

Paper 1 - CALCULATOR INACTIVE

1. Given the line $\frac{x}{3} - \frac{y}{5} = 1$;
 - a. Determine the slope of this line.
 - b. Write the equation of this line in function form.
 - c. Evaluate $f(-12)$.
 - d. Solve for x if $95 = f(x)$.

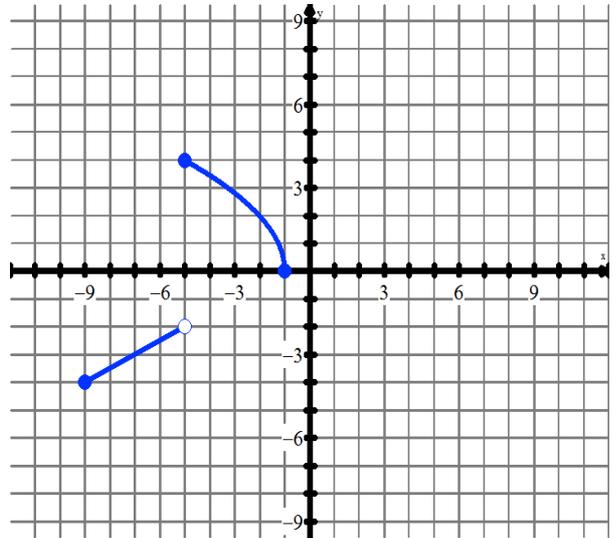
2. Given the line $y - 6 = -\frac{2}{3}(x + 3)$, determine the x- and y-intercepts of this line.
3. Solve the linear system defined by $2x - 4y = 8$ and $y - 2 = -\frac{3}{4}(x + 2)$. Verify answer.
4. Evaluate the following expressions:
 - a. $8^{\frac{4}{3}}$
 - b. $49^{-\frac{1}{2}}$

5. Evaluate the following expressions:
 - c. $\log_3 81$
 - d. $\log_8 2$
 - e. $\log_5 \left(\frac{1}{125} \right)$

6. Evaluate $f(-3)$ if $f(x) = 6(2)^{x+2} + 3$.
7. Give an exact solution to the equation $e^{x+3} = 15$.
8. Solve the equation $f(x) = 58$ for x if $f(x) = -6 + 4(2)^{3-2x}$.
9. Solve the equation $g(x) = 2$ for x if $g(x) = \log_6(3 - x)$.
10. Mr. D has written a population word problem and the equation he used was $P(t) = 5000(0.92)^{4t}$. From the options provided, select the appropriate description of the word problem: (explain the reasoning behind your choice)
 - a. The population was growing at a rate of 8%.
 - b. The population was decreasing at a rate of 92%.
 - c. The population was growing at a rate of 92%.
 - d. The population was decreasing at a rate of 8%.
 - e. We cannot determine the rate of change of population as there is not enough information provided.

11. In this question, you will graph an inverse of a function.

- Explain how to draw the graph of an inverse of any function.
- Graph the line $y = x$ on the graph provided.
- Given your answers to these previous 2 questions, graph the inverse of this function on the graph provided. Show supporting work (if necessary).



12. You are going to work with the linear function $f(x) = -\frac{3}{4}x - 2$. As you work through and answer the following questions, **ALWAYS** show your work OR explain your reasoning.

- | | |
|---|---|
| <ol style="list-style-type: none"> Would the point (12,-11) be on the graph of this linear function? Show necessary work. Determine the equation of the inverse of $f(x) = -\frac{3}{4}x - 2$. Now, use your equation for $f^{-1}(x)$ from Q2c to evaluate $f^{-1}(-11)$. Show your work. | <ol style="list-style-type: none"> What is the x-intercept of the inverse function? Malak PREDICTS the value of $f^{-1}(-11)$ to be 12. Explain the reasoning/thinking for her prediction. What do your answers from Q2d and Q2e mean about your work in Q2c? |
|---|---|

13. The linear function, $f(x)$, can be rewritten in standard form as $4x - Ky + 26 = 0$. This function exists in the domain of $\{x \in \mathbb{R} \mid -2 < x \leq 4\}$ and $f(1) = 10$.

- Show that the value of K is 3.
- Given that $K = 3$, determine the range of $f(x)$.

14. Given the system $4x - 3y = 11$ and $3x - Dy = 7$.

- What does it mean when we say that a system of linear equations has **no solution**.
- Find the value of D such that the system has no solution.

15. Mr. Santowski is studying the relationship between the number of hours that students spend studying, h , and the marks they get on their December Semester exams, M . Included below is a sample data set from his study.

Hours Studied, h	2	5	3.5	1	8	3	9	12
Exam Mark (%), M	68	61	81	48	85	75	93	88

- What does the domain of this relation represent?
- Does this relation HAVE to be a function? Why or why not?
- Evaluate $M(3.5)$ and explain what it means in the context of this problem.
- Solve $M(h) = 68$ and explain what it means in this context.
- After marking all student exams, Ms. A suggests that we apply a transformation to this relation and suggests an equation: $y = M(h) + 5$. Explain what this transformation would do to the graph of the relation and to the student marks.

Omar wonders about the **meaning** of the inverse of this relation.

- What is the **meaning** of the **range** of this inverse relation?
- ESTIMATE** a reasonable value of $M^{-1}(70)$ and **EXPLAIN** what would the result mean?
- Given your answers to the previous 2 questions, how would students make use the equation for the inverse? (What is the point of this inverse relation? (HINT: Think about your answer for Q3(f) and Q3(g))

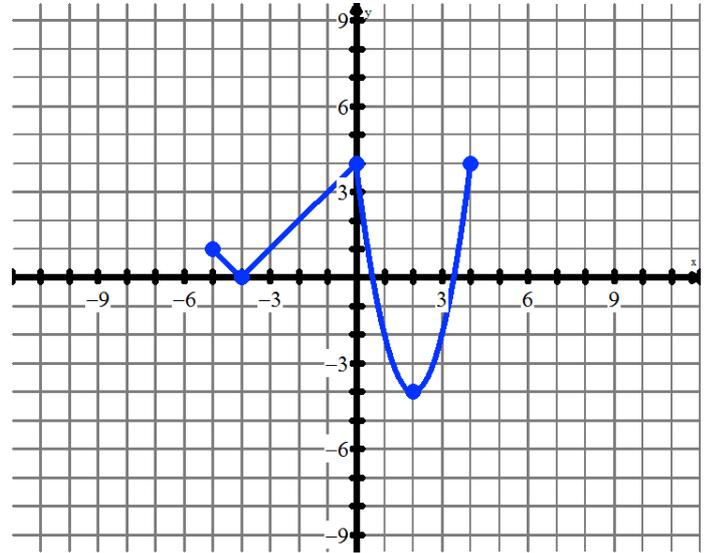
16. Characteristics of Functions: You will produce a sketch of a function that meets the following requirements:

- The domain is to be $\{x \in R \mid x < 4\}$.
- The absolute minimum point must be at $(-2, -9)$.
- The function is to be increasing ONLY on the interval of $\{x \in R \mid -2 < x < 2\}$.
- The function must have an asymptote.

- Label your KEY POINTS.
- Explain why the range of your function is $\{y \in R \mid y \geq -9\}$.
- EXPLAIN what would you have to do with your KEY POINTS in order to graph the new function $y = f(x - 2) + 4$.
- Is inverse of your function also a function? Explain why or why not.

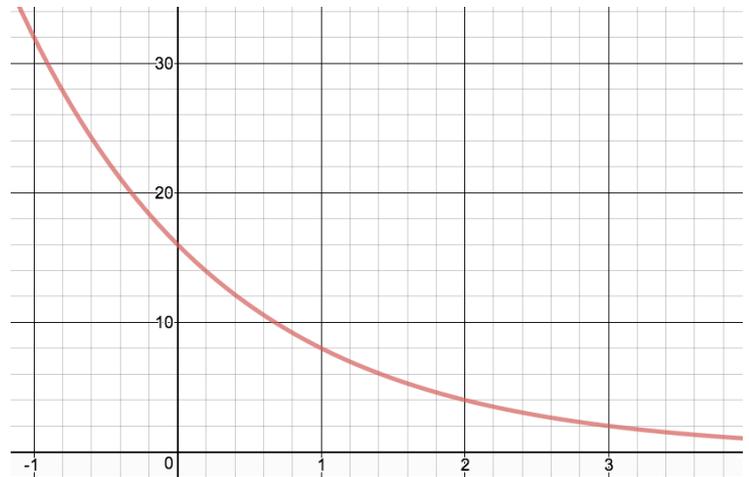
17. You have been provided with a graph of a piecewise function, $y = f(x)$, which is below.

- Given the NEW equation of $y = \frac{1}{2}f(x+1) - 3$, list what transformations will be applied to $y = f(x)$
- Now apply the transformations to $y = f(x)$ and sketch the new transformed function. Show all necessary work to support your sketch of the new function.



18. You have been provided a graph of the function $f(x) = ab^x$. Use the graph to answer the following questions.

- Explain how you know from the graph that $a = 16$.
- Explain how you know from the graph that $b = \frac{1}{2}$.
- State the domain and range of this function.
- State the equation of the asymptote of this function.
- Sketch the inverse of this function and determine its equation.



Mr. S now wants to make a few transformations of the function $f(x) = 16\left(\frac{1}{2}\right)^x$. So he writes a new equation as $f(x) = 20 - 16\left(\frac{1}{2}\right)^{x+1}$.

- List the transformations that Mr. S. made.
- Sketch the new function.
- Write the equation of inverse of this transformed function.

Paper 2 – CALCULATOR ACTIVE

1. A linear function goes through the point A(5,-8) and this linear function is perpendicular to a second line, which has the equation of $3x - 5y - 20 = 0$.
 - a. Determine the slope of the line defined by $3x - 5y - 20 = 0$.
 - b. Hence, determine the equation of the second perpendicular line. Write the equation in point-slope form.
 - c. Write your final answer for the perpendicular line's equation using standard form.
 - d. Sketch a diagram, showing the two lines from this question. The point (5, -8) has already been added into the diagram.

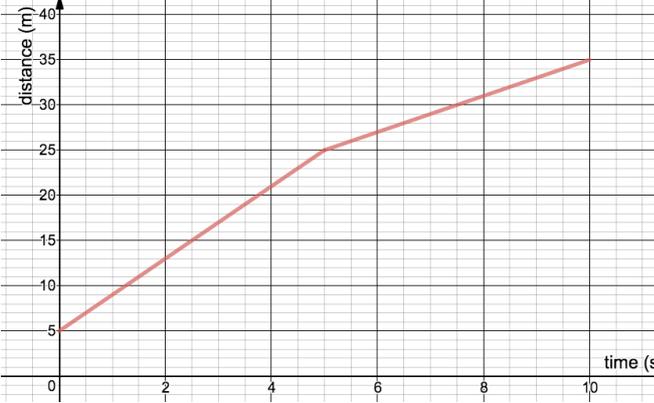
2. A piecewise linear function is defined by the equation $p(x) = \begin{cases} 2x - 3 & x < 3 \\ -\frac{1}{3}x + B & x \geq 3 \end{cases}$
 - a. Evaluate $p(-1)$.
 - b. Draw a diagram of a piecewise function that shows your understanding of functions being **continuous** at $x = 3$.
 - c. Draw a second diagram of a piecewise function that shows your understanding of functions being **discontinuous** at $x = 3$.
 - d. Determine the value of B so that the function $p(x)$ is **continuous** at $x = 3$. Show/explain supporting evidence for your answer.

3. Advika has an investment for college of \$45,000 that is earning 6% per year.
 - a. If this interest of 6% is compounded quarterly for 8 years, determine how much **interest** was earned in those 8 years.
 - b. How much time does it take to triple the value of this investment?

4. Provide an exact and an approximate solution to the equation $\ln(5x + 2) = 3$.

5. Given the function $f(x) = (\sqrt{x+3})(x-5)^2$ and the X window settings of Xmin = -5 and Xmax = 10, determine:
 - a. Appropriate Y window settings.
 - b. The critical point(s).
 - c. The interval(s) of increase and decrease.
 - d. Sketch and label the keys points.

6. Your neighborhood friends have decided to have a running race down the street. Here is the information about the distance, d , in meters, (including a head start in some cases) in terms of time, t , in seconds.

<p>Mitchell</p> <p>Runs 5 meters every 2 seconds and has a 6 meter head start</p>	<p>Kyra</p> <p>Distance is modeled by the equation</p> $d(t) = \frac{9}{2}t + 3$										
<p>Gloria</p> 	<p>Hashem</p> <table border="1" data-bbox="837 705 1388 828"> <tr> <td>20</td> <td>22</td> <td>24</td> <td>26</td> <td>28</td> </tr> <tr> <td>77</td> <td>84</td> <td>91</td> <td>98</td> <td>105</td> </tr> </table>	20	22	24	26	28	77	84	91	98	105
20	22	24	26	28							
77	84	91	98	105							

- Determine an equation for Gloria’s run.
 - Which runner has the fastest pace? Show necessary working to support your answer.
 - How far did Hashem go in the first ten seconds?
 - Who would win the race if the race was 50m long? Show necessary working to support your answer.
7. The population of Whoville in 1980 was 46,750 and in 2010, the population had grown to 67,000. Assume that population growth happens **continuously** and let t represent time in years since 1980.

- Explain why Taeho uses the equation $A(t) = 46,750e^{kt}$ to model this situation.
- Use the information given in this question to show that $k = 0.0120$
- What does k represent in this model?
- According to the model, what was the population of Whoville in 1970?

The population of Smurfytown is modeled with the equation $P(t) = \frac{100,000}{1 + e^{-0.04t}}$, where t is in years since 1980.

- What is the predicted population of Smurfytown in 2040?
- Sketch a graph, showing the graphs of the population models of both towns. Label each curve in your sketch.
- In what year does the population of Whoville first exceed that of Smurfytown?

8. The following graph shows the temperature in degrees Celsius of Fadi's cup of tea, t minutes after pouring it out. The equation of the cooling graph is $f(t) = 20 + 75(1.75)^{-0.15t}$ where $f(t)$ is the temperature and t is the time in minutes after pouring the tea into the cup.



- Find the initial temperature of the tea.
- Write down the equation of the horizontal asymptote and explain what it means in the context of this problem.
- Find the temperature of the tea after 12 minutes.

If the tea is not hot enough it is reheated in a microwave oven. The liquid increases in temperature according to the formula $T = A_o(2)^{1.25t}$ where T is the final temperature of the liquid, A_o is the initial temperature of tea in the microwave and t is the time in minutes after switching on the microwave.

- Given the final temperature of the tea from part (c), find the temperature of Fadi's tea after being heated in the microwave for 30 seconds.
- Calculate the length of time it would take a similar cup of tea, initially at 20°C , heated in the microwave to reach 100°C .

9. Given the function $f(x) = 4 - \sqrt{5 - x}$, determine the:

- the domain and range of $f(x) = 4 - \sqrt{5 - x}$.
- the x-intercept(s) and y-intercept(s).
- sketch** the function and **label** 3 data points.

Probability Worksheet

1. A single die is rolled. Find the probability of rolling a 2 or an odd number.
2. Suppose that 37.4% of all college football teams had winning records in 1998, and another 24.8% broke even. What is the probability that a randomly chosen college football team had a losing record in 1998?
3. A couple plans to have four children. Assuming that boys and girls are equally likely, find the probability that the couple will have at least one boy?
4. A card is drawn from a standard deck of 52 cards.
 - a. What is the probability the 7 of spades is drawn?
 - b. What is the probability that a 7 is drawn?
 - c. What is the probability that a face card is drawn?
 - d. What is the probability that a heart is drawn?
5. A 12-sided die with sides numbered 1 through 12 is rolled. Assuming that all sides are equally likely to be rolled, what is the probability that the number rolled is a multiple of 3? Give your answer as a fraction in lowest terms.
6. One card is drawn at random from a deck of cards. What is the probability that the card will be a red ace?
7. Suppose that 39% of a town's population have type O blood, 15% are Rh-negative, and 7% have type O blood and are Rh-negative. What is the probability that a randomly selected individual in the town will neither have type O blood nor be Rh-negative?
8. Seven hundred and fifteen tickets were sold for a raffle. If you bought 12 of them, what is the probability that you have the winning ticket?
9. Suppose that 54% of a town's population have brown eyes, 51% have black hair, and 37% have both brown eyes and black hair. What is the probability that a randomly selected individual in the town will have brown eyes or black hair?
10. In a class of 28 students, 15 earned an A on the final exam, 10 earned an A for the entire course, and 7 earned an A for both the final exam and the entire course. What is the probability that a randomly selected student in this class did not earn an A on the final exam and did not earn an A for the entire course?
11. Suppose that a town named Sunnyvale had 217 sunny days in 1998. What is the empirical probability that it will be sunny in Sunnyvale on a random day?
12. If a single fair die is rolled, find the probability of rolling:
 - a. a 2, given that the number rolled was odd.
 - b. an even number, given that the number rolled was a 6.
13. If two fair dice are rolled, find the probability of rolling a sum of 6, given the roll was a "double".

14. If two cards are drawn without replacement from a standard deck, find the probability that
- the second is a heart, given that the first is a heart.
 - the second is black, given that the first is a spade.
15. If five cards are drawn without replacement from a standard deck, find the probability that all the cards are
- diamonds.
 - diamonds, given that the first four were diamonds
 - the same suit.
16. Two marbles are drawn without replacement from a jar with four black and three white marbles. Find the probability that
- both are black.
 - the first is black and the second is white.
17. A pet shop has 10 puppies, 6 of them males. There are 3 beagles (1 male) 1 cocker spaniel (male), and 6 poodles. Construct a table and find the probability that one of these puppies, chosen at random, is
- a beagle
 - beagle, given that it is a male.
 - a cocker spaniel, given that it is a female.
 - a female, given that it is a beagle.
18. Suppose S is a sample space and A and B are two events in the sample space. If $P(A) = \frac{3}{4}$, $P(A \cap B) = \frac{3}{8}$, $P(A \cup B) = \frac{7}{8}$, find each of the following.
- $P(A')$ = _____
 - $P(B)$ = _____
 - $P(A | B)$ = _____
 - $P(B | A)$ = _____
 - Odds against A = _____
 - Odds against B = _____
 - Are A and B independent events? Why or why not?
19. The Midtown Bank has found that most customers at the tellers' windows either cash a check or make a deposit. The chart below indicates the transactions for one teller for one day

	Cash Check	No Check	Totals
Make deposit	50	20	70
No deposit	30	10	40
Totals	80	30	110

20. Letting C represent “cashing a check” and D represent “making a deposit,” express each of the following probabilities in words and find its value.
- $P(C | D)$
 - $P((C \cap D)')$
 - $P(C' | D')$
21. Assume that boy and girl babies are equally likely. If a couple have three children, find the probability that all the children are girls given that
- the first is a girl
 - the second is a girl
 - at least two are girls

22. The Motor Vehicle Department has found that the probability of a person passing the test for a driver's license on the first try is .75. The probability that an individual who fails on the first test will pass on the second try is .80, and the probability that an individual who fails the first and second tests will pass the third time is .70. Find the probability that
- an individual fails both the first and second tests.
 - an individual will require at least two tries to pass the test.
23. Suppose that 53.2% of car owners are male and that 12.7% of all car owners change their own oil. If 31.5% of male car owners change their own oil, what is the probability that a randomly selected car owner is male and changes his own oil?
24. Someone thinks of a 2-digit number (the first digit cannot be 0) and asks you to guess what it is. What is the probability that your first guess will be correct?
25. A box of 15 chocolates has 6 that are caramel, 5 that contain nuts, and 4 that are nougat. If you randomly select 3 chocolates, what is the probability that you will get all nuts?
26. A city council has 12 members. Five are Democrats and seven are Republicans. In how many ways can a four member committee be selected?
27. A student organization has 8 freshman members, 5 sophomores, 10 juniors, and 6 seniors. In how many ways can an eight member committee be chosen if there must be two members from each class?
28. A department has 25 members. They must select a chair, an assistant chair, and an advisory committee of three members (neither the chair nor the assistant chair can be on the advisory committee). In how many ways can this be done?
29. The diagram below shows the number of outcomes in a sample space S and outcomes in event A and B . Find each of the following probabilities:
- $P(A \cap B) =$ _____
 - $P(A) =$ _____
 - $P(B) =$ _____
 - $P(A | B) =$ _____
 - $P(B | A) =$ _____
 - Odds against $A =$ _____
 - Odds against $B =$ _____
 - Are A and B independent events? Why or why not?

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