

**Paper 1 - CALCULATOR INACTIVE**

1. In this question, you will work with the linear system defined by the equations  $y - 5 = -\frac{1}{4}(x - 8)$  and  $3x - 5y + 90 = 0$ .
- SHOW THAT  $y - 5 = -\frac{1}{4}(x - 8)$  is the equation  $y = -\frac{1}{4}x + 7$  in slope-intercept form.
  - SHOW THAT  $3x - 5y + 90 = 0$  is the equation  $y = \frac{3}{5}x + 18$  in slope intercept form'
  - Use your graphing calculator to determine a GRAPHIC solution to this linear system
  - State APPROPRIATE window settings that could be used to determine the intersection point.
  - Provide a PROPERLY LABELED SKETCH of your linear system on the grid provided. **Label each function & the intersection point.**

2. State the RANGE of the linear function  $f(x) = 4 - \frac{2}{3}x$  given the domain of  $\{x \in \mathbb{R} \mid -9 \leq x < 12\}$ .  
Show/explain the key steps of your solution.

3. State the DOMAIN of the linear function  $3x - 2y = 14$ , given the range of  $\{y \in \mathbb{R} \mid -10 < y \leq 5\}$ .  
Show/explain the key steps of your solution.

4. ALGEBRAICALLY, verify whether  $(3,1)$  is the solution to the linear system  $6x - 2y = 16$  and  $y = -\frac{4}{3}x + 2$ .

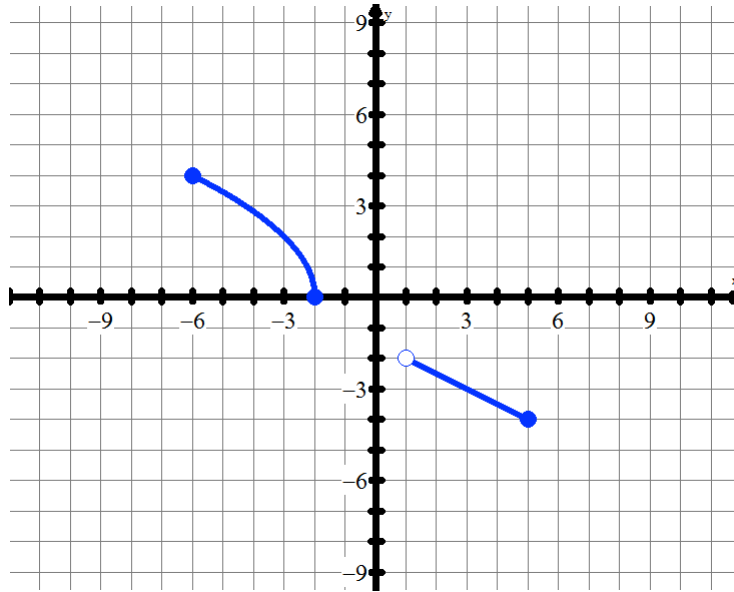
5. Ms. A inherited a sum of money from a relative. She deposits some of the money at 11% p.a. in her ETrade account. Then she deposits \$5000 less than that amount at 8% p.a. in her Bank of America account. She earns \$3576 in interest per year. How much did she invest at each rate?

- One equation is  $y = x - 5000$ . EXPLAIN WHY this equation is correct
- Another equation is  $0.11x + 0.08y = 3576$ . EXPLAIN WHY this equation is correct
- Explain what the variables  $x$  and  $y$  mean in context of this problem
- Use your graphing calculator (in any way) to solve this system. State your solution

System:  $y = x - 5000$   
 $0.11x + 0.08y = 3576$

- Provide an ALGEBRAIC VERIFICATION (using any method) to the linear system you defined to answer this word problem
- State a domain and range that would be reasonable for this context. Explain the reasoning of your domain and range

6. In this question, you will graph an inverse of a function.
- Explain how to draw the graph of an inverse of any function.
  - Graph the line  $y = x$  on the graph provided.
  - Given your answers to these previous 2 questions, graph the inverse of this function on the graph provided. Show supporting work (if necessary).



7. You are going to work with the linear function  $f(x) = -\frac{2}{5}x + 3$ . As you work through and answer the following questions, **ALWAYS** show your work OR explain your reasoning.
- Would the point  $(20, -5)$  be on the graph of this linear function? Show necessary work.
  - What is the x-intercept of the **inverse** function?
  - Determine the equation of the inverse of  $f(x) = -\frac{2}{5}x + 3$ .
  - Malak **PREDICTS** the value of  $f^{-1}(-5)$  to be 20. Explain the reasoning/thinking for her prediction.
  - Now, use your equation for  $f^{-1}(x)$  from Q(c) to evaluate  $f^{-1}(-5)$ . Show your work.
  - What do your answers from Q2(d) and Q(e) mean about your work in Q2(c)?
8. Mr. Santowski is studying the relationship between the number of hours that students spend studying,  $h$ , and the marks they get on their December Semester exams,  $M$ . Included below is a sample data set from his study.

Hours Studied, $h$	2	5	3.5	1	8	3	9	12
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Exam Mark (%), $M$	68	61	81	48	85	75	93	88
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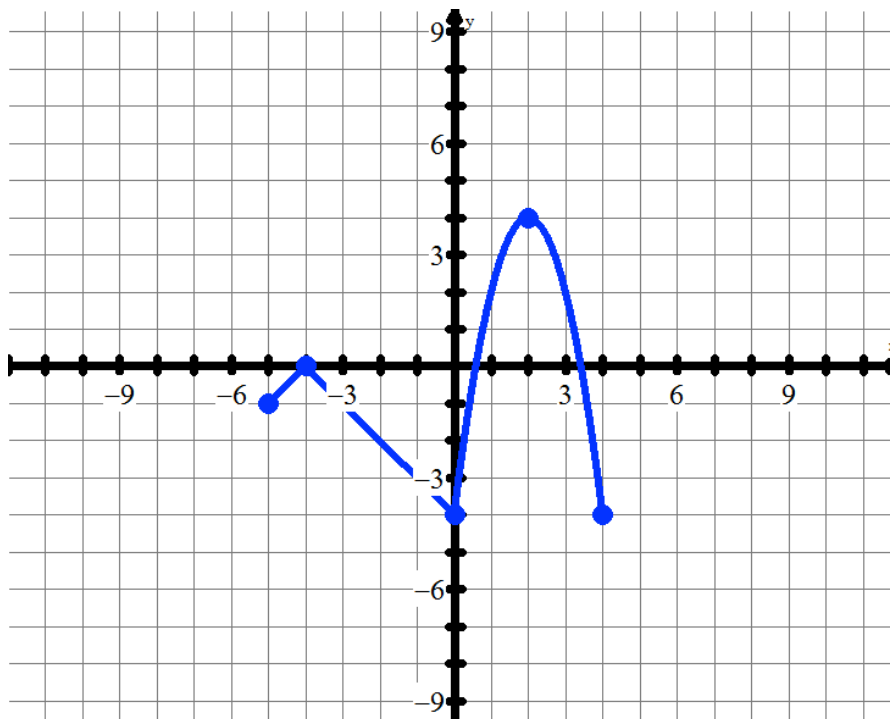
- a. What does the domain of this relation represent?
- b. Does this relation HAVE to be a function? Why or why not?
- c. Evaluate  $M(3.5)$  and explain what it means in the context of this problem.
- d. Solve  $M(h) = 68$  and explain what it means in this context.
- e. After marking all student exams, Ms. A suggests that we apply a transformation to this relation and suggests an equation:  $y = M(h) + 5$ . Explain what this transformation would do to the graph of the relation and to the student marks.

Omar wonders about the **meaning** of the inverse of this relation.

- f. What is the **meaning** of the **range** of this inverse relation?
- g. **ESTIMATE** a reasonable value of  $M^{-1}(70)$  and **EXPLAIN** what would the result mean?
- h. Given your answers to the previous 2 questions, how would students make use of the equation for this inverse? (What is the point of this inverse relation? (HINT: Think about your answers for Q(f) and Q(g) )

9. You have been provided with a graph of a piecewise function,  $y = f(x)$ , which is pictured below.

- a. Given the NEW equation of  $y = 2f(x - 5) - 1$ , list what transformations will be applied to  $y = f(x)$
- b. Now apply the transformations to  $y = f(x)$  and sketch the new transformed function. Show all necessary work to support your sketch of the new function.



10. Characteristics of Functions: You will produce a sketch of a function that meets the following requirements:

- The domain is to be  $\{x \in R \mid x < 5\}$ .
- The function is to be increasing ONLY on the interval of  $\{x \in R \mid -3 < x < 2\}$ .
- The absolute minimum point must be at  $(-3, -12)$ .
- The function must have an asymptote.

- a. Label your KEY POINTS.
- b. Explain why the range of your function is  $\{y \in R \mid y \geq -12\}$ .
- c. EXPLAIN what would you have to do with your KEY POINTS in order to graph the new function
- d. Is inverse of your function also a function? Explain why or why not.

$$y = -\frac{1}{2}f(x) + 4.$$

11. In this unit, you were introduced to three types of transformations: **vertical/horizontal translations**, **vertical/horizontal dilations** (stretches or compressions) as well as **reflections**. In these multiple choice questions, select the choice(s) that best answer these questions concerning generalizations about transformations.

- a. Which transformation type(s) will NOT change the ORIGINAL x-intercepts of a function?
  - i. Vertical Translation
  - ii. Horizontal Translation
  - iii. Vertical Dilation
  - iv. Horizontal Dilation
  - v. Reflection
  - vi. Cannot be predicted. Not enough information.
- b. Which transformation(s) will cause maximums to turn into minimums?
  - i. Vertical Translation
  - ii. Horizontal Translation
  - iii. Vertical Dilation
  - iv. Horizontal Dilation
  - v. Reflection
  - vi. Cannot be predicted. Not enough information.
- c. Which transformation type(s) will cause functions to inverse?
  - i. Vertical Translation
  - ii. Horizontal Translation
  - iii. Vertical Dilation
  - iv. Horizontal Dilation
  - v. Reflection
  - vi. Cannot be predicted. Not enough information.
- d. Which transformation type(s) will cause functions to change their domain?
  - i. Vertical Translation
  - ii. Horizontal Translation
  - iii. Vertical Dilation
  - iv. Horizontal Dilation
  - v. Reflection
  - vi. Cannot be predicted. Not enough information.

12. Solve the following equations, providing EXACT solutions when necessary.

a.  $\log_2\left(\frac{1}{32}\right) = 1 - \frac{x}{2}$

b.  $\log_{x+2}\left(\frac{1}{36}\right) = -2$

c.  $4(e^{x-2}) - 6 = 14$

d.  $(\sqrt{2})^{-x} = 2^{\frac{5}{2}x+3}$

13. There are three towns close to the farm where Mr. S grew up back home in Canada. The populations of each town since 1975 are modelled as follows (where t is time in years since 1975):

Avonmore: $A(t) = 12,000 + 24,000e^{-0.025t}$	Berwick: $B(t) = 12,000(1.02)^t$	Crysler: $C(t) = \frac{45,000}{1 + 3.5e^{-0.07t}}$
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a. Determine the population of Avonmore and Chrysler in 1975.

(i) Avonmore in 1975 :

(ii) Chrysler in 1975 :

b. At what annual rate are the populations of Avonmore and Berwick changing?

(i) Avonmore Rate:

(ii) Berwick Rate:

c. Use your knowledge of “end behaviours” to predict the long term population of each of the three towns.

(i) Avonmore:

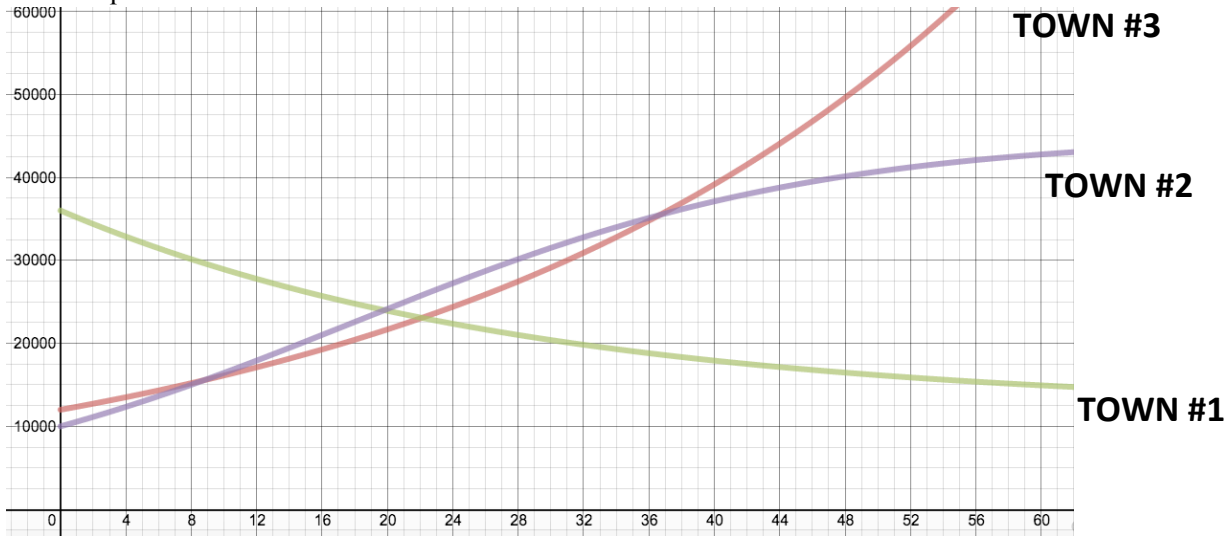
(ii) Berwick:

(iii) Chrysler:

14. There are three towns close to the farm where Mr. S grew up back home in Canada. The population of each town since 1975 are modelled as follows (where  $t$  is time in years since 1975):

Avonmore: $A(t) = 12000 + 24000e^{-0.025t}$	Berwick: $B(t) = 12000(1.02)^t$	Crysler: $C(t) = \frac{45000}{1 + 3.5e^{-0.07t}}$
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Now here is a graph of the population of the three towns. Use the graph to answer the following questions:



- a. Use the graph to estimate the year(s) in which the population of Berwick exceeds the population of Avonmore.
  - b. Use the graph to estimate a solution to the inequality  $B(t) > C(t) > A(t)$  and explain what the solution means.
15. The IM3 H block class is analyzing the function  $f(x) = \frac{x^2 - x - 6}{e^{0.5x}}$ . They begin by making PREDICTIONS, without the benefit of a calculator!!
- a. Marwan is looking for the y-intercept and so he evaluates  $f(0)$ . Where is the y-intercept? (Show supporting evidence).
  - b. Andrew is asked to evaluate  $f(4)$ . He expresses his answer as  $f(4) = 10e^{-4}$ . Explain/show whether Andrew did this correctly or not.
  - c. Rafa is looking for the x-intercept(s) of  $f(x) = \frac{x^2 - x - 6}{e^{2x}}$ , so he solves the equation  $f(x) = 0$ . He gets 2 answers. One answer is  $x = 3$ . State the other x-intercept. (Show supporting evidence).

**Paper 2 – CALCULATOR ACTIVE**

1. In this question, you are being asked to predict the number of solutions for each of the linear systems. CIRCLE the answer from the options listed and then EXPLAIN why you selected the answer you did.

a. You are given the linear system with these two equations:  $-3x + 2y = 4$  and  $12x + ay = b$ . Determine the values of  $a$  and  $b$  such that this system has no solutions. Explain how the graphs of the equations will be related

b. The two linear equations are:  $y = -3x + 5$  and  $7x + 2y = 12$ . This system would have:

- (i) no solutions
- (ii) one solution
- (iii) infinitely many solutions
- (iv) there is not enough information to help make the prediction

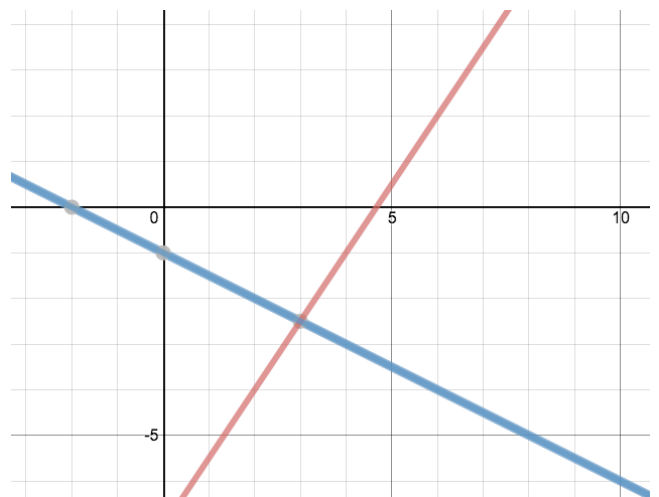
c. Explain/show the reasoning behind your choice.

2. This question concerns the linear system as graphed in the diagram provided. Use the graph to answer the following questions:

a. Estimate the solution of this system (in other words, solve  $f(x) = g(x)$ ).

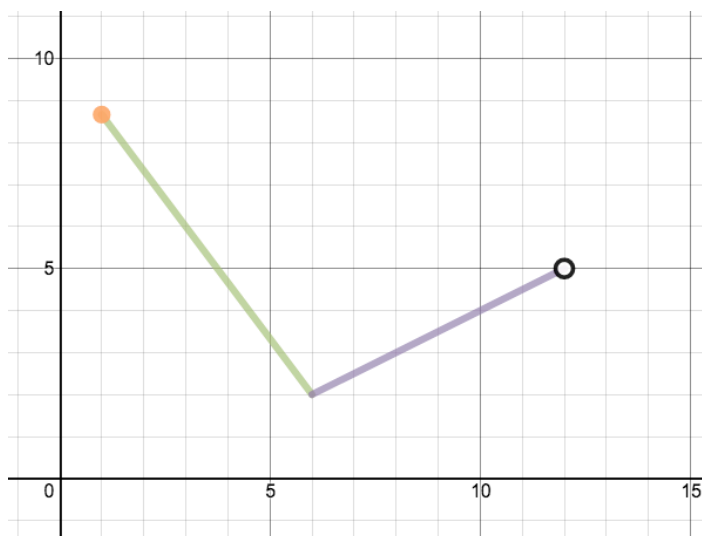
b. Mr. S. thinks the equation of  $f(x)$  is  $3x - 2y - 14 = 0$ . Is he correct? Show/explain your reasoning.

c. The equation of Line 2 is  $y - 1 = -\frac{1}{2}(x + 4)$ . Use an algebraic method of your choosing to determine the exact point at which the two functions intersect.



3. Here is a graph of piecewise function,  $y = f(x)$ .

- State the domain and range of the function as graphed.
- State one point on Line 1 of  $y = f(x)$
- Evaluate  $f(3)$ .
- Solve for  $x$  if  $4 = f(x)$ .
- Calculate the slope of the Line #1
- Write the equation of Line #1 in standard form.
- Write the equation of this piecewise function



4. Ms. Knox is performing a science experiment. She puts 1500 mL of water into a container, but the container has a hole in it, so the water is leaking at a rate of 30 mL per minute.
- How much water remains in the container after 10 minutes?

At the same time, Ms. Brookhart is working on a similar experiment. She puts 2000 mL of water into beaker, but it also has a hole! The water in this beaker **initially** leaks out at a rate of 40 mL per minute. After 20 minutes, though, her container gets a second hole, so now water is leaking out of both holes at a combined rate of 80 ml per minute.

- How much water **remains** in Ms. B's container after 28 minutes?
- At what time does Ms. B's container have 800 mL of water **remaining** in it?
- Sketch a graph showing the relationship between the time and the amount of water **remaining** in Ms. B's container.
- What would be the slope of this linear relation? Explain your answer.

At some point in time, Ms. Knox and Ms. Brookhart have the same amount of water in their containers.

- Determine the value of this intersection point. Show/explain how you determined your answer.

5. Given the function  $f(x) = 2 - \frac{1}{x-3}$ , determine the:

- the domain and range of  $f(x) = 2 - \frac{1}{x-3}$
- the x-intercept(s) and y-intercept(s).
- Sketch** the function and **label** 2 data points as well as the **asymptotes**.



6. Given the function  $g(x) = (2 - |x|)(\sqrt{x + 5})$  and the X window settings of Xmin = -10 and Xmax = 10, determine:

- Appropriate Y window settings
- The critical point(s).
- The interval(s) of increase and decrease.
- Sketch and label the keys points.
- Recall, from Question 5 that  $f(x) = 2 - \frac{1}{x - 3}$  and now from Question 6, that

$g(x) = (2 - |x|)(\sqrt{x + 5})$ . So now, explain what it means to solve the equation  $g(x) = f(x)$  and then solve the equation  $g(x) = f(x)$ .

7. The radioactive isotope carbon-14 is used to determine the approximate date of artifacts found at archaeological sites. Carbon-14 has a half-life of 5370 years. A sample of pottery found at a site at an archaeological site in Cairo and has 73% carbon-14 remaining.

- Shantanav decides to model the decay of the C-14 isotope using the  $A(t) = Pe^{rt}$  “formula”. Show that the approximate value of  $r$  is  $-0.0001291$  (or show that the exact value is  $r = \frac{\ln(0.5)}{5370}$ ).
- Interpret the meaning of “ $r$ ” in this  $A(t) = Pe^{rt}$  “formula”.
- Determine the age of the pottery using any approach you wish. Show supporting evidence in your solution.

8. The HS math teachers were discussing one application of exponential functions → in the analysis of their retirement investments.

- Twenty two years ago, Mr. S invested \$110,000 which has been averaging an annual growth rate of 4.5% **compounded monthly** over these past 22 years. If the investment continues with this same rate, how much money will Mr. S. have when he retires 12 years from now?
- Mr. Smith is starting his retirement investment now. He has inherited \$60,000 and would like the money to grow to a value of \$300,000 in the next 25 years. What annual rate (as a percent) should the investment earn? Assume the investment is **continuously compounded**.
- Ms. A’s retirement investment currently earns her 6.25% p.a. and is compounded quarterly. How long does it take to triple her current value (increase by a factor of 3)?

9. The IM3 H block class is continuing to analyzing the function  $f(x) = \frac{x^2 - x - 6}{e^{0.5x}}$ , only now with the benefit of the TI-84.
- State appropriate view windows so you can see the intercepts and extrema. (Recall that extrema are the maxs & mins)
  - Determine the co-ordinates of the extrema(s) of the function.
  - Sketch and label the extrema, intercepts and asymptotes.
  - In which domain interval(s) is the function **decreasing**?
  - Use the graph to solve for  $x$  given the inequality  $f(x) < -e$ .
10. Next year, my son Ian will start high school here at CAC. So, I have promised to give him a \$20,000 graduation “gift” after his 4 year high school “career”.
- So, I find an investment plan that earns 6% p.a. compounded quarterly. How much must I invest **now**, in order to have \$20,000 for Ian in 4 year’s time?

I do not have that much money available now!! But Ian is pretty smart, so he proposed different idea, wherein we will invest \$5,000 **every year** for the four years of his high school career. In this question, you will analyze Ian’s investment plan, working with the same rate of 6% p.a. compounded quarterly.

- Mr S invests the first \$5,000. Determine the value of the this amount at the **end of the first year**.
- At the beginning of Ian’s second year, Mr. S will now add the second \$5,000. How much money is being invested to **begin the second year**?
- Show that the investment plan **starts** the fourth year with \$21,917.37.
- Using the correct value from part (d) of \$21,917.37, how much money does Ian have at the end of the 4<sup>th</sup> year?

Now, Ian is pretty smart in Math too, so he thinks he notices the total value of the investment could have been determined with a simple “formula”:  $A = 5000(1.015^{16} + 1.015^{12} + 1.015^8 + 1.015^4)$

- What total does Ian’s formula give? Given your answer in part (e), does Ian’s “formula” work?
- To help explain why Ian’s formula works, describe the **pattern** in Ian’s formula.
- Use Ian’s “formula” to predict a new total for running this investment for another 2 years (making it a total of 6 years of investing.)

Now, Alexander makes some observations about Ian’s formula and decides that a newer, easier formula

would have been  $A(n) = \frac{5306.82(1 - 1.06136355^n)}{(1 - 1.06136355)}$

- Test Alexander’s formula for  $n = 4$ . What is the value of  $A(4)$ ?