(A) Lesson Context

BIG PICTURE of this UNIT:	• What is a Polynomial and how do they look? • What are the attributes of a Polynomial? • How do I work with Polynomials? • How predictable is the appearance of a polynomial?		
CONTEXT of this LESSON:	Where we've been We have discussed the basic appearance of graphs of polynomial functions	Where we are Can we predict end & zeroes behaviour as well as the number of zeroes & extrema from an equation	Where we are heading What are the key attributes of a polynomial and how do these affect the shape?

(B) Lesson Objectives:

a. Graph polynomial parent functions and review basic function concepts now in the context of polynomial parent functions.
b. Understand the connection between the degree of a polynomial & it roots & extrema.
c. Define multiplicity of roots & observe function behaviour at the roots given various multiplicities.
d. Continue to realize that the appearance of a polynomial is PREDICTABLE given its equation.

(C) Investigation #1 – Parent Functions & End Behaviour

1. Graph $y = ½ x$ and $y = ½ x^3$ and $y = ½ x^5$ using DESMOS (using the default window setting). Generalize the end behaviours of these 3 functions.

2. Graph $y = ½ x^2$ and $y = ½ x^4$ and $y = ½ x^6$ using DESMOS (using the default window setting). Generalize the end behaviours of these 3 functions.

3. Graph $y = -½ x$ and $y = -½ x^3$ and $y = -½ x^5$ using DESMOS (using the default window setting). Generalize the end behaviours of these 3 functions.

4. Graph $y = -½ x^2$ and $y = -½ x^4$ and $y = -½ x^6$ using DESMOS (using the default window setting). Generalize the end behaviours of these 3 functions.

General Conclusions:

a. What is the general appearance of an even degree polynomial function? An odd degree polynomial function?

b. What are the generalized end behaviours of even degree polynomials?

c. What are the generalized end behaviours of odd degree polynomials?

(D)Investigation #2 – Zeroes & Multiplicity (Use Wolfram-Alpha)

a. Research the meaning of the term "multiplicity of zeroes". Record your findings in your notes.

b. Use Wolframalpha to ***factor and graph*** the following cubic polynomials:

(a) $Y = x^3 - 9x^2 + 27x - 27$ (b) $Y = x^3 - 5x^2 + 3x + 9$ (c) $Y = x^3 + 2x^2 - 11x - 12$

c. Include a sketch of each polynomial in your work.

d. Use Wolframalpha to ***factor and graph*** the following quartic polynomials:

(a) $Y = x^4 - 8x^3 + 24x^2 - 32x + 16$ (b) $Y = x^4 - 2x^3 - 3x^2 + 4x + 4$ (c) $Y = x^4 - 5x^3 + 6x^2 + 4x - 8$

(d) $Y = x^4 - 8x^3 + 15x^2 + 4x - 20$ (e) $Y = x^4 - x^3 - 18x^2 + 16x + 32$ (f) $Y = x^4 + 14x^2 + 40$

e. Include a sketch of each polynomial in your work.

f. Use Wolframalpha to ***factor and graph*** the following quintic polynomials:

(a) $Y = x^5 - 8x^4 + 24x^3 - 32x^2 + 16x$ (b) $Y = x^5 + x^4 - 5x^3 - x^2 + 8x + 4$ (c) $Y = x^5 - x^4 - 9x^3 + 5x^2 + 16x - 12$

(d) $Y = x^5 - x^4 - 10x^3 + 10x^2 + 9x - 9$ (e) $Y = x^5 - 2x^4 - 13x^3 + 14x^2 + 24x$ (f) $Y = x^5 - 2x^4 + 3x^3 - 6x^2 + 2x - 4$

g. Include a sketch of each polynomial in your work.

General Conclusions:

(A) Conclusion #1: How can you predict the maximum number of zeroes of a polynomial?

(B) Conclusion #2: How do you predict the appearance of a function near the x-axis if the multiplicity of its zeroes is even?

(C) Conclusion #3: How do you predict the appearance of a function near the x-axis if the multiplicity of its zeroes is odd?

(E) Investigation #3 – Extrema

a. Use DESMOS to graph the following cubic polynomials.

(a) $Y = x^3 + 3x^2 + 2$ (b) $Y = x^3 - 5$ (c) $Y = x^3 + 3x^2 + 2x - 2$

b. Include a sketch of each polynomial in your work. Label the extrema.

c. Use DESMOS to graph the following quartic polynomials.

(a) $Y = x^4 + x^3 - x^2 + 2x - 2$ (b) $Y = x^4 - 2x^3 - 3x^2 + 2x - 2$

(d) $Y = x^4 - 3x^2 - 2x - 2$ (e) $Y = 2x^4 + x^3 - 3x^2 + 5x$

d. Include a sketch of each polynomial in your work. Label the extrema.

e. Use DESMOS to graph the following quintic polynomials.

(a) $Y = x^5 + 4x^4 + x^3 - 3x^2 + 5x + 1$ (b) $Y = x^5 + 4x^4 + x^3 - 3x^2 - 3$ (c) $Y = -2x^5 + 4x^4 + x^3 - 3x^2 + x + 3$

(d) $Y = -x^5 + 2x^4 + 2x^3 - 3x^2 + x + 3$ (e) $Y = -x^5 + 4x^4 - 7x^3 - x^2 - 3x - 2$ (f) $Y = x^5 - 5x^4 + 5x^3 + 10x^2 - 20x - 4$

f. Include a sketch of each polynomial in your work. Label the extrema.

General Conclusions:

(A) How can you predict the maximum number of extrema in a polynomial?

General Conclusions – Parent Functions and End Behaviour:

(A) What is the general appearance of an even degree polynomial function? An odd degree polynomial function?

(B) What are the generalized end behaviours of even degree polynomials?

(C) What are the generalized end behaviours of odd degree polynomials?

General Conclusions – Zeros and Multiplicity:

(A) How can you predict the maximum number of zeroes of a polynomial?

(B) How do you predict the appearance of a function near the x-axis if the multiplicity of its zeroes is even?

(C) How do you predict the appearance of a function near the x-axis if the multiplicity of its zeroes is odd?

General Conclusions – Extrema:

(A) How can you predict the maximum number of extrema in a polynomial?