

**(A) Lesson Context**

BIG PICTURE of this UNIT:	<ul style="list-style-type: none"> <li>• What is a Polynomial and how do they look?</li> <li>• What are the attributes of a Polynomial?</li> <li>• How do I work with Polynomials?</li> </ul>		
CONTEXT of this LESSON:	Where we've been  We have discussed the basic appearance of graphs of polynomial functions and the process of synthetic division	Where we are  How can we determine the zeroes of a polynomial function when the equation is in standard form?	Where we are heading  What are the key attributes of a polynomial and how do these affect the shape?

**(B) Lesson Objectives:**

- Use the process of Synthetic Division to develop new understandings about polynomial roots and factors
- Work toward developing an understand the connection between roots, zeroes and factors.
- Factor and sketch polynomial functions when equations are presented in standard form.

**(C) KEY POINTS FROM Lesson 4.4**

- The process of synthetic division can be used to evaluate a polynomial at a given value for  $x$
- If  $P(x)$  is divided by  $(x - R)$  and there is NO remainder, then  $(x - R)$  is a FACTOR of  $P(x)$
- If  $(x - R)$  is a linear factor of the polynomial  $P(x)$ , then then the graph has an  $x$ -intercept at  $x = R$

**(D) Opening Exercises: Working with the Divisor  $(ax+b)$** 

Divide $P(x) = 4x^3 - 2x^2 - 6x - 1$ by $2x - 1$ using SD	Show that $2x - 3$ is a factor of $P(x) = 2x^3 + x^2 - 22x - 24$
Divide $P(x) = 2x^3 - x^2 - 7x + 6$ by $2x + 3$ using SD	Show that $3x - 5$ is a factor of $P(x) = 6x^3 - 7x^2 - 14x + 15$

**(E) Factoring Cubics where a = 1: Practicing the BASICS**

Use synthetic division to help answer the following questions:

Example #1

Factor  $x^3 + 2x^2 - 11x - 12$

Example #3

Factor  $P(x) = x^3 - 4x^2 - 17x + 60$

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Example #2

Factor  $P(x) = x^3 + 4x^2 - 4x - 16$

Example #4

Factor  $P(x) = x^3 - x^2 - 5x - 3$

**(F) Extending Skills: Factoring Cubics where a ≠ 1: Extending the BASICS**

Use synthetic division to help answer the following questions:

Example #1

Factor  $2x^3 - x^2 - 7x + 6$

Example #3

Factor  $P(x) = 2x^3 + x^2 - 22x + 24$

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Example #2

Factor  $P(x) = 3x^3 + x^2 - 22x - 24$

Example #4

Factor  $P(x) = 6x^3 - 7x^2 - 14x + 15$

**(G) Extending Skills: Factoring Cubics: Irrational roots: Extending the BASICS**

Use synthetic division to help answer the following questions:

Example #1

Factor  $x^3 - 9x + 10$

Example #3

Factor  $P(x) = 2x^3 + 5x^2 + x - 2$

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Example #2

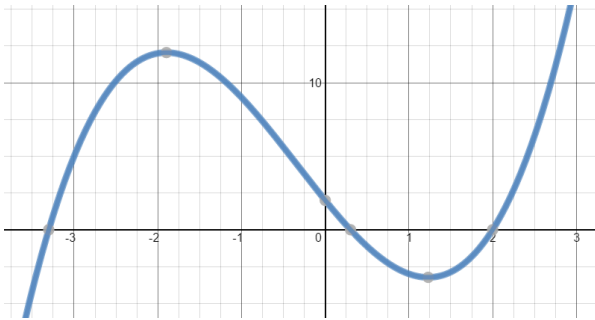
Factor  $P(x) = x^3 + 5x^2 - 2x - 6$

Example #4

Factor  $P(x) = 3x^3 - 7x^2 + 2x + 2$

**(H) Further Examples**

- Which binomials are factors of  $P(x) = 2x^3 - x^2 - 7x + 6$ ? (a)  $x + 3$  (b)  $2x - 3$
- Which binomials are factors of  $P(x) = -2x^4 - 7x^3 + 22x^2 + 63x - 36$ ? (a)  $x - 1$  (b)  $x - 3$
- Given the polynomial  $P(x) = x^3 - 2x^2 - 21x - 18$ , is  $x = 6$  a zero of  $P(x)$ ? is  $x = -2$  a zero of  $P(x)$ ?
- Given the polynomial  $P(x) = x^4 - 3x^3 + 3x^2 - 3x + 2$ , is  $x = 2$  a root of  $P(x)$ ? is  $x = -2$  a root of  $P(x)$ ?
- Given the polynomial  $g(x) = 2x^3 + x^2 - 27x - 35$ , one factor of  $x + 3$  is given. Determine the other factors.
- Factor  $P(x) = x^4 - 6x^3 + 22x^2 - 30x + 13$  given that  $x = 1$  is a double root.
- Given the polynomial  $h(x) = x^4 + 3x^3 - x - 3$ , one of the roots is  $x = -3$ . Determine the other roots.
- Determine all roots of the polynomial  $A(x) = x^3 + x^2 - 7x + 2$ , given the following graph of  $A(x)$



- Factor using the Factor Theorem:  $f(x) = x^3 - 3x^2 - 10x + 24$
- Factor using the Factor Theorem:  $f(x) = x^4 + 2x^3 - 23x^2 - 24x + 144$
- Sketch the function  $g(x) = 6 + 5x - 2x^2 - x^3$
- Sketch the function  $f(x) = 3x^3 + x^2 - 22x - 24$  & label all intercepts.
- Sketch the function  $f(x) = x^4 + x^3 - 7x^2 - x + 6$  & label all intercepts.

## More on Factors, Zeros, and Dividing

**Factor each and find all zeros. One factor has been given.**

1)  $f(x) = x^3 + 9x^2 + 23x + 15; x + 5$

2)  $f(x) = x^3 - x^2 - 14x + 24; x - 3$

3)  $f(x) = x^4 + 3x^3 - 13x^2 - 15x; x - 3$

4)  $f(x) = x^3 - 12x^2 + 47x - 60; x - 3$

5)  $f(x) = x^3 - 7x^2 + 2x + 40; x - 5$

6)  $f(x) = x^3 - 3x^2 - 9x + 27; x - 3$

7)  $f(x) = 10x^3 + 37x^2 + 37x + 6; 5x + 1$

8)  $f(x) = 25x^3 + 150x^2 + 131x + 30; 5x + 3$

9)  $f(x) = 5x^3 + 21x^2 - 21x - 5; x + 5$

10)  $f(x) = 3x^3 - 4x^2 - 9x + 10; x - 2$

$$11) f(x) = 5x^3 + 9x^2 - 26x - 24; x + 3$$

$$12) f(x) = 6x^3 + 7x^2 - 1; 2x + 1$$

**Factor each and find all zeros. One zero has been given.**

$$13) f(x) = 5x^3 + 4x^2 - 20x - 16; 2$$

$$14) f(x) = 25x^4 - 40x^3 - 19x^2 - 2x; -\frac{1}{5}$$

$$15) f(x) = 3x^4 + 5x^3 + 81x + 135; -\frac{5}{3}$$

$$16) f(x) = 2x^4 - x^3 - 18x^2 + 9x; -3$$

$$17) f(x) = 10x^3 - 41x^2 + 32x + 20; \frac{5}{2}$$

$$18) f(x) = 3x^3 + 4x^2 - 35x - 12; 3$$

## More on Factors, Zeros, and Dividing

**Factor each and find all zeros. One factor has been given.**

1)  $f(x) = x^3 + 9x^2 + 23x + 15; x + 5$

Factors to:  $f(x) = (x + 1)(x + 3)(x + 5)$   
Zeros:  $\{-1, -3, -5\}$

2)  $f(x) = x^3 - x^2 - 14x + 24; x - 3$

Factors to:  $f(x) = (x - 2)(x + 4)(x - 3)$   
Zeros:  $\{2, -4, 3\}$

3)  $f(x) = x^4 + 3x^3 - 13x^2 - 15x; x - 3$

Factors to:  $f(x) = x(x + 1)(x + 5)(x - 3)$   
Zeros:  $\{0, -1, -5, 3\}$

4)  $f(x) = x^3 - 12x^2 + 47x - 60; x - 3$

Factors to:  $f(x) = (x - 4)(x - 5)(x - 3)$   
Zeros:  $\{4, 5, 3\}$

5)  $f(x) = x^3 - 7x^2 + 2x + 40; x - 5$

Factors to:  $f(x) = (x + 2)(x - 4)(x - 5)$   
Zeros:  $\{-2, 4, 5\}$

6)  $f(x) = x^3 - 3x^2 - 9x + 27; x - 3$

Factors to:  $f(x) = (x + 3)(x - 3)^2$   
Zeros:  $\{-3, 3 \text{ mult. } 2\}$

7)  $f(x) = 10x^3 + 37x^2 + 37x + 6; 5x + 1$

Factors to:  $f(x) = (2x + 3)(x + 2)(5x + 1)$   
Zeros:  $\left\{-\frac{3}{2}, -2, -\frac{1}{5}\right\}$

8)  $f(x) = 25x^3 + 150x^2 + 131x + 30; 5x + 3$

Factors to:  $f(x) = (5x + 2)(x + 5)(5x + 3)$   
Zeros:  $\left\{-\frac{2}{5}, -5, -\frac{3}{5}\right\}$

9)  $f(x) = 5x^3 + 21x^2 - 21x - 5; x + 5$

Factors to:  $f(x) = (5x + 1)(x - 1)(x + 5)$   
Zeros:  $\left\{-\frac{1}{5}, 1, -5\right\}$

10)  $f(x) = 3x^3 - 4x^2 - 9x + 10; x - 2$

Factors to:  $f(x) = (3x + 5)(x - 1)(x - 2)$   
Zeros:  $\left\{-\frac{5}{3}, 1, 2\right\}$

$$11) f(x) = 5x^3 + 9x^2 - 26x - 24; x + 3$$

$$\text{Factors to: } f(x) = (5x + 4)(x - 2)(x + 3)$$

$$\text{Zeros: } \left\{ -\frac{4}{5}, 2, -3 \right\}$$

$$12) f(x) = 6x^3 + 7x^2 - 1; 2x + 1$$

$$\text{Factors to: } f(x) = (3x - 1)(x + 1)(2x + 1)$$

$$\text{Zeros: } \left\{ \frac{1}{3}, -1, -\frac{1}{2} \right\}$$

**Factor each and find all zeros. One zero has been given.**

$$13) f(x) = 5x^3 + 4x^2 - 20x - 16; 2$$

$$\text{Factors to: } f(x) = (5x + 4)(x + 2)(x - 2)$$

$$\text{Zeros: } \left\{ -\frac{4}{5}, -2, 2 \right\}$$

$$14) f(x) = 25x^4 - 40x^3 - 19x^2 - 2x; -\frac{1}{5}$$

$$\text{Factors to: } f(x) = x(5x + 1)^2(x - 2)$$

$$\text{Zeros: } \left\{ 0, -\frac{1}{5} \text{ mult. } 2, 2 \right\}$$

$$15) f(x) = 3x^4 + 5x^3 + 81x + 135; -\frac{5}{3}$$

$$\text{Factors to: } f(x) = (x + 3)(x^2 - 3x + 9)(3x + 5)$$

$$\text{Zeros: } \left\{ -3, \frac{3 + 3i\sqrt{3}}{2}, \frac{3 - 3i\sqrt{3}}{2}, -\frac{5}{3} \right\}$$

$$16) f(x) = 2x^4 - x^3 - 18x^2 + 9x; -3$$

$$\text{Factors to: } f(x) = x(2x - 1)(x - 3)(x + 3)$$

$$\text{Zeros: } \left\{ 0, \frac{1}{2}, 3, -3 \right\}$$

$$17) f(x) = 10x^3 - 41x^2 + 32x + 20; \frac{5}{2}$$

$$\text{Factors to: } f(x) = (5x + 2)(x - 2)(2x - 5)$$

$$\text{Zeros: } \left\{ -\frac{2}{5}, 2, \frac{5}{2} \right\}$$

$$18) f(x) = 3x^3 + 4x^2 - 35x - 12; 3$$

$$\text{Factors to: } f(x) = (3x + 1)(x + 4)(x - 3)$$

$$\text{Zeros: } \left\{ -\frac{1}{3}, -4, 3 \right\}$$