

(A) Lesson Context

BIG PICTURE of this UNIT:	<ul style="list-style-type: none"> • How can we visualize events and outcomes when considering compound events ? • How can we calculate probabilities when considering compound events ? 		
CONTEXT of this LESSON:	<p>Where we've been</p> <p>We've seen the difference between experimental and theoretical probability as well as the difference between calculating AND/OR probabilities</p>	<p>Where we are</p> <p>When dealing with compound events, how can we visually represent probability problems and extend our knowledge with conditional probabilities?</p>	<p>Where we are heading</p> <p>Can we predict how likely is an event to occur?</p> <p>How can we use that knowledge?</p>

(B) Lesson Objectives:

- a. be able to use visualizations like Venn Diagrams and Tree diagrams to visually represent problems in probability
- b. be able to use visualizations like Venn Diagrams and Tree diagrams to extend our understandings & applications of probability

Questions for More Complex Concepts in Probability. Determine the probability of the event described in each exercise. Unless stated otherwise, assume all items of chance (dice, coins, cards, spinners, etc.) are fair.

Problem 1

In the Grade 10 class of 100 kids, there are two sports commonly played, soccer and basketball. 76 students do only one of the sports, 14 students don't do any sport and 46 students play basketball. What is the probability when a student is selected that they play soccer?

Problem 3

The probability that a person has a deadly virus is 5 in one thousand. A medical test will CORRECTLY diagnose the disease 95% of the time, but INCORRECTLY diagnose the disease 20% of the time. Find the probability of this test giving a correct diagnosis.

Problem 2

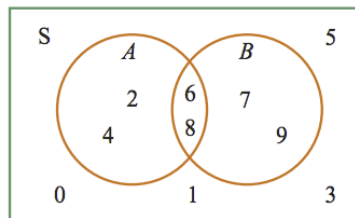
A bag of MM's contains 6 red, 4 green and 2 brown candies. Mario takes one MM and then offers another one to his friend Sponge Bob. What is the probability that they both have the same color MM's.

Problem 4

Three friends decide to meet up at High Street one day during the school week, but have not decided which day. Given that the phone lines are all dead, internet is not working and school is on holiday, what is the probability that all three girls meet at High Street on the same day?

1.

Numbers are written on cards, the cards are shuffled and one is selected at random. The outcomes for this experiment and events A and B are shown in the Venn diagram. Complete the following.



- a** The sample space $S = \{\text{_____}\}$ (outcomes inside the rectangle)
- b** Event $A = \{\text{_____}\}$ (outcomes inside the circle A)
- c** Event 'not A ' = $\{\text{_____}\}$ (outcomes outside the circle A)
- d** Event $B = \{\text{_____}\}$ (outcomes inside circle B)
- e** Event 'not B ' = $\{\text{_____}\}$ (outcomes outside circle B)
- f** Event ' A and B ' = $\{\text{_____}\}$ (outcomes in the intersection of the circles)
- g**
 - i** Event ' A or B or both' = $\{\text{_____}\}$ (outcomes within both circles, including the intersection)
 - ii** Is this an 'inclusive or' event or an 'exclusive or' event?
- h**
 - i** Event ' A or B but not both' = $\{\text{_____}\}$ (outcomes within both circles, excluding the intersection)
 - ii** Is this an 'inclusive or' event or an 'exclusive or' event?
- i** Event 'neither A nor B ' = $\{\text{_____}\}$ (outcomes outside both circles)

2.

1.1 Some Simple Games

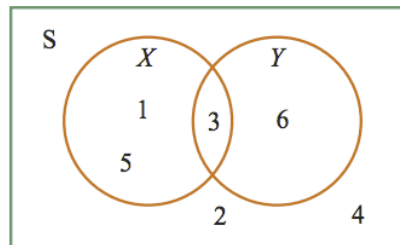
Lets say you play a game where you roll a fair die (what does this mean?) and get paid according to your roll:

Roll	Payout
6	\$ 4
5	\$ 2
4	\$ 1
3	\$ 0
2	\$ 0
1	\$ 0

You have to pay \$1 to play this game. Is it worth it? What do you expect to happen in the long run?

3.

A six-sided die is rolled. The outcomes for this experiment and the events X and Y are shown in the Venn diagram.



- a** List the sample space.
- b** List the set of outcomes for event X .
- c** List the set of outcomes for the event 'not X '.
- d** List the set of outcomes for event Y .
- e** List the set of outcomes for the event 'not Y '.
- f** List the set of outcomes for event ' X and Y '.
- g**
 - i** List the set of outcomes for the event ' X or Y or both'.
 - ii** Is this an 'inclusive or' event or an 'exclusive or' event?
- h**
 - i** List the set of outcomes for the event ' X or Y but not both'.
 - ii** Is this an 'inclusive or' event or an 'exclusive or' event?
- i** List the outcomes that belong to neither X nor Y .

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4. Take a card from a normal deck, without looking at it.

- a. What is the probability you drew a spade? _____
- b. Now assume that you peeked just a little and know that the card is black. What is the probability you drew a spade? _____
- c. Assume you didn't peek but your friend did and tells you that the card is not a heart. What is the probability you drew a spade? _____

The proper notation for the last question is $P(\text{spade} \mid \text{not a heart})$. This means "the probability of a spade given that the card is not a heart." The symbol for a *known* or *given* condition is a _____ and the given fact determines the _____ of the probability.

Definition

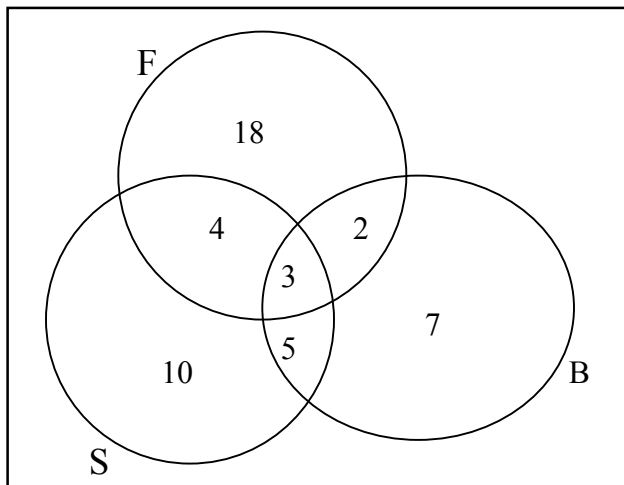
The conditional probability of an event B in relationship to an event A is the probability that event B occurs given that event A has already occurred.

Notation $\rightarrow P(B \mid A)$ is the conditional probability that B occurs given that you have knowledge that A has occurred.

5. A group of 60 students were asked if they played field hockey (F), basketball (B) or soccer (S). The diagram below displays the results.

Determine the probability that a student plays:

- (a) field hockey **given that** they play basketball?
- (b) soccer **given that** they play field hockey?
- (c) field hockey **given that** they play soccer or basketball?
- (d) basketball **given that** they play field hockey & soccer?
- (e) neither of the three sports?
- (f) only 1 sport?

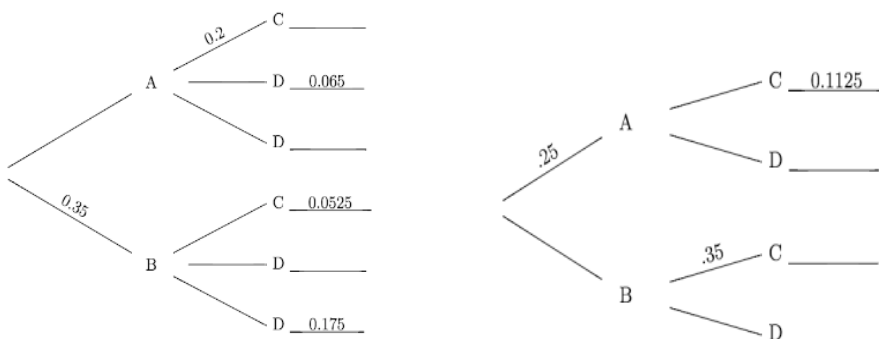


6.

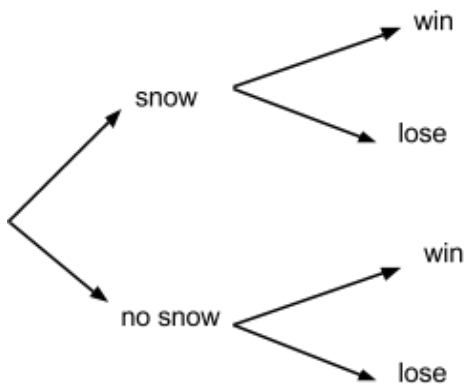
EXAMPLE 5 Expected Value as Average Payoff

A game is played using one die. If the die is rolled and shows 1, 2, or 3, the player wins nothing. If the die shows 4 or 5, the player wins \$3. If the die shows 6, the player wins \$9. If there is a charge of \$1 to play the game, what is the game's expected value? Describe what this means in practical terms.

7. Fill in each path for the tree diagram.



8. A football team has a 70% chance of winning when it does not snow, but only a 40% chance of winning when it does snow. Suppose there is a 50% chance of snow. Create a tree diagram to find the probability that the team will win.



9. Five percent of my students suffer from a terrible malady called Lazybrain(LB). A blood test detects LB accurately 90% of the time. Yusuke is told that his blood test is positive for LB. Yusuke hopes that this is a “false positive” and he actually doesn't have Lazybrain.

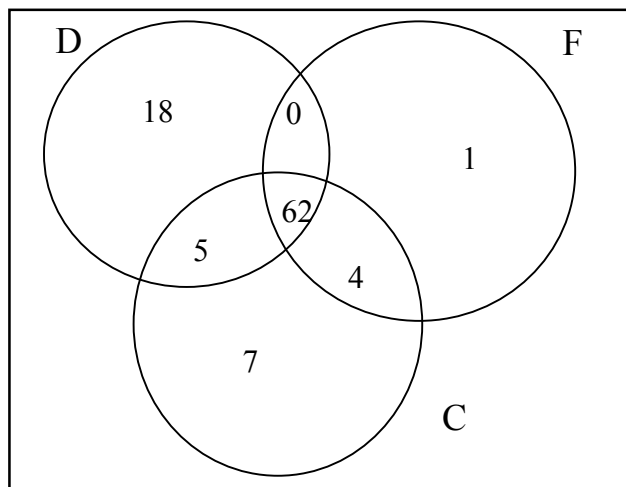
- i. Draw the tree with all the probabilities. Indicate which branches are the *false positive*, *false negative*, *correct positive*, and *correct negative*.
- ii. Find the probability that Yusuke is OK even though his blood test was positive.

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10. The Venn diagram displays the results of a survey of 100 families regarding technology in their homes. Computer (C), DVD player (D) and fax machine (F)

How probable is it that a family has:

- iii. a computer at home?
- iv. all three machines?
- v. none of the machines in their home?
- vi. no fax machine?
- vii. a computer and a DVD?
- viii. a DVD or a computer?
- ix. A computer **given that** they have a fax machine?
- x. A DVD players **given that** they have a computer?



11. A small school has 24 boys graduating. Half of them are funny and 7 are good dancers. Eight of them are neither funny nor good dancers. One boy is selected at random. Translate the following into conditional probability notation, then find the probabilities.

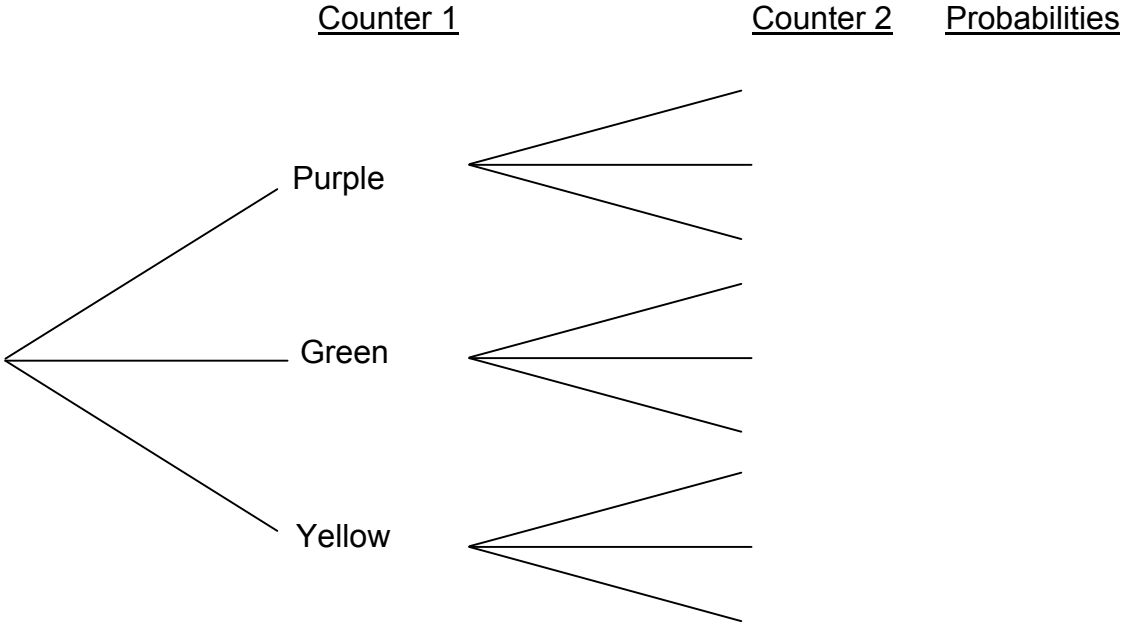
- i. Probability he is a good dancer given that he is funny.
- ii. Probability he is funny given that he is a good dancer.
- iii. If he is a good dancer, what is the probability he is not funny?
- iv. If he is not a good dancer, what is the probability he is funny?

12. Isabel goes to school by one of two routes, A or B. The probability of going by route A is 30%. If she goes by route A, the probability of being late is 5% and if she goes by route B, the probability of being late is 10%

- i. Draw a tree diagram
- ii. Find the probability that Isabel is late for school.
- iii. Given that she is late for school, find the probability that she went to school using route A

13. In a group of 35 children, 10 have blonde hair, 14 have brown eyes, and 4 have both blonde hair and brown eyes. If a child is selected at random, find the probability that the child has blonde hair or brown eyes.

14. A bag contains 3 purple counters, 2 green ones and 5 yellow ones. A counter is taken from the bag at random and then replaced. A second counter is then drawn from the bag. Complete the tree diagram:



15. Of the 28 students in a class, 12 have a part time job, 22 have a part time job or do regular volunteer work, and 4 of the students have a part time job and do regular volunteer work.

- a. Display the data in a Venn diagram.
- b. How many of the students do not have a part time job or do not volunteer regularly?
- c. How probable is it that a student does volunteer work given that they have a part time job?

16.

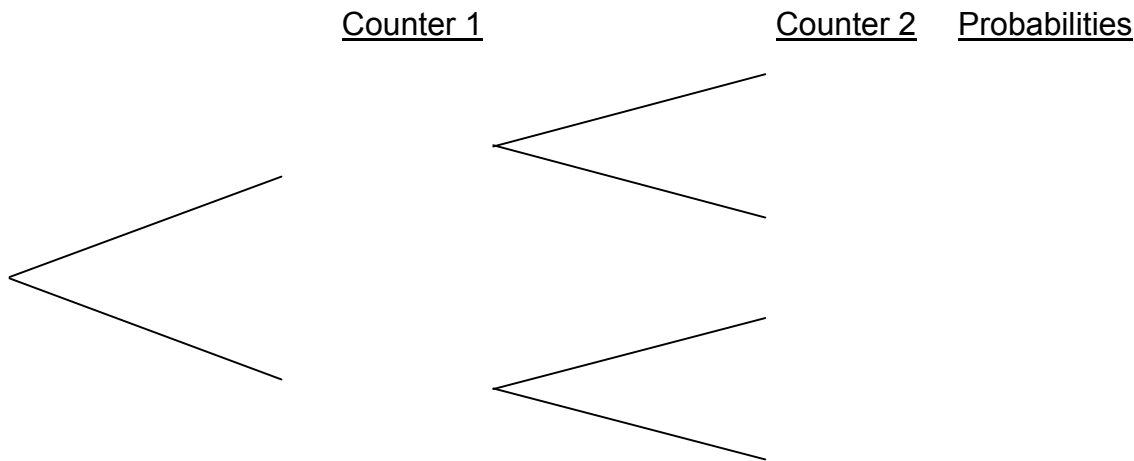
EXAMPLE 6 Expected Value and Roulette

One way to bet in roulette is to place \$1 on a single number. If the ball lands on that number, you are awarded \$35 and get to keep the \$1 that you paid to play the game. If the ball lands on any one of the other 37 slots, you are awarded nothing and the \$1 that you bet is collected. Find the expected value for playing roulette if you bet \$1 on number 20. Describe what this means.

17. Amber, a college senior, interviews with Acme Corp. and Mills, Inc. The probability of receiving an offer from Acme is 0.35, from Mills is 0.48, and from both is 0.15. Find the probability of receiving an offer from either Acme Corp. or Mills, Inc., but not both.

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18. A bag contains 7 red counters and 4 blue counters. A disc is taken at random from the bag and not put back in. A second disc is then removed from the bag. Complete the tree diagram below:



Find the probability that:

- a. Both discs are red
- b. Both discs are blue
- c. Both discs are different

19. Ten students are trying out for three positions on a coed soccer team. The students include four boys (Adam, Alex, Anthony and Arnold) and six girls (Abbey, Aurora, Agnes, Alice, Amanda and Anna). All the students have an equal chance of being selected for the team.

(a) How many different three-member teams can be formed? _____

(b) Determine the probability that the team would include:

Three boys: _____

One boy and two girls: _____

At most one girl: _____

Adam, Anthony and Alice: _____

Agnes and two other students: _____

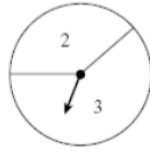
(c) How is the number of ways to select the team affected if the three openings on the team are specifically for positions of forward, midfield and defense? That is, with each selection, a student's name is attached to a specific position.

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20.

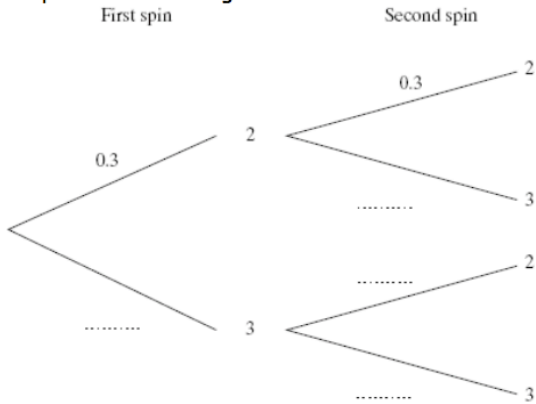
The diagram shows a spinner.

When the arrow is spun the probability of scoring 2 is 0.3



The arrow is spun twice and the scores are added.

(a) Complete the tree diagram



(b) What is the probability that the total score is 4?

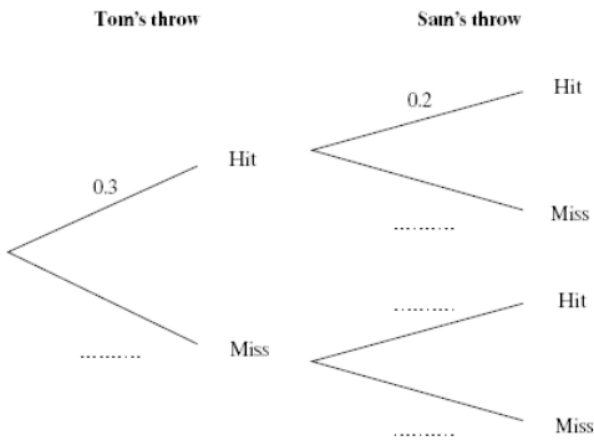
21. 100 people were asked if they liked Math, Science, or Social Studies. Everyone answered that they liked at least one. The results were that 56 like Math, 43 like Science, 35 like Social Studies, 18 like Math and Science, 10 like Science and Social Studies, 12 like Math and Social Studies and finally 6 like all three subjects. A student is chosen at random. How probable is it that:

they like Math only?	they like Science only?	they like Social Studies only?	They like social studies and Science?
They like math or science?	They like science given that they like math?	They like social sciences given that they do not like math	

22.

Tom and Sam take turns to throw a dart at a target. The probability that Tom hits the target is 0.3 and the probability that Sam hits the target is 0.2

(a) Complete the tree diagram



(b) What is the probability that both Tom and Sam hit the target?

23. Widgets are made in Factories K and P. Factory K makes 60% of all widgets and Factory P makes the other 40%. K has a 20% defective rate and P has a 25% defective rate.

- i. What is the probability that a defective widget came from K?
- ii. What is the probability that a defective widget came from P?
- iii. Who has the lower defective rate? _____
- iv. Who is most likely to have made a defective widget? _____
- v. Explain the paradox _____

24. Hannah is going to play one badminton match and one tennis match. The probability that she will win the badminton match is 0.9. The probability that she will win the tennis match is 0.4. Determine the probability that Hannah wins both matches.

25. Each member of a sports club plays at least one of soccer, rugby or tennis. The following is known: 43 members play tennis, 11 play tennis & rugby, 7 play tennis & soccer, 6 play soccer & rugby, 84 play rugby or tennis, 68 play soccer or rugby and 4 play all three sports.

- (i) How many members does the club have?
- (ii) Two members are chosen at random. How probable is it that both play rugby?
- (iii) Two members are chosen at random. How probable is it that both play rugby, given that neither plays tennis?

26.

1.2 Some Exercises for You

Determine the expected value for the games.

1. Charge \$1 to play. Roll one die, with payouts as follows:

Roll	Payout
6	\$ 2
5	\$ 2
4	\$ 1
3	\$ 0
2	\$ 0
1	\$ 1.50

2. Charge: \$1 to toss 3 coins. Toss the coins. If you get all heads or all tails, you receive \$5. If not, you get nothing.
3. Charge: \$1. Roll 2 dice. If you roll 2 odd numbers, like a 3 and a 5, you get \$2. If you roll 2 even numbers, like 4 and 6, you get \$2. Otherwise, you get nothing.
4. Charge: \$5. Draw twice from a bag that has one \$10 and 4 \$1 bills. You get to keep the bills.

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27. Suppose you throw a pair of fair 6-sided dice. One is white and the other is black. Let T = total showing on both dice, and B = number showing on the black die.

a) Find $P(T = 5 | B = 2)$

b) Find $P(B = 2 | T = 5)$

28. Jar A has 4 red and 5 black candies. Jar B has 6 red and 2 black candies. A fair die is rolled and jar A is selected if a number divisible by 3 comes up, otherwise, Jar B is selected. One candy is drawn from the jar.

a) What is the probability you selected Jar A and got a red candy?

b) What is the probability you selected Jar B and got a red candy?

c) What is the probability you got a red candy?

d) Suppose a red candy is drawn, what is the probability it came from jar A?

e) What is the probability the candy was red, given that the candy came from jar A?

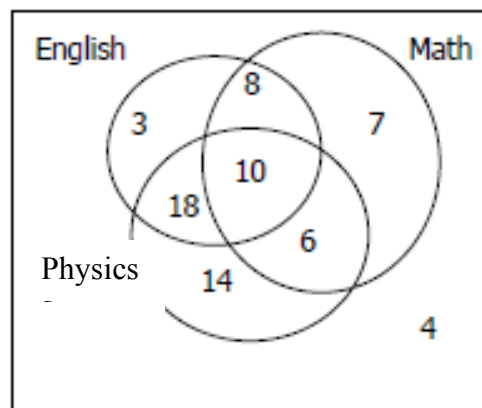
f) What is the probability Jar B was selected if a black candy is drawn?

29. Use the Venn Diagram given at the right to find the following:

(a) $P(\text{Physics} | \text{English})$

(b) $P(\text{Math and English} | \text{Physics})$

(c) $P(\text{English} | \text{Math or Physics})$



30. Box X contains 2 red and 3 white marbles. Box Y contains 1 red and 3 white marbles. A marble is randomly chosen from Box X and put into box Y. A marble is then randomly chosen from box Y.

a) What is the probability that the marble chosen from box Y is red?

b) If the marble from Y is red, what is the probability that the marble moved from X was white?

31. Both 9 and 10 can be made up in 2 different ways by adding pairs of the numbers 1, 2, ... 6, that is

$$9 = 3 + 6 = 4 + 5$$

$$10 = 4 + 6 = 5 + 5$$

Explain why it is that when you throw two dice you are more likely to get a score of 9 than of 10. What about the case of 3 dice? Is a score of 9 more likely than a score of 10 with 3 dice?

32. Participants in a study of a new medication received either medication A or a placebo. Make a tree diagram of the results of the study. Of all those who participated in the study 80% received medication A

- Of those who received medication A, 76% reported an improvement
- Of those who received the placebo, 62% reported no improvement

Then find (i) P(placebo and improvement), (ii) P(improvement), (iii) P(placebo was received given that improvement was noticed)

33. Create a tree diagram for the following. There is a 40% chance of heavy snow and a 60% chance of light snow. If there is a heavy snow then there is a 80% chance that the school closes. If there is a light snow there is only a 30% chance that the school closes. Answer the following from your tree diagram:

- P(school closes)
- P(heavy snow and school remains open)
- P(heavy snow given that the school closes)
- P(light snow given that the school remains open)
- P(if school closes then there is light snow)
- P(if school remains open then there is heavy snow)

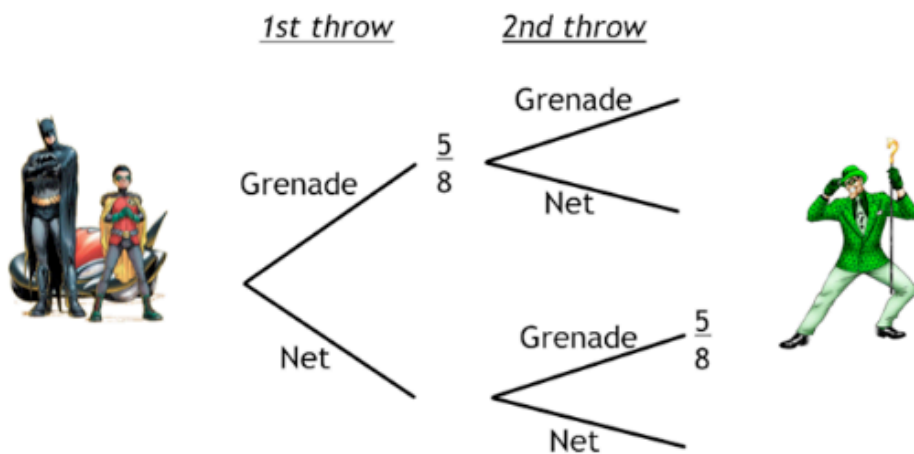
34. Micah has just graduated with a hospitality degree from VCC. She has applied for a job in Ontario at a four star hotel. The probability that she will get the job is 0.35. The probability that she will move to Ontario if she receives a job offer is 0.85. The probability that she will move to Ontario if she does not receive a job offer is 0.40. Create a tree diagram for all possible outcomes. What is the probability that she will be offered the job and not move to Ontario (to 4 decimal places)?

35. The owner of a local yoga studio tells you that the probability of a visitor buying a membership is 65%. The probability that someone will buy a membership and sign up for a yoga class is 26%. The probability that someone will not sign up for a class given that they did not buy a membership is 75%. (a) What is the probability that a visitor to the studio will sign up for a class, given that they bought a membership? (b) What is the probability that someone does not purchase a membership and signs up for a class?

36.

Batman always carries his weapons in a bag in the Batmobile. In that bag he keeps 5 stun grenades and 3 nets. As soon as he uses an item Robin immediately replaces it so that there are always 5 stun grenades and 3 nets.

Batman just picks an item from the bag and throws it towards the villain each time - he does not know which item it is until that item is thrown. Copy and complete the tree diagram of Batman's battle with the Riddler in which he used just two weapons:

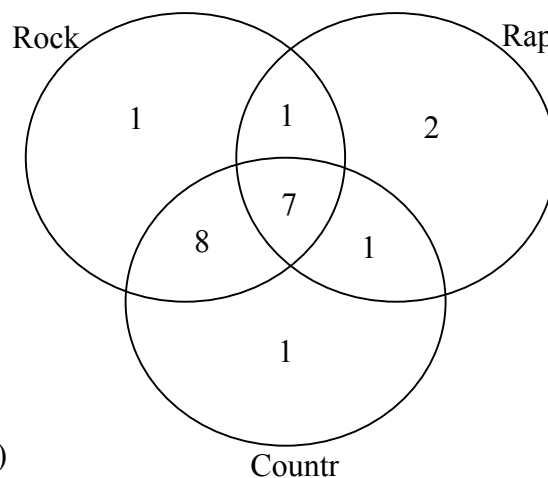


Calculate the probability that Batman throws the following:

- a. Two grenades.
- b. One of each weapon (in either order).

37. Use the Venn Diagram below to answer question (a) – (f).

- a. How many total people are represented in the diagram?
- b. How many people like country?
- c. If one person is chosen at random, what is the probability that:
 - i. they will like rap music?
 - ii. They will like rap or country music?
 - iii. They will not like rock nor country?
- iv. They will like rock **given that** they like country?
- v. They will not like country **given that** they like rock?
- vi. They will not like rap **given that** they do not like country?



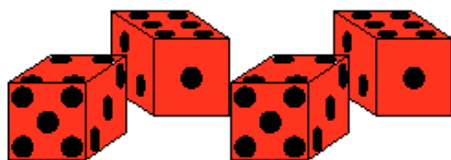
BONUS: (RESEARCH HOW TO DETERMINE odds)

- d. If one person is chosen at random, what are the odds for picking a person who likes country?
- e. If one person is chosen at random, what are the odds against picking a person who likes all three types of music?
- f. Odds against all three = ?

38.

A Dicey Paradox

Stage: 3 ★★



Four fair dice are marked on their six faces, using the mathematical constants e , π and ϕ as follows:

A: 4 4 4 4 0 0

B: $\pi\pi\pi\pi\pi\pi$ where π is approximately 3.142

C: e e e e 7 7 where e is approximately 2.718

D: 5 5 5 $\phi\phi\phi$ where ϕ is approximately 1.618

The game is that we each have one die, we throw the dice once and the highest number wins. I invite you to choose first ANY one of the dice. Then I can always choose another one so that I will have a better chance of winning than you. You may think this is unfair and decide you want to play with the die I chose. In that case I can always choose another one so that I still have a better chance of winning than you. Investigate the probabilities and explain the choices I make in all possible cases.

Q39:

Probability vs. Odds

The probability of an event is defined as the number of ways the event can happen successfully divided by the number of ways it can possibly happen (successes + failures).

The odds in favor of an event are defined as the number of ways the event can happen successfully divided by the number of ways it can fail to happen.

If the odds in favor of an event are $\frac{a}{b}$, or a to b , then the probability of the event is $\frac{a}{a+b}$, and the odds against an event are $\frac{b}{a}$, or b to a ,

A single die is tossed.

1. What is the probability that the number of spots showing is:
 - a. 6
 - b. even
 - c. odd
 - d. less than 3?

2. What are the odds that the number of spots showing is:
 - a. 6
 - b. even
 - c. Odd
 - d. less than 3?

3. One letter is selected at random from the first 10 letters of the alphabet. What is the probability that the letter is:
 - a. a vowel
 - b. a consonant
 - c. before E in the alphabet
 - d. in the word SIDEWALK

4. What are the odds in favor of each event in #3?
 - a. a vowel
 - b. a consonant
 - c. before E in the alphabet
 - d. in the word SIDEWALK

5. Two dice are thrown. Refer to the 2 die roll chart to the right to decide the probability of each of the following events.

(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

The sum of the numbers showing is 7.

Both dice show the same number.

The dice show different numbers.

The sum of the numbers showing is 4 or 6.

6. What are the odds in favor of each event in #5?

The sum of the numbers showing is 7.

Both dice show the same number.

The dice show different numbers.

The sum of the numbers showing is 4 or 6.