(A) Lesson Context

BIG PICTURE of this UNIT:	• How do algebraically & graphically work with growth and decay applications? • What are logarithms and how do we invert or undo an exponential function? • How do we work with simple algebraic and graphic situations involving the use of logarithms (or inversing exponentials?)		
CONTEXT of this LESSON:	Where we've been We have seen what logarithms are and how to convert between exponential and logarithmic equations	Where we are Solving Exponential equations using logarithms	Where we are heading How do work with the mathematically model $f(x) = aB^{k(x+c)} + d$?

(B) Lesson Objectives:

a. Use logarithms to solve simple and complex exponential equations.
b. Solve word problems using exponents & logs

(C) BOARDWORK: Examples: Solve w/ & w/o calculator (approx vs exact)

(a) Solve $2^x = 7$	(b) Solve $4 + 2^x = 7$	(c) Solve $2^{x-2} = 7$
(d) Solve $3(2^x) = 7$	(e) Solve $2^{3x} = 7$	(f) Solve $1 - 2^{3(x-2)} = -7$
(g) Solve $2^x = -7$	(h) Solve $2e^{3x+2} = 14$	(i) Solve $1 + 2e^{3(x-2)} = 9$
(j) Solve $\dfrac{16}{2+4e^{-x}} = 4$	(k) Solve $\dfrac{24e^{-2x}}{2+4e^{-2x}} = 2$	(l) Solve $e^{\ln(x-2)} = 2$

PRACTICE

http://cdn.kutasoftware.com/Worksheets/Alg2/Solving%20Exponential%20Equations%20with%20Logarithms.pdf

(D)**Word Problems – Solve Using Logarithms**

For each: Write the equation that can be used to model the problem, show substitution into the formula, solve until in calculator ready form. Round answers to 3 decimal places. Write the answer in a complete sentence.

1) If Tanisha has $100 to invest at 8% per year, how long will it be before she has $150? If the compounded is continuous, how long will it be?

2) How long does it take for an investment to double in value if it is invested at 6% per year compounded monthly? Compounded continuously?

3) A business purchased for $650,000 in 1994 is sold in 1997 for $850,000. What is the annual rate of return on this investment? (Hint: this is an appreciation problem)

4) The number N of bacteria present in a culture at time "t" (in hours) obeys the equation $N = 1000e^{0.01t}$ After how many hours will the population equal 1500?

5) Jerome will be buying a new car for $15,000 in 3 years. How much money should he ask his parents for now so that, if he invests it at 5% compounded continuously, he will have enough to buy a car?

6) How many years will it take for an initial investment of $10,000 to grow to $25,000? Assume a rate of interest of 6% compounded continuously.

7) Jason uses his car for his job. He is allowed to depreciate the car 8% per year. IF the car was worth $23,000 new, in about how many years will the car be worth $3000?

8) At any time $t \geq 0$, in days, the number of bacteria present, y, is given by $y = Ce^{kt}$ where k is a constant. The initial population is 1000 and the population triples during the first 5 days.
   a) Write an expression for y at any time $t \geq 0$.
   b) By what factor will the population have increased in the first 10 days?
   c) At what time, t, in days, will the population have increased by a factor of 6?

9) Four months ago after it stopped advertising, a manufacturing company noticed that its sales per unit "y" had dropped each month according to the function $y = 100{,}000e^{-0.05x}$ where "x" is the number of months after the company stopped advertising.
   a) Find the projected drop in sales per unit six months after the company stops advertising.
   b) How many months until sales per unit had dropped $50,000?

10) Let P(t) represent the number of wolves in a population at time t years, when $t \geq 0$. The population of wolves is given by $P(t) = 800 - Ce^{-kt}$.
   a) If $P(0) = 500$, find P(t) in terms of t and k.
   b) If $P(2) = 700$, find k
   c) As time increases without bound, what happens to the population of wolves? Support your answer with a graph and a written explanation.

11) There are 80 grams of Cobalt-58, which has a half-life of 71 days. How long will it take to have 13 grams remaining?

12) During a certain epidemic, the number of people that are infected at any time increases at a rate proportion to the number of people that are infected at that time. If 1000 people are infected when the epidemic is first discovered and 1200 are infected 7 days later, how many people are infected 12 days after the epidemic is first discovered?

13) The following data represents the price and quantity supplied in 1997 for IBM personal computers.

Price ($/Computer)	2300	2000	1700	1500	1300	1200	1000
Quantity Supplied	180	173	160	150	137	130	113

a. Use your calculator draw the scatter plot. Use price as the independent variable. Label your axes.
b. Use your calculator to fit a logarithmic model to this data.

14) The half-life of radium is 1690 years. If 10 grams are present now, how much will be present in 50 years?

15) A hard-boiled egg has a temperature of 98 degrees Celsius. If it is put into a sink that maintains a temperature of 18 degrees Celsius, its temperature x minutes later is given by the formula $T(x) = 18 + 80e^{-0.29x}$. Hilda needs her egg to be exactly 30 degrees Celsius for decorating. How long should she leave it in the sink?

16) Ten years ago Michael paid $250 for a rare 1823 stamp. Its current value is $1000. Find the average annual rate of growth.

17) A new car that cost $12,000 decreased in value to $4000 in 5 years. Find the average annual rate of depreciation.

18) Buzz Lightyear is returning to Earth in his spaceship when he detects an oxygen leak. He knows that the rage of change of the pressure id directly proportional to the pressure of the remaining oxygen. a) Write an equation that expresses this fact. b) Solve you equation subject to the initial condition that the pressure is 3000 psi at time t = 0 when Buzz discovers the leak. c) Five hours after he discovers the leak, the pressure has dropped to 2300 psi. At the time, Buzz is still 15 hours from Earth. Will he make it home before the pressure drops to 800 psi?

19) Tanya has just inherited a diamond ring appraised at $5000. If diamonds have appreciated in value at an annual rate of 8%, what was the value of the ring 10 years ago when the ring was purchased?

20) Bacteria in a certain cultures increase at a rate proportion to the number present. If the number of bacteria doubles in three hours, in how many hours will the number of bacteria triple?

21) How long will it take an investment of $1000 to triple in value if it is invested at an annual rate of 12% compounded quarterly?

(E) **Further CHALLENGE Problems**

Exercise #1: Working with the function $f(x)=\dfrac{8}{1+3e^{-0.5x}}$,

(a) Determine the value of $f(0)$.
(b) Determine the "end behaviour" as $x\to+\infty$ (say, "evaluate" $f(1000)$).
(c) Determine the "end behaviour" as $x\to-\infty$ (say, "evaluate" $f(-1000)$).
(d) Is $f(x)=\dfrac{8}{1+3e^{-0.5x}}$ an even/odd/neither function? Explain how you know.
(e) Graph the function $f(x)=\dfrac{8}{1+3e^{-0.5x}}$. This function is called a "logistic growth function" and can be used to model population growth. Explain why.

Exercise #2: Working with the population function $P(t)=\dfrac{A}{1+Be^{-0.2t}}$, it is known that $P(0)=4$ and $P(10)=23.54$. Determine the values of A and B.

Exercise #3: Solve the following equations algebraically: (i) $\dfrac{e^{x}+e^{-x}}{e^{x}-e^{-x}}=3$ and then (ii) $e^{2x}-5e^{x}+6=0$

Exercise #4: Given the function $g(x)=e^{-\frac{1}{2}x}+e^{\frac{1}{2}x}$,

(a) Determine the value of $f(0)$.
(b) Determine the "end behaviour" as $x\to+\infty$ (say, "evaluate" $f(1000)$).
(c) Determine the "end behaviour" as $x\to-\infty$ (say, "evaluate" $f(-1000)$).
(d) Is $g(x)=e^{-\frac{1}{2}x}+e^{\frac{1}{2}x}$ an even/odd/neither function? Explain how you know.
(e) Graph $g(x)=e^{-\frac{1}{2}x}+e^{\frac{1}{2}x}$

Exercise #4: Given the function $g(x)=e^{-x^{2}}$,

(a) Determine the value of $f(0)$.
(b) Determine the "end behaviour" as $x\to+\infty$ (say, "evaluate" $f(1000)$).
(c) Determine the "end behaviour" as $x\to-\infty$ (say, "evaluate" $f(-1000)$).
(d) Is $g(x)=e^{-x^{2}}$ an even/odd/neither function? Explain how you know.
(e) Graph $g(x)=e^{-x^{2}}$

(F) **Exploring PATTERNS & Logarithms**

Pattern Set #1

$\ln\left(e^2\right)$	$\ln\left(e^3\right)$	$\ln\left(e^4\right)$	$\ln\left(e^0\right)$	$\ln\left(e^{-2}\right)$	$\ln\left(\frac{1}{e^4}\right)$	$\ln\left(\sqrt{e}\right)$	In general?
$\ln\left(2^2\right)$	$\ln\left(2^3\right)$	$\ln\left(2^4\right)$	$\ln\left(2^0\right)$	$\ln\left(2^{-1}\right)$	$\ln\left(\frac{1}{2^4}\right)$	$\ln\left(\sqrt{2}\right)$	In general?
$\log_3 2$	$\log_3 4$	$\log_3 8$	$\log_3 16$	$\log_3 32$	$\log_3 0.5$	$\log_3 0.25$	In general?

Generalization in ALL cases →

Pattern Set #2

Set A	$\ln(2)$	$\ln(3)$	$\ln(6)$	$\ln(12)$	$\ln(18)$
Set B	$\log_3(8)$	$\log_3(5)$	$\log_3(40)$	$\log_3(200)$	$\log_3(1.6)$
Set C	$\log_8 2$	$\log_8 24$	$\log_8 48$	$\log_8 96$	$\log_8 12$

Generalization in ALL cases →

Consolidating Number Patterns from Set 1 and Set 2:

If ln(2) = 0.693 and ln(3) = 1.0986 and ln(5) = 1.609, predict the values of:

(a) ln(100)	(b) ln(1.5)	(c) ln(150)	(d) ln(0.1)
(e) ln(135)	(f) ln (1.2)	(g) $\ln\left(\sqrt[3]{200}\right)$	