Pre-Calculus – Unit 2 TEST – Polynomial Functions | Name:

1. A graph of a polynomial function has been provided for you. Answer the following questions about this polynomial function **and provide a brief justification/reasoning for your answer**. (15 marks)

a. The degree of this polynomial could be: (a) even   (b) odd   (c) cannot be determined

   WHY?

b. The multiplicity of the root x = 4 is: (a) 1   (b) 2   (c) 3   (d) 4   (e) cannot be determined

   WHY?

c. The sign of the leading coefficient is: (a) positive   (b) negative   (c) cannot be determin ed

   WHY?

d. Is x – 1 a factor of p(x)? _____.   WHY?

e. The solution(s) to p(x) = 0 is/are _____.   WHY?

f. If p(x) is divided by x – 2, then the remainder of the division is: _____.   WHY?

g. As $x \to \infty, p(x) \to$ _____.

h. On the interval of $\{x \in R \mid -2 \le x \le 7\}$, the interval(s) of decrease is/are:

2. Answer the following True or False questions and for each answer, **provide a brief justification/reasoning for your selection**: (6 marks)
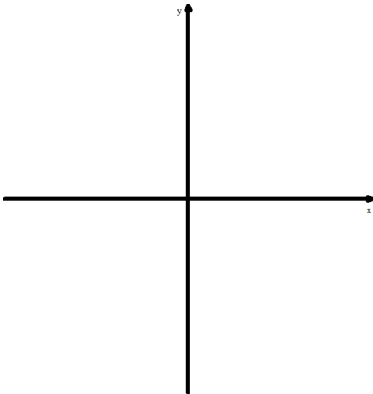
   a. All cubic functions MUST have at least one real root.   TRUE   FALSE

   b. All quartic functions MUST have at least one real root.   TRUE   FALSE
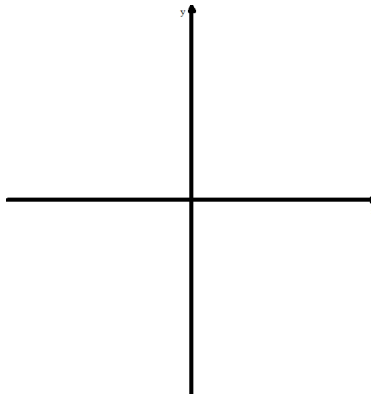
   c. The roots of a cubic function could be $x = 2, x = 1$ and $x = 1 + 3i$.   TRUE   FALSE

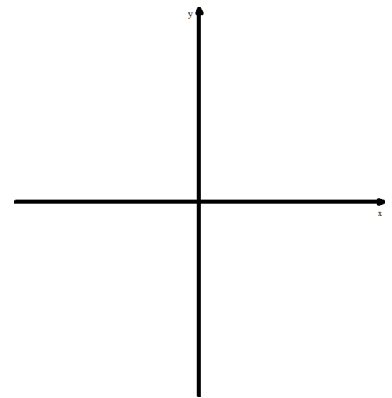3. Provide a sketch of a polynomial function that meets the following description: (7 marks)

a. A quartic polynomial where all 4 roots are real, but only has 3 x-intercepts	b. A quintic polynomial with only 3 distinct real roots and $x \to -\infty, p(x) \to \infty$.	c. The equation is $f(x) = (1-x)^5(x+2)$.

4. List all possible rational roots of the polynomial $g(x) = 4 - x + 3x^4$.

(2 marks)

5. Use appropriate algebraic analysis to show whether or not $(x + 1)$ is a factor of $p(x) = -2x^4 - 2x^3 + 38x^2 - 12x - 10$. Justify/explain your conclusion.

(3 marks)

6. Find the value of k so that when $x^3 - x^2 + kx + 11$ is divided by x - 2, the remainder is 3

(3 marks)

7. Given $p(x) = x^3 - 4x^2 + 1$, use the partial table of values to predict where the roots of p(x) would be. Explain your reasoning.

(3 marks)

$y = x^3 - 4x^2 + 1$

x	$x^3 - 4x^2 + 1$
−2	-23
−1	-4
0	1
1	-2
2	-7

8. Fully factor $f(x) = -6x^3 + 7x^2 - 1$ for $x \in R$

(6 marks)

9. For the cubic function $p(x) = x^3 - 5x^2 + 12x - 12$:

   a. Find all roots for $x \in C$.
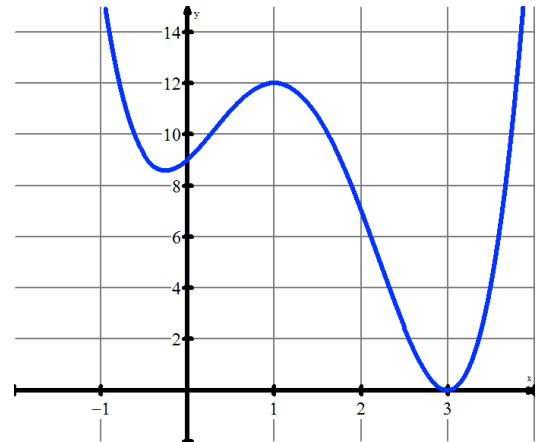
(8 marks)

   b. Write the polynomial as a product of linear factors.

   c. Determine the x-intercepts of $p(x)$.

10. For the quartic function $p(x) = x^4 - 5x^3 + 4x^2 + 3x + 9$, you have been provided with a graph. Show complete algebraic workings in order to:

(11 marks)

a. find all roots for $x \in C$.

b. then, write the polynomial as a product of linear factors.

BONUS QUESTION: In this question, you are given that 2 of the roots of a quartic polynomial are $2i$ and $3 - i$

(4 marks)

(i)   find the remaining roots,

(ii)   write the equation as a product of linear factors,

(iii)   and finally in standard form.the roots of a quartic

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Honor Pledge: I have neither given nor received any unauthorized aid on this assessment and will not share information about it until authorized to do so.

Signature_____

PART A – CALCULATOR INACTIVE QUESTIONS (35 minutes)

Show complete algebraic solutions that are clear, concise, and properly presented

1. In this question, you will analyze and sketch the rational function $f(x) = \dfrac{4x - 5}{x - 2}$.

(a) State the equations of the vertical and horizontal asymptotes.

(2 marks)

(b) State the domain and range of $y = f(x)$.

(2 marks)

(c) Determine the x- and y-intercepts of $y = f(x)$.

(2 marks)

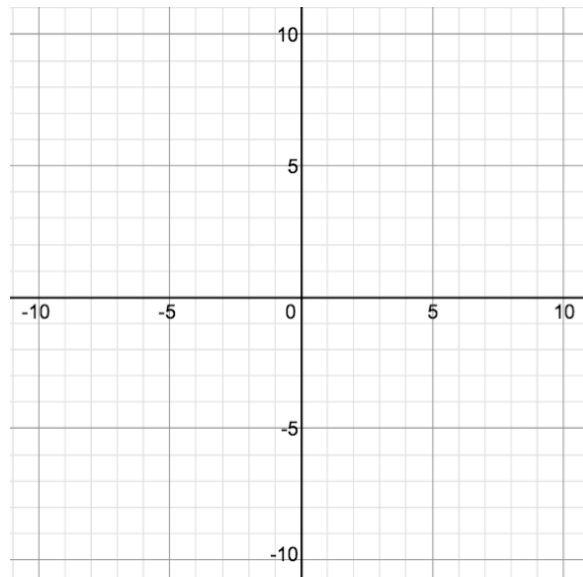(d) Solve the equation $f(x) = 3$.

(2 marks)

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(e) The function $y = f(x)$ is a transformation of $y = \dfrac{1}{x}$. Rewrite the equation $f(x) = \dfrac{4x - 5}{x - 2}$ in the form

of $f(x) = \dfrac{a}{b(x - c)} + d$ and identify the transformations of $y = \dfrac{1}{x}$.

(5 marks)

(f) Graph $f(x) = \dfrac{4x - 5}{x - 2}$ on the grid provided,

including ALL KEY features found in the previous steps.

(3 marks)

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2. In this question, you will analyze and sketch the rational function $R(x) = \dfrac{-2x^2 + 11x + 6}{x - 4}$.

| (a) Factor the numerator. (2 marks)	(b) Rewrite the equation for the rational function $y = R(x)$ in factored form. (1 mark)
(c) Determine the asymptotes (vertical & non-vertical) of $y = R(x)$ and show the analysis that leads to your answers. (3 marks)	(d) Determine the values of the intercepts. (2 marks)

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(e) Determine the values of at least 2 other points to help you in sketching $R(x) = \dfrac{-2x^2 + 11x + 6}{x - 4}$

(2 marks)

(f) Sketch $y = R(x)$ on the grid provided. (4)
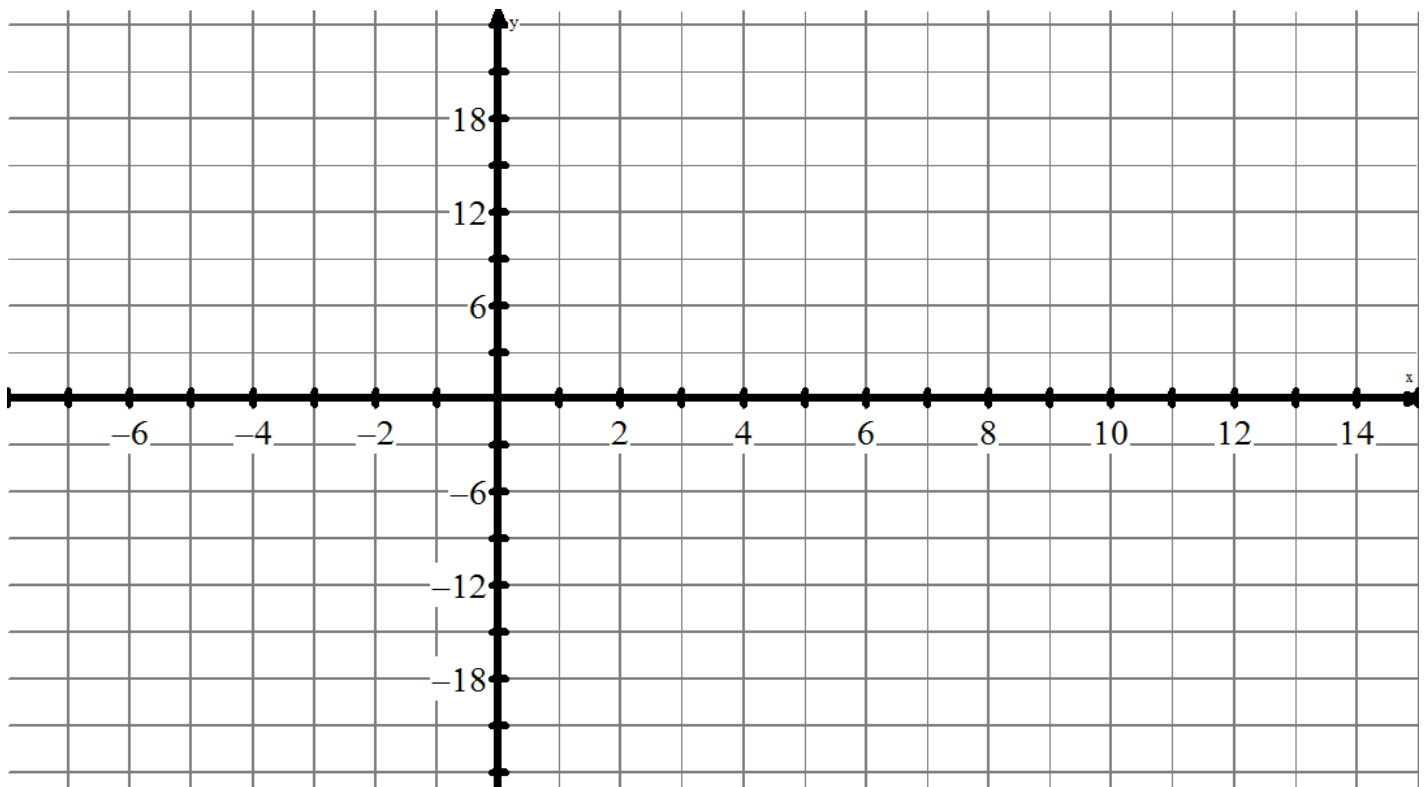
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PART B – CALCULATOR ACTIVE QUESTIONS (35 minutes)

This section is calculator ACTIVE and *__all solutions/work will be done on a separate sheet of paper__*. However, some work may be done quicker and more easily using algebraic methods. In either case, show or present supporting "evidence" to your solutions, or briefly describe what you did on the calculator (point form acceptable).

3. Use the view window x_{min} = -10, x_{max} = 20, y_{min} = -10, y_{max} = 20 to graph and view the rational

   function $H(x) = \dfrac{x^3 + 4x^2 + x - 6}{x^2 - 4}$:   (14 marks)

   a. What are the factors of the quadratic function in the denominator? (2 mark)

   b. Determine the zeroes of the rational function. (2 marks)

   c. What are the factors of the cubic function in the numerator? Show/Explain the analysis that led to your answer. (4 marks)

   d. Find the **coordinates** of any holes. (2 marks)

   e. Determine the equation of the slant asymptote. Show/explain the analysis that leads to your equation. (2 marks)

   f. Determine the end behavior of H(x). Use proper notation in expressing the end behavior. (2 marks)

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4. Use your graphing calculator and the view window x_{min} = -20, x_{max} = 20, y_{min} = -200, y_{max} = 400 to

   graph and view the rational function $H(x) = \dfrac{x^4 + 2x^3 - 4x^2 + 8x - 32}{x^2 - x - 12}$. This function can be factored

   to $H(x) = \dfrac{(x^2 + 4)(x - 2)(x + 4)}{(x - 4)(x + 3)}$.   (11 marks)

   a. Explain how you would use your calculator to verify that $H(7) = 97\dfrac{1}{6}$. (1 mark)

   b. Evaluate $H(-3.5)$. Round to one decimal place. (1 mark)

   c. Describe the asymptotic behaviour of $H(x)$ at $x = -3$. (2 marks)

   d. Provide a sketch of what you see on your calculator, given the window settings. (5 marks)

   e. Use the graph, data table and algebraic analysis as necessary to predict the end behaviour
      of $H(x)$. (2 marks)

5. Graph the function $g(x) = \dfrac{2x}{9 + x^2}$ on your calculator:

   a. Determine the range of the function and explain how you determined this from your
      graphing calculator. (3 marks)

   b. Explain WHY the function has no vertical asymptotes. (1 mark)

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