(A) Lesson Context

BIG PICTURE of this UNIT:	• How do algebraically & graphically work with growth and decay applications? • What are logarithms and how do we invert or undo an exponential function? • How do we work with simple algebraic and graphic situations involving the use of logarithms (or inversing exponentials?)		
CONTEXT of this LESSON:	Where we've been We have seen simple algebra skills (exponent laws & solving exponential eqns) in IM2 as well as graphs of simple exp fcns in the form of $y = AB^x$	Where we are Review fundamental algebra & graphic skills associated with exponential functions & their applications	Where we are heading How do work with the mathematically model $f(x) = aB^{k(x+c)} + d$?

(B) Lesson Objectives:

a. Review KEY algebra skills associated with exponential functions
b. Review the application of exponential models to assorted real world contexts.

(C) REVIEW: Analyzing Data Sets

Given the following data sets, analyze the pattern, predict (i) the next three terms of the sequence; (ii) the previous 3 terms and then (iii) the 20th term in the sequence.

Data Set #1

x	y
1	9
2	27
3	81
4	243
5	729
6	2187

Data Set #2

x	y
2	-22.5
3	-33.75
4	-50.625
5	-75.9375
6	-113.90625
7	-170.859375

Data Set #3

x	y
1825	200
1850	252
1875	318
1900	401
1925	504
1950	635
1975	800

Data Set #4 – Variations:

x	y
1	19
2	37
3	91
4	253
5	739
6	2197

Data Set #5 – Variations:

x	y
2	177.5
3	166.25
4	149.375
5	124.0625
6	86.09375
7	29.140625

(D) REVIEW: Graphs of Exponential Functions

(a) Graph/sketch and analyze (domain, range, x- and y-intercept(s), asymptote) the function $f(x) = 2^x$. Label 5 points on your graph/sketch.

(b) Graph/sketch and analyze (domain, range, x- and y-intercept(s), asymptote) the function $f(x) = 3^x$. Label 5 points on your graph/sketch.

(c) Graph/sketch and analyze (domain, range, x- and y-intercept(s), asymptote) the function $f(x) = \left(\frac{1}{2}\right)^x$. Label 5 points on your graph/sketch.

(d) Graph/sketch and analyze (domain, range, x- and y-intercept(s), asymptote) the function $f(x) = 2^{-x}$. Label 5 points on your graph/sketch.

(e) CONNECTIONS: If $f(x) = 2^x$, graph $y = f^{-1}(x)$

(f) CONNECTIONS: If $f(x) = 2^x$, graph $y = \frac{1}{f(x)}$.

(E) REVIEW: Simplfying & Evaluating (Q2,4,6,7)

2. Evaluate.

(a) 2^8 (b) 3^5 (c) $(-8)^3$ (d) -4^2

(e) -6^{-2} (f) $350(1.02)^3$ (g) $(-2)^{-5}$ (h) $(-1)^{-100}$

(i) $\left(\frac{2}{3}\right)^2$ (j) $\left(\frac{3}{4}\right)^{-3}$ (k) $5^{-1} + \left(\frac{-4}{9}\right)^2$ (l) $\frac{7}{8} - 4^{-2}$

3. Evaluate to two decimal places where necessary.

(a) $4^{\frac{1}{2}}$ (b) $(-8)^{\frac{1}{3}}$ (c) $\left(\frac{1}{16}\right)^{\frac{1}{2}}$ (d) $216^{-\frac{1}{3}}$

(e) $2^{\frac{1}{2}}$ (f) $\left(\frac{5}{6}\right)^{\frac{1}{3}}$ (g) $200^{-\frac{2}{5}}$ (h) $(17.4)^{-\frac{3}{7}}$

4. Write each expression as a power of 2.

(a) $2^4 \times 2^7$ (b) $2^6 \div 2^{-3}$ (c) $4^2 \div 2^m$ (d) 8^n

5. Evaluate.

(a) $49^{\frac{1}{2}} + 16^{\frac{1}{4}}$ (b) $16^{\frac{3}{2}} + 16^{-0.5}$

(c) $\left(\frac{1}{8}\right)^{\frac{1}{3}} - \sqrt[3]{\frac{27}{125}} + 4(8^{-\frac{2}{3}})$ (d) $(2^2 \times 5)^{-1}$

(e) $\left(\frac{3^{-1}}{2^{-1}}\right)^{-2}$ (f) $(5^0 \times 5^4 \div 5^3)^{-4}$

6. Evaluate each expression for $x = 2$ and $y = -2$.

(a) $\frac{4x^{-2}y^3}{x^3}$ (b) $(3x^2y^{-3})^{-2}$

7. Simplify.

(a) $\frac{x^4y^2}{x^2y}$ (b) $\left(\frac{-1}{a}\right)^2(a^4b^2)^5$ (c) $(-d^3)^4\left(\frac{c}{d}\right)^6$ (d) $\left(\frac{x^5}{y^2}\right)^4\left(\frac{y^3}{x}\right)^2$

(F) REVIEW: Solving Exponential Equations

(a) Solve $3^x = 27$

(b) Solve $2^x = \frac{1}{4}$

(c) Solve $5^{2x-1} = \frac{1}{125}$

(d) Solve $3(2^x) = 24$

(e) Solve $3 - (2^x) = -29$

(f) Solve $9^{-x-2} = \left(\frac{1}{27}\right)^{x+3}$

(G) NEW SKILL - EXPLORATION: How do we work with exponents in the form of a/b?

Part 1 ➔ use your TI-84 to help answer these questions ➔ try to figure out HOW to come up with the answer without the use of the TI-84

$16^{\frac{1}{2}}$ $144^{\frac{1}{2}}$ $\left(\frac{1}{625}\right)^{\frac{1}{2}}$ $8^{\frac{1}{2}}$

$27^{\frac{1}{3}}$ $343^{\frac{1}{3}}$ $\left(\frac{1}{1000}\right)^{\frac{1}{3}}$ $100^{\frac{1}{3}}$

Part 2 ➔ use your TI-84 to help answer these questions ➔ try to figure out HOW to come up with the answer without the use of the TI-84

$8^{\frac{1}{3}}$ $8^{\frac{2}{3}}$ $8^{\frac{4}{3}}$ $8^{\frac{5}{3}}$

$16^{\frac{3}{4}}$ $25^{\frac{3}{2}}$ $27^{\frac{2}{3}}$ $\left(\frac{1}{8}\right)^{\frac{2}{3}}$

Part 3 ➔ do NOT use a calculator as you work these out

$4^{\frac{3}{2}}$ $27^{\frac{4}{3}}$ $32^{\frac{3}{5}}$ $9^{-\frac{3}{2}}$

$\left(\frac{16}{81}\right)^{\frac{3}{4}}$ $\left(\frac{1}{4}\right)^{-\frac{3}{2}}$ $(-8)^{-\frac{2}{3}}$ $\left(\frac{-343}{1000}\right)^{\frac{2}{3}}$

(H) Modeling with Exponential Functions in the form of $f(x)=a(B)^x$

a. **Example 1**: The number of a certain bacteria doubles every hour. The initial population in a sample of this bacteria is 36.
   1. Determine the number of bacteria after 8 hours.
   2. Determine an exponential model for *N*, the number of bacteria after *t* hours i.e. N(t) = ???

b. **Example 2**: The number of bacteria in a different culture doubles every 2 hours. The initial pop. is again 36.

   1. Determine the number of bacteria after 8 hours.
   2. Determine an exponential model for *N*, the number of bacteria after *t* hours i.e. N(t) = ???
   3. Determine the growth rate PER HOUR.

c. **Example 3**: The population of a small town appears to be increasing exponentially. Town planners need a model for predicting the future population. In the year 2000, the population was 35,000, while in the year 2010, the population grew to 57,010.

1. PREDICT: What will be the town's population in 2030?
2. Create an **exponential** algebraic model for the town's population growth.
3. Check your population model by using the fact that the town's population was 72,825 in 2015.
4. CALCULATE: What will be the town's population in 2030?

d. **Example 4**: Three years ago, the fish population in Loon Lake was 2500. Due to the effects of acid rain, there are now about 1950 fish in the lake. Assume that the decline of the fish population is exponential. Find the predicted fish population 5 years from now.

e. **Example 5**: What is the average annual rate of inflation if a loaf of bread cost $1.19 in 1991 but costs $1.50 in 2001?