Paper 2 – CALCULATOR ACTIVE

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Section B

Do NOT write solutions on this page. Answer all questions on the answer sheets provided.

1. Reem is playing baseball in her PE class. When she hits the baseball, its height, h, in meters, t seconds since being hit, is modelled by the equation $h(t) = -4.9(t-3)^2 + 45$.

(15 marks)

a. What is the height of the ball at the instant the ball is hit?

(2)

b. Find the maximum height of the ball and the time when this height is reached.

(2)

c. The ball hits the wall of the Middle School building 5.9 seconds after it gets hit. How high up the wall does the ball hit?

(2)

d. Determine the value(s) of the zeroes and interpret their meaning.

(3)

e. State the domain and range for this function and explain your reasonings.

(3)

f. Determine the equation of the inverse **function** and explain **why we could use** this inverse relationship.

(3)

2. The function below models the expenses for the MathiMagics company. In this model, x represents the number of textbooks sold (in thousands) and $E(x)$ represents the expenses, in thousands of Euros.

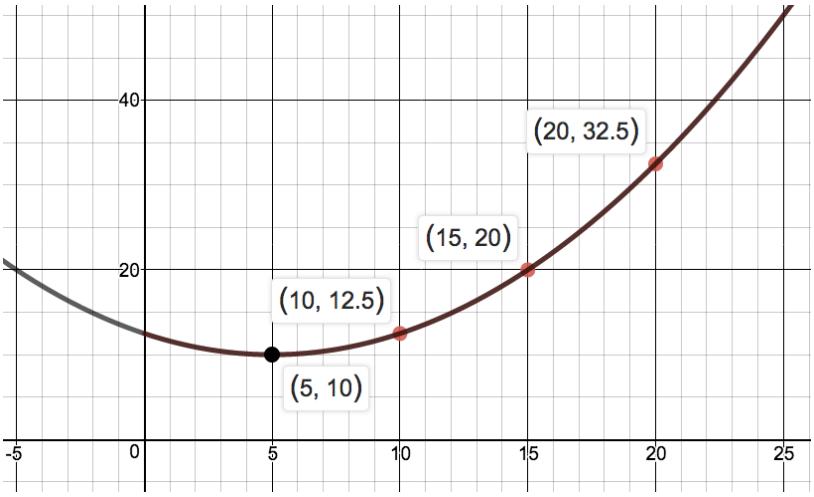
(11 marks)

a. Determine the equation of the function and express the equation in vertex form.

(3)

b. Express the equation in standard form.

(2)



The revenue of the MathiMagics company are modelled by the function $R(x) = 5 + 1.75x$, where x represents the number of textbooks sold (in thousands) and $R(x)$ represents the expenses, in thousands of Euros.

c. Determine the number of textbooks that must be sold so that the MathiMagics company **breaks even**.

(3)

d. How many books need to be sold so that MathiMagics **optimizes its profits**?

(2)

e. What is the optimal profit?

(1)

3. Almost a perfect parabolic arch, the opening of Rainbow Bridge is 309 feet and extends 275 feet across Bridge Creek. It is considered to be the largest natural bridge in the world.

(4 marks)

Write an equation that can be used to model the Rainbow Bridge (near Lake Powell in Utah, USA)

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Section B

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1. To make extra money in the summer, Mr S runs a tour company called Mr. Math's Magical Mathematical Mystery Tours. This company prepares summer adventures for high school students. The company's **profit per student** (P), is represented by the quadratic equation $P(n) = -0.6n^2 + 36n - 405$, where n represents the number of students taking the tour.

(9 marks)

a. Determine the maximum profit of the company. Interpret your answer.

(2)

b. Determine the break-even points of this company.

(2)

Last year, the company decided to restructure and now the new profit per student equation is $P(n) = -0.2n^2 + 15n - 200$.

c. Determine the maximum profit of the company now.

(1)

d. Determine the break-even points of this company now.

(2)

e. Is this a "successful" restructuring? Explain why or why not. You must give TWO REASONS when you justify your answer. (HINT: include some specific numbers in your explanation.) (2)

2. Remember the water balloon projectile competition at Spirit Week? Of course, Mr. S decided to model the Sophomore class' launch with the following equation $h(d) = -0.1d^2 + 2.4d + 3.1$. In this equation, h is the balloon's **height** in meters and d is the **horizontal distance** from the launching position, also in meters.

(10 marks)

a. How far horizontally has the balloon travelled when its height is 10 m above the ground?

(3)

b. What is the maximum height of the water balloon?

(2)

c. If the old MS building is 35 m away from your launch point, does the balloon from the Sophomore class hit the MS building?

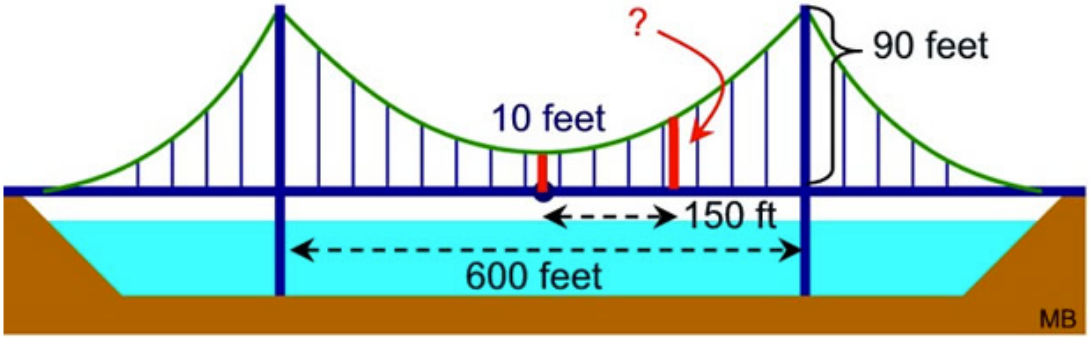
(2)

d. To make things easier (given your poor performances in the contest), Mr. S moves the target ring (remember, it's on the grass of the MS field) to a position 25 m away from your launch point. Assuming the **same initial launch height** of the balloon, write an equation of one possible trajectory for your the balloon, so that it hits this new target.

(3)

3. The cables of a suspended-deck suspension bridge are in the shape of a parabola. The pillars supporting the cable are 600 feet apart and rise 90 feet above the road. The lowest height of the cable, which is 10 feet above the road, is reached halfway between the pillars.

(6 marks)



a. Determine an equation that could be used to model the cables on this suspension bridge.

(4)

b. Hence, or otherwise, determine the height of the cable from the road at a point 150 feet (horizontally) from the center of the bridge?

(2)

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SECTION A

1. A polynomial is given as $p(x) = 2x^4 - 5x^3 - 20x^2 + 5$. Find the domain interval(s) in which the values of $p(x)$ are greater than 0 – that is solve $p(x) > 0$. (You may wish to show a quick sketch to support your answer).

(3)

2. Given the polynomial $p(x) = 3x^3 - 6x^2 + x + 2$.

   a. Write the equation $p(x) = 3x^3 - 6x^2 + x + 2$ in factored form. If necessary, round numbers to two decimal places.

(3)

   b. This polynomial can also be rewritten as a product of linear function $(x + D)$ with a quadratic function $Ax^2 + Bx + C$, where A,B,C and D are each integers. Determine the values of A, B, and C.

(2)

Section B

Do NOT write solutions on this page. Answer all questions on the answer sheets provided.

1. Near my hometown in Canada, the government created a federal game park in the year 1920. In this park, both deer and moose were introduced in the first year of 1920 and their populations (in hundreds) over the years are modeled by the following functions. The number of deer, $D(t)$, in hundreds, since 1920 is modeled by $D(t) = \frac{800 + 64t}{20 + 0.8t}$ and the number of moose, $M(t)$ in hundreds, since 1920 is modeled by $M(t) = \frac{3}{200{,}000}t^4 - 0.00316t^3 + 0.205t^2 - 4.2t + 70$.

(18 marks)

A graph of the functions (from DESMOS) has been provided for you, to make sure that your TI-84 graphs are programmed correctly

a. How many deer and moose were initially put in the park in 1920? (2)

b. You should find that $M(80) \approx 48$ Explain what $M(80) \approx 48$ means in the context of this problem. (2)

c. State a reasonable domain and range for this problem. Explain your reasoning. (3)

d. Find the number of deer in 1967 (the year of Canada's centennial celebration!!!) (2)

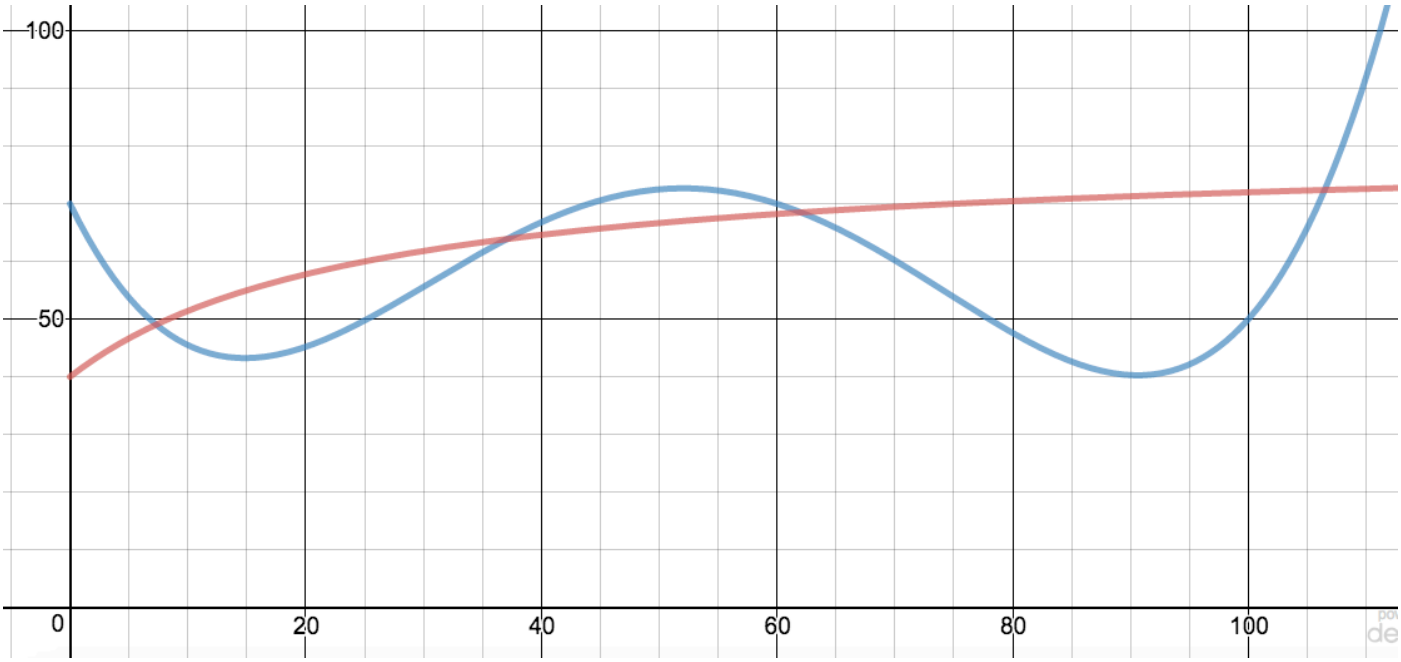
e. Find the horizontal asymptote of the graph of $D(t)$. (1)

f. In what years is the moose population increasing? (3)

g. Determine the minimum number of deer and the minimum number of moose in this park. (2)

h. According to the model, what is the largest possible deer population? (1)

i. In what years does the deer populations exceed the moose population? (2)


100
50
0
20
40
60
80
100

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Section A

Answer all questions in the spaces provided.

1. Find the domain interval(s) in which $f(x) = -2x^4 + 5x^3 + 15x^2 - 5$ meets **BOTH** these conditions: the polynomial $f(x)$ must be less than 0 **AND** $f(x)$ must be increasing as well. (You may wish to show a quick sketch to support your answer).

(5 marks)

Section B

Do NOT write solutions on this page. Answer all questions on the answer sheets provided.

1. Given the rational function $r(x) = \dfrac{3x-4}{2x-5}$ and the cubic function $p(x) = 3x^3 - 6x^2 + x + 2$:

(9 marks)

a. Rewrite $p(x)$ in factored form.

(3)

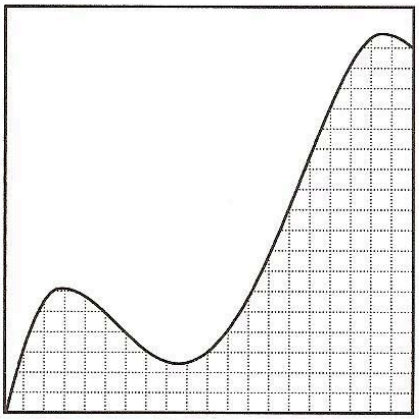
b. This polynomial, $p(x)$, can also be rewritten as a product of linear function $(x + D)$ with a quadratic function $Ax^2 + Bx + C$, where A,B,C and D are each integers. Determine the values of A, B, and C.

(2)

c. Find the domain interval(s) in which $\dfrac{3x-4}{2x-5} < 3x^3 - 6x^2 + x + 2$. Include a diagram/sketch in your solution.

(4)

2. Next weekend, Fred, Elena, Michael, and Diane will visit an amusement park and will ride on a roller coaster. Elena snaps a picture of part of the coaster from the park entrance. The diagram at the right represents this part of the coaster.



a. Given the picture, do you think quadratic, cubic, or quartic function would be the best model for this part of the coaster? Clearly explain your choice.

(2)

The part of the coaster captured by Elena on film is modeled by the function $h(t) = -0.2t^4 + 4t^3 - 24t^2 + 48t$

b. Use your graphing calculator to approximate relative maxima and minima of this function. Round your answers to three decimal places.

(3)

c. Identify the domain interval(s) in which the roller coaster's height is increasing.

(2)

d. Clearly describe the end behavior of this function and the reason for this behavior.

(2)

e. When does the roller coaster get back to "ground level"?

(2)

f. Suppose that this coaster is a 2-minute ride. Do you think that $h(t) = -0.2t^4 + 4t^3 - 24t^2 + 48t$ is a good model for the height of the coaster throughout the ride? Clearly explain and justify your response.

(2)

g. Elena wants to find the time(s) when the height of the coaster is 25m. Find these time(s).

(2)

(Total 15 marks)

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SECTION A

Answer all questions in the spaces provided.

1. In ΔABC, ∠A = 32°, ∠C = 81° and side BC is 24.1 cm.

(6 marks)

a. Prepare a sketch of this triangle and correctly label the given information in your diagram. (1)

b. Find the measure of side AC. (3)

c. Determine the area of ΔABC. (2)

2. An angle, θ, is drawn in standard position and it is known that $\cos(\theta) = -\frac{7}{\sqrt{53}}$, where $0° \leq \theta \leq 360°$.

(6 marks)

a. Sherif tells you that the principal angle θ lies in EITHER quadrant 2 or quadrant 3. How does he know this? (To help explain Sherif's reasoning, draw a diagram of the angle θ in standard position).

(2)

b. Calculate the values of the two possible principal angles of θ, to the nearest degree.

(2)

c. State the EXACT value of the tangent ratio of the angle θ, if $90° \leq \theta \leq 180°$. (HINT: express final answer as a fraction)

(2)

3. The function $J(t) = 4.25\sin(6.92t) + 5.8$ is used to model the number of weekly job applications (in hundreds) for GOOOGLE MATH Company, where t is time in weeks since Jun 1st, 2014.

(6 marks)

a. Interpret the meaning of $J(46) = 2.97$.

(2)

b. State a domain for this function. Explain your reasoning.

(2)

c. Since June 1st, 2016, for what values for t have their been **at least** 800 weekly job applicants?

(2)

SECTION B

Do NOT write solutions on this page. Answer all questions on the answer sheets provided.

Mr. S and Mr D are sailing in the Philippines and have seen an island upon which are two mountains. They would like to determine the distance from one end of the mountain to the other end of the second mountain, so that they can determine the width of the island.

So, Mr D has used GEOGEBRA to make the following diagram of the island and the 2 mountains.

From this work in GEOGEBRA, the relevant measures are:

In ΔEFG, side EF = 4.52 units side FG = 2.04 units ∠EFG = 137°	In between the two mountains: ∠FGH = 128°	In ΔGHI, side HI = 3.84 units ∠GHI = 131°

1. To determine the width of the island (side EI, from point E to point I), answer these questions:

(12 marks)

   a. Label all the relevant information on the diagram. (1)

   b. In ΔEFG, determine the length of side EG. (3)

   c. In ΔEFG, determine the measure of ∠FGE. (3)

   d. In ΔGHI, determine the measure of side GI. (3)

   e. Given your answers in Q(b) and Q(d), determine the length of side EI. (1)

   f. If one unit on our diagram represents 250 m, estimate the length of the island. (1)

2. Mr. S and Mr. D have now landed their sailboat on the island. At 2:00 p.m. on May 19, the tide is in (i.e., high tide ➔ the water is at its deepest). At that time we find that the depth of the water at the end of the breakwater is 15 meters. At 8:00 p.m. the same day when the tide is out (low tide), we find that the depth of the water is 11 meters. Assume that the depth of the water varies **sinusoidally** with time.

(12 marks)

To help you write an equation, we have started the following data table

Time	2:00 pm	8:00 pm	??	??	??
Water Depth (m)	15 m	11 m	??	??	??

   a. Hence, or otherwise, derive an equation expressing depth in terms of the number of hours that have elapsed since 2:00 pm on May 19. (4)

   b. Use your mathematical model to predict the depth of the water at (5)

      i. 4:00 p.m. on May 19,

      ii. 7:00 a.m. on May 20,

      iii. 4:36 p.m. on May 20.

   c. What is the earliest time on May 22 that the water will be at 12.7 meters deep? (3)

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SECTION A

Answer all questions in the spaces provided.

1. The function $J(t) = 5.25\sin(6.92t) + 7.8$ is used to model the number of weekly job applications **(in hundreds)** for the GOGEL MATH Company, where t is the time **in weeks from Jun 1st, 2014 until today's date (late May of 2017)**.

(6 marks)

a. Interpret the meaning of $J(29) = 5.95$.

(2)

b. State a domain for this function. Explain your reasoning.

(2)

c. Since June 1st, 2016, for what values of t have there been **at most** 1000 weekly job applications?

(2)

2. In ΔABC, side AB is 17 cm, side AC is 20 cm and ∠BAC = 70°.

(6 marks)

a. Prepare a sketch of this triangle and correctly label the given information in your diagram.

(1)

b. Find the **perimeter** of this triangle.

(3)

c. Determine the area of ΔABC.

(2)

3. An angle, θ, is drawn in standard position and its terminal arm goes through the point A(4,-6).

(8 marks)

a. Draw a diagram of this angle θ and show the principal angle and the related acute angle.

(3)

b. Explain **why** the value of the cosine ratio of this angle is: $\cos(\theta) = -\frac{6}{\sqrt{52}}$.

(2)

c. Determine the value of the principal angle of θ, to the nearest degree.

(1)

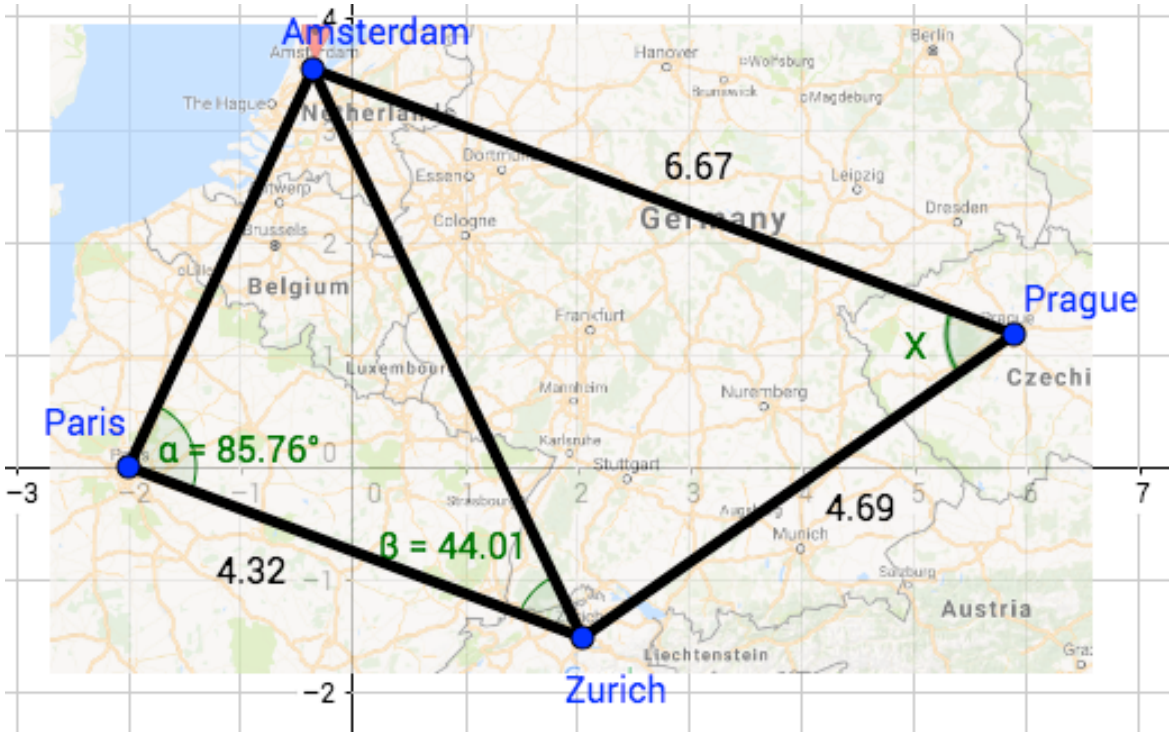
d. State the measures of two other angles whose terminal arms would also go through the point A(4,-6). Explain/show your thinking.

(2)

SECTION B

Do NOT write solutions on this page. Answer all questions on the answer sheets provided.

1. Mr. S and Mr. D are vacationing in Europe this summer and need to know some information to help with their travel plans. We used GEOGEBRA to help us work out distances, so here is a diagram showing the cities of Amsterdam, Prague, Paris and Zurich. Major flight paths are shown between the cities and shown in the form of 2 triangles below.



a. To solve for the required angle (at Prague - labelled as x in the diagram):

(6)

i. In ΔAZPa to find side AZ

ii. Then, in ΔAZPr, find the measure of ∠Pr (the angle at Prague, marked as X in the diagram)

b. To help you find the direct distance from Prague to Paris,

(6)

i. Show that the **entire** angle at Amsterdam (∠PaAPr) is 94.1°.

ii. Hence, or otherwise, find the distance between Paris and Prague.

2. Mr. S and Mr. D are vacationing in the city of Prague (Czech Republic). The number of hours (and minutes) of daylight on the 15th of each month, beginning in January, is shown for Prague.

(11 marks)

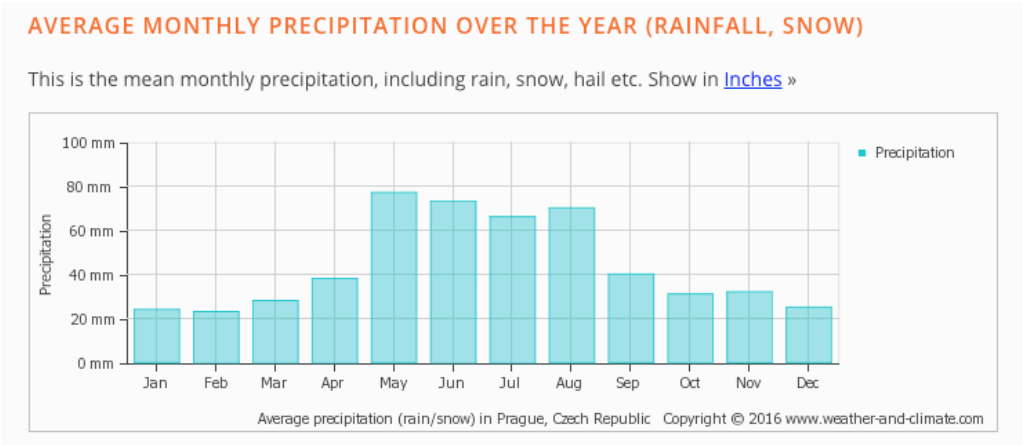
Month	Hours of Daylight
1	8:30
2	10:07
3	11:48
4	13:44
5	15:04
6	16:21
7	15:38
8	14:33
9	12:42
10	10:47
11	9:06
12	8:05

a. Explain why Mr S changes the time of 8:30 hours into 8.5 hours. (1)

b. Use your curve of best fit and the data table to write a model for the number of hours of daylight as a function of the month (where m = 1 represents January). (3)

c. Use your **model** to predict the number of hours of daylight on July 31st. (2)

Likewise, the following bar graph gives us data for the amount of rainfall in Prague.



Mr. S will use $R(m) = 27\sin(30(m-4)) + 50$ to model the amount of rainfall in Prague.

d. To help justify that the model is **appropriate**, explain why the parameters of A = 27 and D = 50 are **appropriate and correct** in this model. (2)

e. To help justify that Mr. S's model is **inappropriate**, use your model to predict how much rain we could expect in Prague in May (month 5). State the value for the rainfall from the chart for the month of May. (2)

f. Finally, is Mr. S's model appropriate? Justify your answer. (1)

Scatterplot for Question #2(a)