A. Lesson Context

BIG PICTURE of this UNIT:	• How & why do we build NEW knowledge in Mathematics? • What NEW IDEAS & NEW CONCEPTS can we now explore with specific references to QUADRATIC FUNCTIONS? • How can we extend our knowledge of FUNCTIONS, given our BASIC understanding of Functions?		
CONTEXT of this LESSON:	Where we've been In previous lessons, you reviewed methods for solving for roots and for the optimal points of quadratic models	Where we are NOW we will focus on analysing quadratic models in a variety of contextual and algebraic contexts	Where we are heading How do we extend our knowledge & skills of the algebra of quadratic functions, and build in new ideas & concepts involving functions.

B. *Lesson Objectives*

a. Analyze quadratic models in context – work through some optimization problems & some where we need to find the roots
b. Work with quadratic models in system applications

C. *REVIEW: Solving & Optimizing with Quadratic Models*

a. To solve an equation means ➔

b. Quadratic equations can be solved by:

   i. ➔

   ii. ➔

   iii. ➔

c. To make a graphic connection, what are we finding (graphically) when we solve the eqn $0 = ax^2 + bx + c$?

d. To find an optimal value means ➔

e. We can determine optimal values by: :

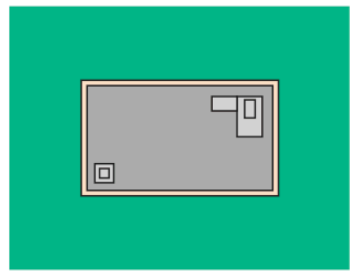
   i. ➔

   ii. ➔

   iii. ➔

D. <u>Modeling with Zeroes of Quadratic Functions</u>

A factory is to be built on a lot that measures 80 m by 60 m. A lawn of uniform width, equal to the area of the factory, must surround it. How wide is the strip of lawn, and what are the dimensions of the factory?



9. A rectangle has an area of 330 m^2. One side is 7 m longer than the other. What are the dimensions of the rectangle?

10. The sum of the squares of two consecutive integers is 685. What could the integers be? List all possibilities.

11. A right triangle has a height 8 cm more than twice the length of the base. If the area of the triangle is 96 cm^2, what are the dimensions of the triangle?

12. **T** Jackie mows a strip of uniform width around her 25 m by 15 m rectangular lawn and leaves a patch of lawn that is 60% of the original area. What is the width of the strip?

E. Modeling Optimization with Quadratic Functions

Example #1:

Lila is creating dog runs for her dog kennel. She can afford 30 m of chain-link fence to surround four dog runs. The runs will be attached to a wall, as shown in the diagram. To achieve the maximum area, what dimensions should Lila use for each run and for the combined enclosure?

Example #2:

A computer software company models the profit on its latest game (MATH-R-FUN) using the quadratic relation ***P*** = $-2x^2 + 28x - 90$, where ***x*** is the number of games it produces in hundreds of thousands and ***P*** is the profit in millions of dollars.

a. What is the maximum profit that the company can earn?
b. How many games must it produce to earn this profit?
c. What are the break-even points for the company?

Example #3: Soundz Inc makes Blu-Ray DVD players. Last year, the equation ***P*** = $-5x^2 + 60x - 135$ was used to model the company's profits, where ***P*** is the profit in hundreds of thousands of dollars and ***x*** is the number of DVD players made, in hundreds of thousands. In an effort to become "more efficient", SoundZ Inc "restructured" its operations by eliminating some employees and reducing costs. This year, the company's profits were modelled by the equation ***P*** = $-7x^2 + 70x - 63$. Was Soundz Inc.'s restructuring effective? Justify your answer.

Example #4: Alice is a manager at a hardware store. Her research shows that an increase of 10 cents on the price of a package of batteries will cause a drop in sales of 10 packages per day. The store normally sells 600 packages of batteries per day at $4.95 per package. How many packages of batteries must be sold to maximize the revenue? What is the expected maximum revenue?

Example #5: A dance club has a $5 cover charge and averages 300 customers on Friday nights. They have determined that a price increase of $0.50 on the cover charge, the number of customers decreases by 30. Use an algebraic model to determine the cover charge that will maximize the revenue.

Example #6: A movie theatre can accommodate a maximum of 450 people in one day. The theatre operators have changed admission prices on several occasions to find out how price affects attendance, daily revenue and profit . After reviewing their data and using the formula profit = revenue – expenses, the operators found they could express the relation between profit, P, and ticket price, t as $P = t(450 - 30t) - 790$.

(a) What does the expression 450 – 30t represent?

(b) What does the 790 represent?

(c) What is the ticket price that maximizes the daily profit?

(d) How many tickets will be sold at this price?

(e) What is the maximum profit?

(f) Determine the break-even ticket price and required minimum ticket sales

(g) However, the operators forgot to take into account the revenue from concession sales. They expect this to be $3.50 per ticket. As a result, the new profit equation is $P = (t + 3.50)(450 - 30t) - 790$.

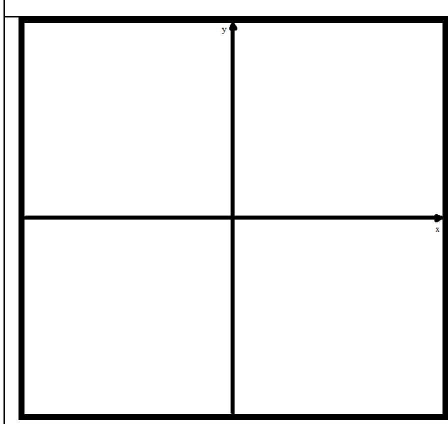
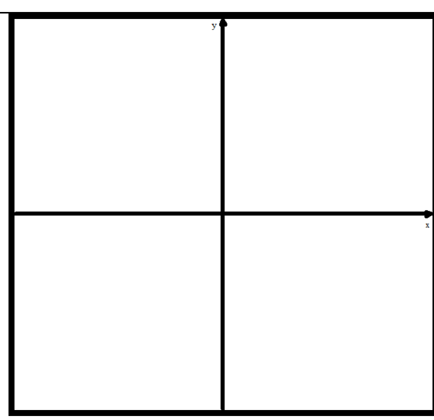
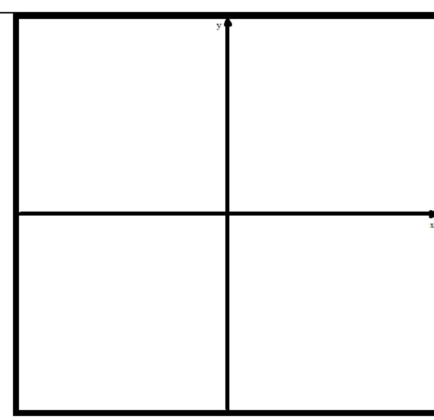
i. Explain what the expression $t + 3.50$ represents.

ii. What new ticket price will maximize the profit?

iii. What will the maximum profit be?

iv. Determine the new break even ticket price and the required minimum ticket sales

F. ***Applying Systems to Quadratic Functions***

$y_1 = x^2 - x + 5$ and $y_2 = 2x^2 - x + 3$	$y_1 = x^2 + 3x - 4$ and $y_2 = 5x + 4$	$y_1 = -2x^2 + x + 5$ and $y_2 = x^2 - 2x + 3$
		

G. ***Applying Quadratic Systems – Profit, Revenue, Expenses***

Ex 1. A company prints and sells math textbooks. Their revenues are modelled by the quadratic equation $R(b) = -0.1b^2 + 15b - 120$, where ***R*** is revenue in tens of thousands of dollars for the sale and printing of ***b*** thousands of textbooks. The expenses for printing and selling the ***b*** thousands of textbooks (***E***, in tens of thousands of dollars) are given by the linear equation $E(b) = 100 + b$.

d. What is the profit/loss if 30,000 books are printed & sold? If 130,000 books are printed & sold?

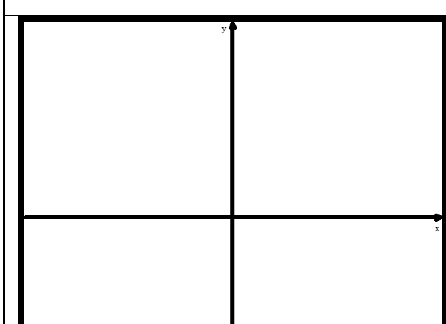
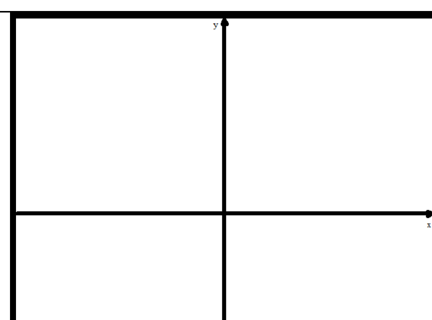
e. How many books must be printed and sold is the profit is to be $1,800,000?

f. How many books must be printed & sold if the company is to break even?

g. When does the company achieve its maximum profit? What is the maximum profit?

h. When does the company lose money? Explain how you know.

H. Working with Quadratic Inequalities

If $f(x) = x^2 - x + 5$ and $g(x) = 2x^2 - x + 3$, solve $f(x) > g(x)$	If $f(\text{x}) = x^2 + 3x - 4$ and $g(x) = 5x + 4$, solve $f(x) \le g(x)$
	

I. Connecting Function Concepts: Inverse of Quadratic Functions