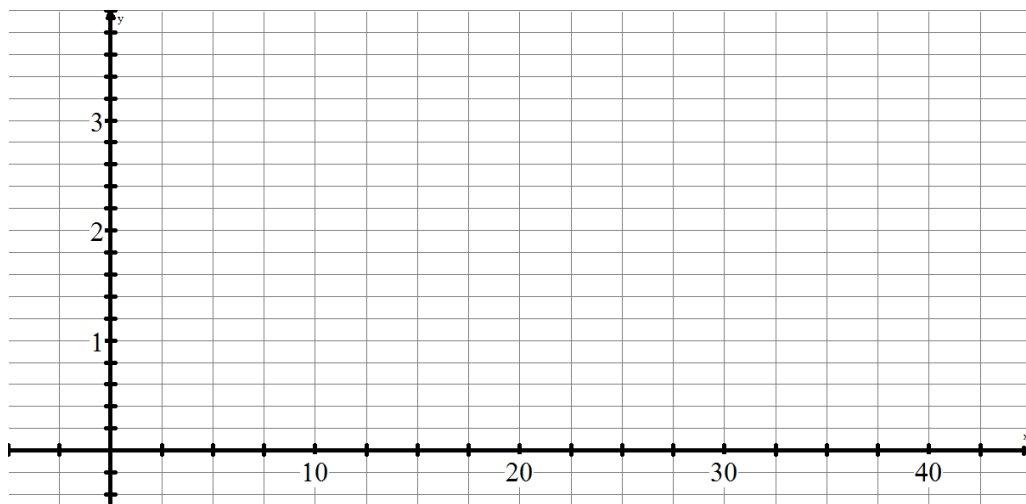
- (j) Write an equation for the following NEW scenario → After 10 years, the value of the cottage increases annually by \$12,000. Now evaluate $V(15)$ and re-determine the doubling time.
- (k) The \$8,000 increase in the first year is percent increase of $6\frac{2}{3}\%$. If the change in value were to be modeled with an EXPONENTIAL function, determine (i) a new equation, (ii) $V(10)$ and (iii) the “doubling time”

Homework: Modeling with Piecewise Linear Functions

A long distance calling plan charges \$1.29 for any call up to 20 minutes in length and 7 cents for each additional minute (or each part of a minute)

- What is the independent variable (input)? What would the domain be?
- What is the dependent variable (output)? What would the range be?
- Would you expect this relation to be a function? Why/why not?
- Evaluate $C(50)$ and interpret.
- Evaluate $\$2.41 = C(m)$ and interpret.
- To help draw a graph, complete the following table of values. Then graph this relation.

Time (min)	0	5	10	15	20	25	30	35	40
Cost (\$)									



Now, how would you write an equation for this relation?