Lesson 7 : Periodic Phenomenon | Unit 5 – Trigonometric Functions

(A) Lesson Context

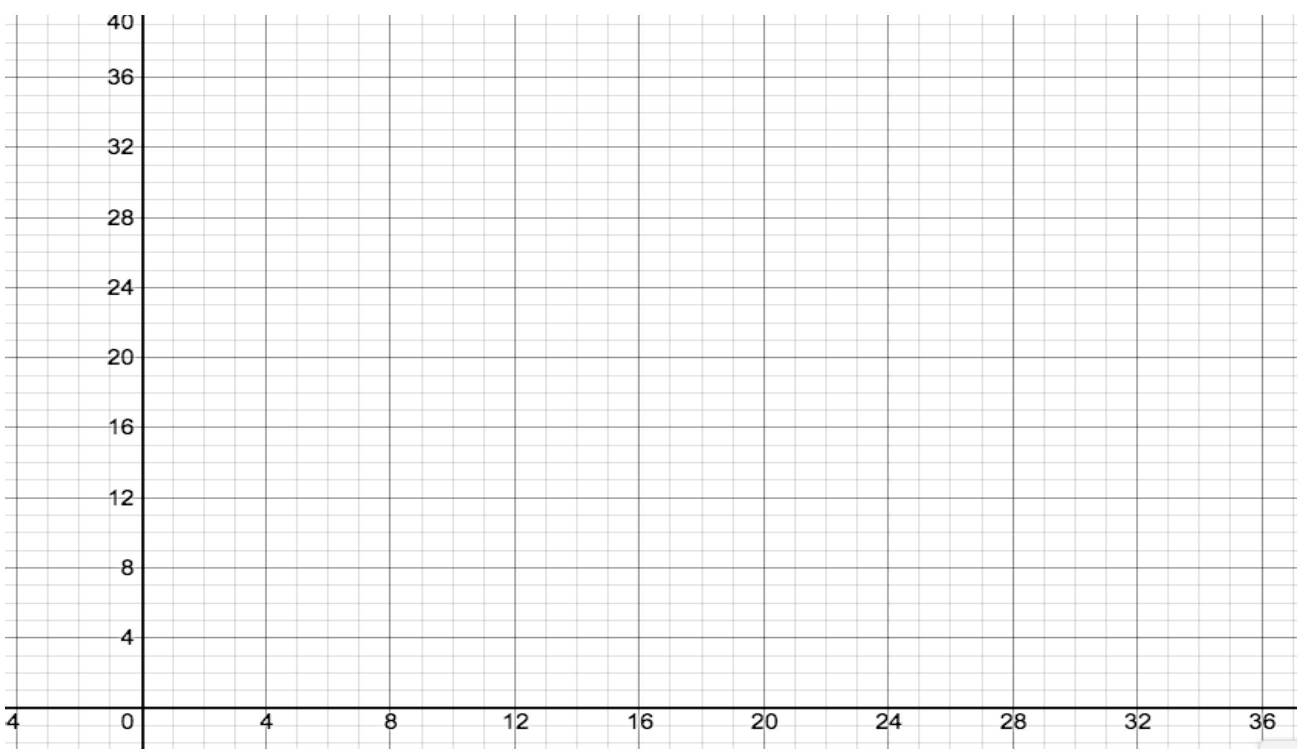
BIG PICTURE of this UNIT:	• How do work through geometry based problems, wherein triangles are used to model the problem • How do we model phenomenon that are periodic in nature		
CONTEXT of this LESSON:	Where we've been We have practiced & reviewed working with triangles using RTT, Sine Law & Cosine Law	Where we are What do graphs of periodic/cyclic phenomenon look like & what are their key features?	Where we are heading How do we mathematically model phenomenon that are periodic in nature)

(B) Lesson Objectives:

a. Identify situations that can be modeled using sinusoidal and other periodic functions
b. Interpret graphs of sinusoidal and other periodic phenomenon.
c. Introduce key terms used in the analysis and understanding of periodic phenomenon

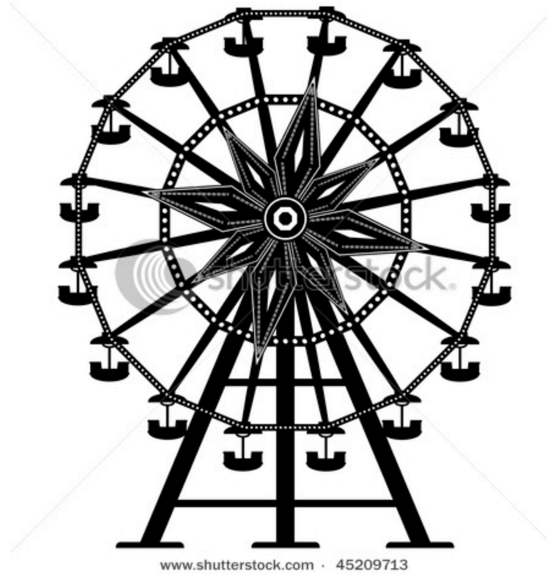
(C) Fast Five:

A carnival Ferris wheel with a radius of 14 m makes one complete revolution every 16 seconds. The bottom of the wheel is 2.0 m above the ground. A person starts a ride at the bottom of the Ferris wheel when a stop watch is started. Draw a sketch showing the riders height as a function of time. Estimate how high above the ground that person will be after 1 minute and 8 seconds.



(D) Example #1 - Modelling Periodic Phenomenon – Riding on a Ferris Wheel

You are going for a ride on a Ferris wheel. The Ferris wheel rotates at a constant speed. It has a radius of 15 meters and the bottom of the wheel is 5 meters off the ground. It takes 60 seconds to go around the Ferris wheel one time. Again, you will be graphing the dependent variable, height (H), of your carriage in meters above the ground, at time (t) seconds.



1. You just get into your carriage at the ***MIDDLE*** of the wheel. What is your height when t=0? Plot this point on your graph.

2. What is the highest you will go? When will this happen? Plot this point on your graph.

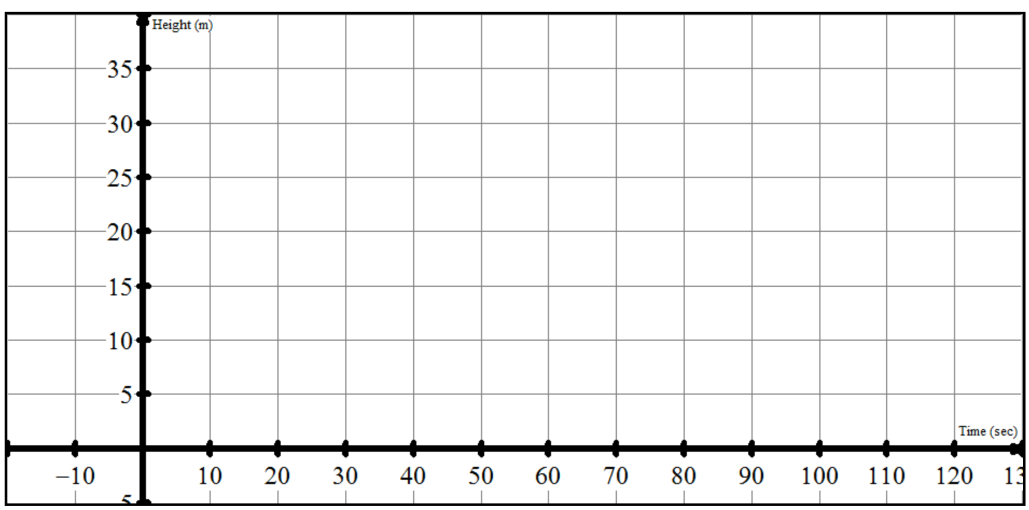
3. How high will you be after 30 seconds? Plot this point on your graph.

4. Is there another time (t) when you will be at the same height as above at 30 seconds? When will this be? Plot this point on your graph.

5. When will your height (h) be 5 meters? Plot this point on your graph.

6. Expand your graph to show your height on the Ferris wheel over 2 cycles of rotation.

Time (sec)	Height (m)
0 sec	
15 sec	
30 sec	
45 sec	
60 sec	



a. Find the **period** of this function. Label it.

b. Find the **range** of this function. Label it.

c. Find the **domain** of this function. Label it.

d. What is the minimum point… call that the **Trough.** Label it!

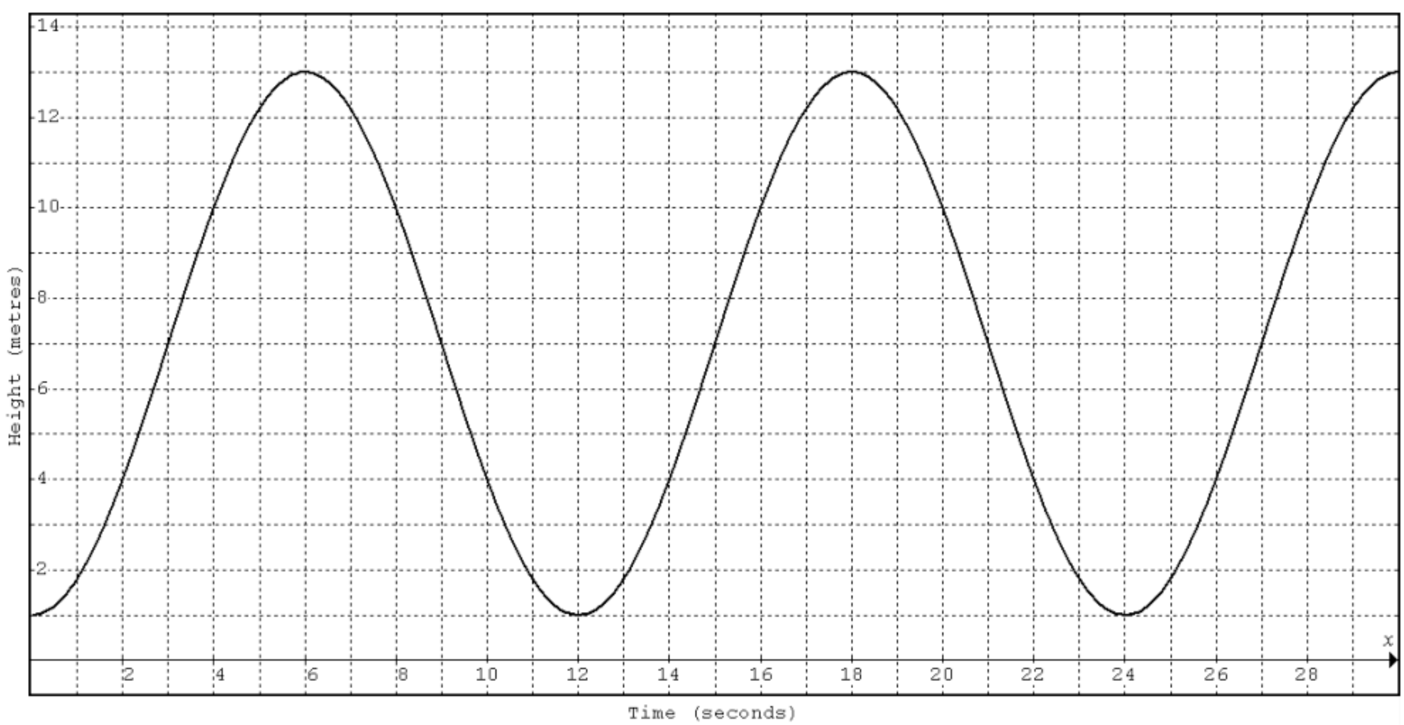
e. What is the maximum point… call that the **Peak.** Label it!

f. **Equation of the Axis of the Curve/Equilibrium Axis:** The equation of the horizontal line halfway between the minimum and maximum…Find if for the graph. $y = \dfrac{Max.\,Value + Min.\,Value}{2}$

g. **Amplitude:** Half the distance between the maximum and the minimum. Find it for the graph. Label it.

(E) Example #2 - Modelling Periodic Phenomenon – Riding on a Ferris Wheel

Victoria rode on a Ferris wheel at Cluney Amusements. The graph models Victoria's height above the ground in metres in relation to time in seconds. The data were recorded while the ride was in progress.



a. What is the height of the axle on the Ferris wheel?

b. What is the radius of the Ferris wheel?

c. What is the maximum height of the Ferris wheel?

d. How long does it take for the Ferris wheel to complete one revolution?

e. Victoria boards the Ferris wheel at its lowest point. How high above the ground is this?

f. Within the first 20 seconds, how many times is Victoria at a height of 7 m above the ground?

g. What is Victoria's approximate height above ground at 16 seconds?

h. What is Victoria's approximate height above the ground at 57 seconds?

i. Find the **period** of this function. Label it.

j. Find the **range** of this function. Label it.

k. What is the minimum point… call that the **Trough.** Label it!

l. What is the maximum point… call that the **Peak.** Label it!

m. **Equation of the Axis:** The equation of the horizontal line halfway between the minimum and maximum…Find if for the graph. $y = \dfrac{Max.\ Value + Min.\ Value}{2}$

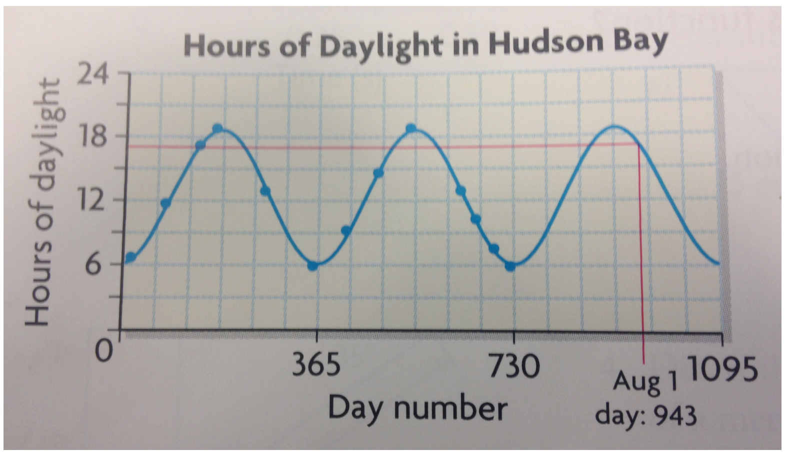
n. **Amplitude:** Half the distance between the maximum and the minimum. Find it for the graph. Label it.

(F) Example #3 - Modelling Periodic Phenomenon – Daylight Hours

The number of hours of daylight in any particular location changes with the time of the year. The table shows the average number of hours of daylight for approximately a two year period at Hudson Bay, Nunavut. Day 15 is January 15th 2010. Day 74 is March 15 2010… day 441 is February 15 of 2011…etc.

Day	15	74	135	166	258	349	411	470	531	561	623	653	684	714
Hours	6.7	11.7	17.2	18.8	12.9	5.9	9.2	14.6	18.8	18.1	12.9	10.2	7.5	5.9

This is a scatter plot of the situation. Lets discuss a few key ideas before we move onto the next periodic functions?



a. Why does it make sense to call a graph of the hours of daylight a **Periodic Function?** ______________________

b. Define Periodic Function: ______________________

c. How can we find the Period of the graph...

   a. From the table:

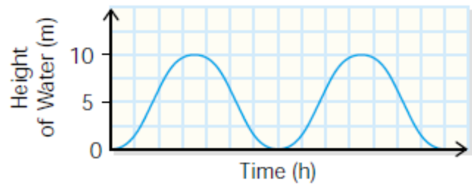
   b. From the graph:

d. Which points on the graph could help you determine the range (y distance) of the graph?______________________

(G) Further Exampes

Example 3

The Bay of Fundy, which is between New Brunswick and Nova Scotia, has the highest tides in the world. There can be no water on the beach at low tide, while at high tide the water covers the beach.

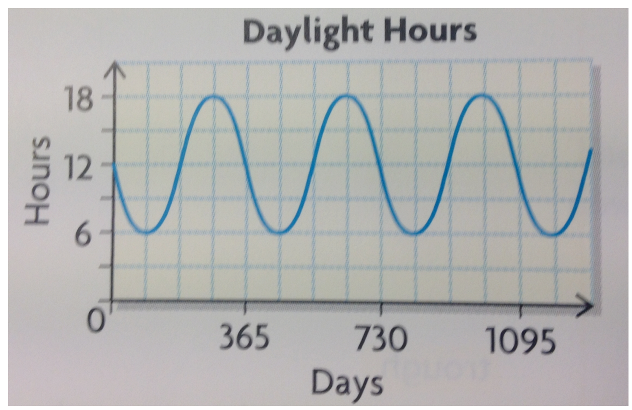


(a) Why can you use periodic functions to model the tides?

(b) What is the change in depth of water from low tide to high tide?

(c) Determine the equation of the axis of the curve.

(d) What is the amplitude of the curve?

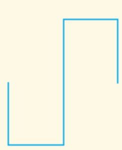


1. Given the graph included, explain why a PERIODIC model is appropriate.

2. Find the **period**, **equation of axis** and **amplitude**.

(H) Independent Practice

5. Sketch periodic graphs to satisfy the given properties.

Shape	Period	Amplitude	Equation of Axis	Number of Cycles
	4	6	$y = 2$	2
	3	4	$y = 1$	3
	$\frac{1}{2}$	5	$y = -3$	2