(A) Lesson Context

BIG PICTURE of this UNIT:	• How do algebraically & graphically work with growth and decay applications? • What are logarithms and how do we invert or undo an exponential function? • How do we work with simple algebraic and graphic situations involving the use of logarithms (or inversing exponentials?)		
CONTEXT of this LESSON:	Where we've been We have introduced the natural base e & then developed the exponential function $f(t) = Pe^{rt}$	Where we are Consolidate & extend our ability to work with exponential eqns $f(n) = aB^n$ & $f(t) = Pe^{rt}$	Where we are heading How do work with the mathematically model $f(x) = aB^{k(x+c)} + d$?

(B) Lesson Objectives:

a. Consolidate & extend our ability to work with exponential eqns $f(n) = aB^n$ & $f(t) = Pe^{rt}$

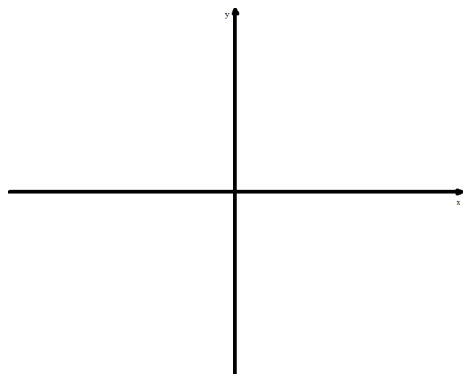## (C) Graphing with Base e (Time Permitting)

Graph Set A

SET #1

$y = e^x$

$y = e^{-x}$

$y = -e^x$

$y = -e^{-x}$

HL EXTENSION SET #3

$y = \dfrac{1}{1 + e^{-x}}$

$y = e^{-x^2}$
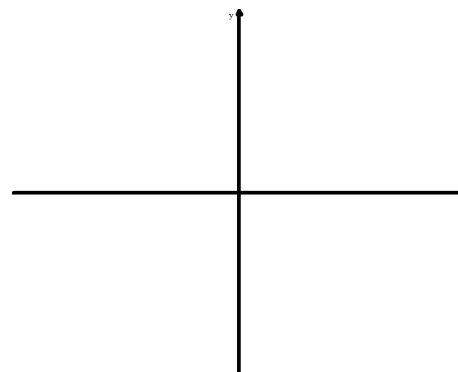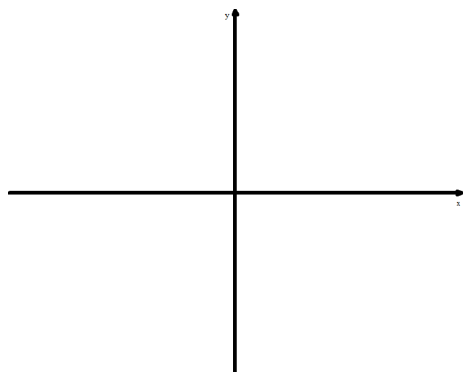
$y = e^x + e^{-x}$

$y = 1 - e^{-x}$

Graph Set B

Set #2

$y = e^x + 5$

$y = e^x - 5$

$y = 5 - e^x$

$y = 5 - e^{-x}$

Probem Set # 1 – Exponential Functions

Example # 1: The population of a town is modeled by the function $P(t) = 15,752(1.045)^t$ where t is time in years since 1990. Answer the questions below using the function given.

(a) Is the population of the town growing or shrinking... by what percentage?

(b) Find P(5). Interpret.

(c) What was the population of the town in 1990?

(d) When will the population of the town become more that 50,000?

(e) What assumptions are we making in questions (d)?

Example # 2: You invest 6,500 Euro in a bank that gives you a 5.5% annual interest rate, compounded continuously.

(a) Create an equation to model the growth of your principle investment.

(b) What is your investment worth after 7 years?

(c) How long will it take to triple your investment?

(d) If you keep the same interest rate, and want to have 30,000 Euro after 20 years... how much should your Principle Investment be? Round to the nearest dollar.

Example # 3: There has been an accident at the local Nuclear Plant and a new radioactive material (Santoagen) has been spilled. This radioactive material begins to decay exponentially. Assume that this decay is **continuous**. There were 1820 Bq (becquerels) of Santoagen which is way too radioactive to work with. 8 hours later there were 576 Bq

(a) What is the decay rate of Santoagen?

(b) This material becomes non-leathal when there is a max of 20 Bq. When will it be safe for workers to enter the space and clear it out?

(c) Will there ever be 0 Bqs left of the Santoagen material?

(d) What is the half life of Santoagen?

Problem Set # 2 – Working with Half Life and Doubling

Example #1: The half life of caffeine in a child's system when a child eat or drinks something with cafine is 1.5 hours. How much caffine would remain in a child's body if the child ate a chocolate bar with 30 mg of caffine 8 hours before?

Example # 2: The half-life of Carbon-14 is about 5370 years. What percentage of the original carbon-14 would you expect to find in a sample after 2500 years?

Example #3: The population of a certain bacteria grows exponentially and can be modeled by $P(t) = 18(2)^{\frac{t}{3.5}}$ where t is time in hours.

   (a) What was the population of the bacteria when the observations started?

   (b) What is the percentage growth of this population? What do we also call this kind of growth?

   (c) What is the doubling period of this bacteria? Explain what this means.

   (d) How much bacteria will be present in 35 hours?

   (e) When will the bacteria reach a population of 294,900?

   (f) Let's now make the assumption that the bacteria population was changing **continuously**. Knowing the doubling period from Q(c), show that the equation can also be written as $P(t) = 18e^{0.198042t}$ or $P(t) = 18e^{\ln \sqrt[3.5]{2}\, t}$

   (g) Graph BOTH equations ($P(t) = 18(2)^{\frac{t}{3.5}}$ as well as $P(t) = 18e^{0.198042t}$. Observations?

Problem Set # 3 – Application of the Natural Base

Example #1: The number of bacteria in a culture is given by the function $n(t) = 10e^{0.22t}$

(a) What is the rate of growth of this bacterium population? Express your answer as a percentage.

(b) What is the initial population of the culture (at t = 0)?

(c) Evaluate and interpret n(15).

(d) Solve and interpret 500 = n(t).

(e) What is the doubling time for this bacterial population?

Example #2:

60. **Population Growth.** If the population in Mexico is around 100 million people now and if the population grows continuously at an annual rate of 2.3%, what will the population be in 8 years? Compute the answer to two significant digits.

Example #3:

62. **Population Growth.** In 1996 the population of Germany was 84 million and the population of Egypt was 64 million. If the populations of Germany and Egypt grow continuously at annual rates of −0.15% and 1.9%, respectively, when will Egypt have a greater population than Germany?

Problem Set #4 – Applications of the Natural Base

Example #1:

You invest $5250 in an account that pays 4.38% compounded continuously.

(a) Determine the value of this investment after 7 years.

(b) Mr. S would like to invest in an account that doubles his money in 7 years. What interest rate should the invest have if: (i) his money was compounded quarterly, (ii) his money was compounded continuously

(c) CHALLENGE: This account is special because the interest rate is **increased** by 0.1% every year. Determine the new value of the investment after 7 years.

Example #2

Three years ago, the fish population in Loon Lake was 2500. Due to the effects of acid rain, there are now about 1950 fish in the lake. Assume that the decline of the fish population is exponential and is **continuous**. Find the predicted fish population 5 years from now.

Example #3

Advertising. A company is trying to expose a new product to as many people as possible through television advertising in a large metropolitan area with 2 million potential viewers. A model for the number of people N, in millions, who are aware of the product after t days of advertising was found to be

$$N = 2(1 - e^{-0.037t})$$

(A) How many viewers are aware of the product after 2 days? After 10 days? Express answers as integers, rounded to three significant digits.

(B) How many days will it take until half of the potential viewers will become aware of the product? Round answer to the nearest integer.

CHALLENGE QUESTIONS

58. Investigate the behavior of the functions $g_1(x) = xe^x$, $g_2(x) = x^2e^x$, and $g_3(x) = x^3e^x$ as $x \to \infty$ and as $x \to -\infty$, and find any horizontal asymptotes. Generalize to functions of the form $g_n(x) = x^ne^x$, where n is any positive integer.

57. Investigate the behavior of the functions $f_1(x) = x/e^x$, $f_2(x) = x^2/e^x$, and $f_3(x) = x^3/e^x$ as $x \to \infty$ and as $x \to -\infty$, and find any horizontal asymptotes. Generalize to functions of the form $f_n(x) = x^n/e^x$, where n is any positive integer.

C∫ *It is common practice in many applications of mathematics to approximate nonpolynomial functions with appropriately selected polynomials. For example, the polynomials in Problems 53–56, called **Taylor polynomials,** can be used to approximate the exponential function $f(x) = e^x$. To illustrate this approximation graphically, in each problem graph $f(x) = e^x$ and the indicated polynomial in the same viewing window, $-4 \le x \le 4$ and $-5 \le y \le 50$.*

53. $P_1(x) = 1 + x + \frac{1}{2}x^2$

54. $P_2(x) = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3$

55. $P_3(x) = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4$

56. $P_4(x) = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5$

C **19.** It can be proved that $e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots + \frac{1}{n!} + \cdots$.

Approximate e by using the first five terms. (*Note:* $n!$, read "n factorial," denotes $n(n-1)(n-2)\cdots 2 \cdot 1$. For example, $3! = 3 \cdot 2 \cdot 1 = 6$.)

82. Training. A trainee is hired by a computer manufacturing company to learn to test a particular model of a personal computer after it comes off the assembly line. The learning curve for an average trainee is given by

$$N = \frac{200}{4 + 21e^{-0.1t}}$$

(A) How many computers can an average trainee be expected to test after 3 days of training? After 6 days? Round answers to the nearest integer.

(B) How many days will it take until an average trainee can test 30 computers per day? Round answer to the nearest integer.

(C) Does N approach a limiting value as t increases without bound? Explain.

77. Newton's Law of Cooling. This law states that the rate at which an object cools is proportional to the difference in temperature between the object and its surrounding medium. The temperature T of the object t hours later is given by

$$T = T_m + (T_0 - T_m)e^{-kt}$$

where T_m is the temperature of the surrounding medium and T_0 is the temperature of the object at $t = 0$. Suppose a bottle of wine at a room temperature of 72°F is placed in the refrigerator to cool before a dinner party. If the temperature in the refrigerator is kept at 40°F and $k = 0.4$, find the temperature of the wine, to the nearest degree, after 3 hours. (In Exercise 4-7 we will find out how to determine k.)

CHALLENGE: Newton's Law of Cooling PROBLEM

(http://mrsantowski.tripod.com/2015IntegratedMath3/Homework/Newtons_Law_of_Cooling.pdf)