Lesson 5.6:
Angles in Standard Position

IM3 - Santowski

Fast Five – Opening Exercises

- Use your TI – 84 calculator:

- Evaluate sin(50°) ➔ illustrate with a diagram
- Evaluate sin(130°) ➔ Q ➔ How can a right triangle have a 130° angle?

- Evaluate sin(230°) ➔ illustrate with a diagram?? Q ➔ How can a right triangle have a 230° angle?

- Evaluate sin(310°) ➔ illustrate with a diagram??
- Evaluate sin(770°) ➔ illustrate with a diagram??

- Evaluate sin(-50°) ➔ illustrate with a diagram?? Q ➔ How can a right triangle have a -50° angle?

QUIZ

- Draw the following angles in standard position

- 50°
- 130°
- 230°
- 310°
- 770°
- -50°

BIG PICTURE

- The first of our keys ideas as we now start our Trig **Functions** Lessons:

- (1) **How do we use current ideas to develop new ones**

BIG PICTURE

■ The first of our keys ideas as we now start our Trig **Functions** Lessons:

■ (1) **How do we use current ideas to develop new ones** ➔ We will use RIGHT TRIANGLES and CIRCLES to help develop new understandings

BIG PICTURE

- The first of our keys ideas as we now start our Trig **Functions** Lessons:

- (2) **What does a TRIANGLE have to do with SINE WAVES**

BIG PICTURE

- The first of our keys ideas as we now start our Trig **Functions** Lessons:

- (2) **What does a TRIANGLE have to do with SINE WAVES** ➔ How can we REALLY understand how the sine and cosine ratios from right triangles could ever be used to create function equations that are used to model periodic phenomenon

(A) Angles in Standard Position

- Angles in standard position are defined as angles drawn in the Cartesian plane where the initial arm of the angle is on the *x* axis, the vertex is on the origin and the terminal arm is somewhere in one of the four quadrants on the Cartesian plane

(A) Angles in Standard Position

- To form angles of various measure, the terminal arm is simply rotated through a given angle

a positive angle

a negative angle

(A) Angles in Standard Position

■ We will divide our Cartesian plane into 4 quadrants, each of which are a multiple of 90 degree angles

The x-y plane is divided into four **quadrants** by the x- and y-axes. If θ is a positive angle, then the terminal arm lies in

- quadrant I when $0° < \theta < 90°$
- quadrant II when $90° < \theta < 180°$
- quadrant III when $180° < \theta < 270°$
- quadrant IV when $270° < \theta < 360°$

(A) Coterminal Angles

- **Coterminal angles** share the same terminal arm and the same initial arm.

- As an example, here are four different angles with the same terminal arm and the same initial arm.

If $\theta_1 = 120°$, then

$\theta_2 = 360° + 120°$
$\quad = 480°$

$\theta_3 = 720° + 120°$
$\quad = 840°$

$\theta_4 = -360° + 120°$
$\quad = -240°$

(A) Principle Angles and Related Acute Angles

■ The **principal angle** is the angle between 0° and 360°.

■ The coterminal angles of 480°, 840°, and 240° all share the same principal angle of 120°.

■ The **related acute angle** is the angle formed by the terminal arm of an angle in standard position and the *x*-axis.

■ The related acute angle is always positive and lies between 0° and 90°.

(B) Examples

■ **Example 1**

Determine the principal angle and the related acute angle for $\theta = -225°$.

(B) Examples

■ **Example 1**

Determine the principal angle and the related acute angle for $\theta = -225°$.

Solution

Sketch $\theta = -225°$ terminating in quadrant II. Label the principal angle and the related acute angle.

The principal angle is the smallest positive angle that is coterminal to $-225°$. In this case, $360° - 225° = 135°$. The related acute angle lies between the terminal arm and the x-axis. It is positive but less than $90°$. In this case, $\left| -225° - (-180°) \right| = 45°$. Or, using the principal angle, $180° - 135° = 45°$.

(B) Examples

- **Example 2**

  Determine the next two consecutive positive coterminal angles and the first negative coterminal angle for 43°.

(B) Examples

Example 2

- Determine the next two consecutive positive coterminal angles and the first negative coterminal angle for $43°$.

Solution

Sketch each situation, showing the principal angle of $43°$.

(a) The first positive coterminal angle for $43°$ is $360° + 43° = 403°$.

(b) The second coterminal angle is $360° + 360° + 43° = 763°$.

(c) The first negative coterminal angle is $-360° + 43° = -317°$.

(B) Examples

- For the given angles, determine:

- (a) the principle angle
- (b) the related acute angle (or reference angle)
- (c) the next 2 positive and negative co-terminal angles

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(i) $143°$

(ii) $-132°$

(iii) $419°$

(iv) $-60°$

(C) Ordered Pairs & Right Triangle Trig

- To help discuss angles in a Cartesian plane, we will now introduce ordered pairs to place on the terminal arm of an angle

$90° < \theta_1 < 180°$
θ_1 terminates in quadrant II.

$180° < \theta_2 < 270°$
θ_2 terminates in quadrant III.

$P(x, y)$ lies in the negative y-axis.
$\theta_3 = 270°$

(C) Ordered Pairs & Right Triangle Trig

- So to revisit our trig ratios now in the context of the xy co-ordinate plane:

- We have our simple right triangle drawn in the first quadrant

$$\sin\theta = \frac{o}{h} = \frac{y}{r} \qquad \csc\theta = \frac{h}{o} = \frac{r}{y}$$

$$\cos\theta = \frac{a}{h} = \frac{x}{r} \qquad \sec\theta = \frac{h}{a} = \frac{r}{x}$$

$$\tan\theta = \frac{o}{a} = \frac{y}{x} \qquad \cot\theta = \frac{a}{o} = \frac{x}{y}$$

(C) Ordered Pairs & Right Triangle Trig

- Point *P* (-3, 4) is on the terminal arm of an angle, θ, in standard position.

- **(a)** Sketch the principal angle, θ and show the related acute/reference angle

- **(b)** Determine the values of all six trig ratios of θ.

- **(c)** Determine the value of the related acute angle to the nearest degree and to the nearest tenth of a radian.

- **(d)** What is the measure of θ to the nearest degree and to the nearest tenth of a radian?

(C) Ordered Pairs & Right Triangle Trig - Examples

■ Point $P(-9, 4)$ is on the terminal arm of an angle in standard position.
   (a) Sketch the principal angle, θ.
   (b) What is the measure of the related acute angle to the nearest degree?
   (c) What is the measure of θ to the nearest degree?

Point $P(-5, -3)$ is on the terminal arm of an angle, θ, in standard position.
   (a) Sketch the principal angle, θ.
   (b) What is the measure of the related acute angle to the nearest degree?
   (c) What is the measure of θ to the nearest degree?
   (d) What is the measure of the first negative coterminal angle?

Point $P(-5, -8)$ is on the terminal arm of an angle, θ, in standard position.
Determine all values of θ for $-540° \leq \theta \leq 270°$.

(C) Ordered Pairs & Right Triangle Trig - Examples

- Determine the angle that the line 2y + x = 6 makes with the positive x axis