A. Lesson Context

BIG PICTURE of this UNIT:	• How & why do we build NEW knowledge in Mathematics? • What NEW IDEAS & NEW CONCEPTS can we now explore with specific references to QUADRATIC FUNCTIONS? • How can we extend our knowledge of FUNCTIONS, given our BASIC understanding of Functions?		
CONTEXT of this LESSON:	Where we've been In Lessons 2 & 3, you worked with quadratic graphs and equations using the form of $y = \boldsymbol{a}(x - \boldsymbol{h})^2 + \boldsymbol{k}$	Where we are HOW do we apply the vertex form of quadratic models in contextual problems	Where we are heading How do we extend our knowledge & skills of quadratic functions, given the new ideas & concepts we now know about functions.

B. Lesson Objectives

a. Apply the equation $y = \boldsymbol{a}(x - \boldsymbol{h})^2 + \boldsymbol{k}$ to geometrical contexts and to modeling contexts as well as data contexts (scatter plots)

C. Fast Five (Skills Review Focus)

Example 1: Given the quadratic relation $y = x^2 - 8x + 12$;

a. Sketch a graph of this parabola and label all key points/features.

b. Solve $0 = x^2 - 8x + 12$. What does your solution mean?

c. Solve $-3 = x^2 - 8x + 12$. What does your solution mean?

d. Solve $5 + 8x = x^2 + 12$. What does your solution mean?

Example 2: Given the quadratic relation $y = 6x^2 + x - 15$

a. Factor $y = 6x^2 + x - 15$

b. Solve $0 = 6x^2 + x - 15$

c. Solve $36 = 6x^2 + x - 15$

d. Find the vertex and re-write the equation in vertex form.

Example 3: Given the quadratic relation $y = 6x^2 + 5x - 4$

a. Factor $y = 6x^2 + 5x - 4$

b. Solve $0 = 6x^2 + 5x - 4$

c. Solve $65 = 6x^2 + 5x - 4$

d. rewrite the equation in VERTEX FORM.

Example 4: Given the quadratic relation $y = 2x^2 - 20x + 50$

a. Factor $y = 2x^2 - 20x + 50$

b. Solve $0 = 2x^2 - 20x + 50$

c. Solve $72 = 2x^2 - 20x + 50$

d. rewrite the equation in VERTEX FORM.

D. Business Modeling with Quadratic Equations

Mr Santowski runs a clothing business and models how his revenues on sales of denim jeans are related to price changes. He uses the quadratic equation R = 300 + 20x – x^2, where R represents his daily revenue in dollars and x represents an increase or decrease in price. (So x = +1 represents a price increase of 1 dollar and x = -2 represents a price decrease of 2 dollars)

a. Determine the price change that will result in maximum revenues. What is the maximum revenue

b. Factor the equation R = 300 + 20x – x^2.

c. Solve the equation 0 = 300 + 20x – x^2 and interpret what the answers mean, given the context.

d. Solve the equation 300 = 300 + 20x – x^2 and interpret what the answers mean, given the context.

e. Make a sketch of the relation.

f. Solve the equation 375 = 300 + 20x – x^2 and interpret what the answers mean, given the context

g. Solve the equation 144 = 300 + 20x – x^2 and interpret what the answers mean, given the context

The profits of a company in its first 13 months of operations are modelled by the quadratic function $P(m) = -0.25m^2 + 3m - 5$ where m is the number of months (and m = 1 represents January) and P(m) is measured in billions of pesos. (CALC INACTIVE)

a. Determine when the company "breaks even".
b. Determine in which month the company maximizes its profits.
c. What are the company's maximum profits?
d. Solve and interpret P(m) < 0 given that the domain is $D: \{m \in Z \mid 0 \leq m \leq 13\}$
e. For what values of m are the profits DECREASING? Explain how you determined your answer.
f. Solve P(m) = -12 and interpret

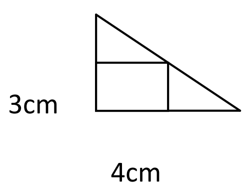
E. <u>Modeling with Quadratic Functions – Writingb your own Equations</u>

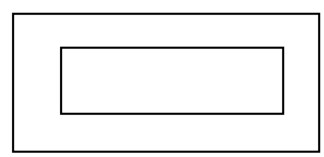
1. If the length of one side of a square is tripled and the length of an adjacent side is increased by 10, the resulting rectangle has an area that is 6 times the area of the original square. Find the length of a side of the original square.

2. The length of a rectangle is 7 units more than its width. If the width is doubled and the length is increased by 2, the area is increased by 42 square units. Find the dimensions of the original rectangle.

3. Among all rectangles that have a perimeter of 20 feet, find the dimensions of the one with the largest area.

4. Find the area of the largest rectangle that can be inscribed in a right triangle with legs of lengths 3 cm and 4 cm if two sides of the rectangle lie along the legs as shown in the figure. (Hint: set the triangle with the right angle at the origin of a graph and write the equation of the line containing the hypotenuse)





5. A frame for a picture is 2½ cm wide. The picture enclosed inside the frame is 5 cm longer than it is high. If the area of the picture itself is 300 cm^2, what are the dimensions of the outer frame? (see diagram above)

6. A farmer has 3000 feet of fence available to enclose a rectangular field. Assuming that he uses all of his fence material, find the length of each of the sides of the rectangle which will maximize the area. What is the maximum area he can enclose?

7. A farmer with 4000 meters of fencing material wants to enclose a rectangular plot that borders on a river. If the farmer does not fence the side along the river, what is the largest area that he can enclose? What will the dimensions be?

8. In a trapezoid, the smaller base is 3 more than the height, the larger base is 5 less than 3 times the height, and the area of the trapezoid is 45 square centimeters. Find, in centimeters, the height of the trapezoid.

F. Modeling with Quadratic Functions (Example #6 – Modeling Bridges)

The underside of a concrete railway underpass forms a parabolic arch. The arch is 24 m wide at the base and 8.0 high in the center. The upper surface of the underpass is 40 m wide. The concrete is 2 m thick at the center. Can a truck that is 5 m wide and 7.5 tall get through this underpass if the time is 3:30pm?

(a) Visualize the information by drawing a diagram wherein you label the relevant information.
(b) Recall that our general starting point would be a quadratic model and that the vertex form of the equation is $y = a(x - h)^2 + k$
(c) How can you use the quadratic equation to address the problem of the truck passing through the underpass?

G. Modeling with Quadratic Functions (Example #7)

The Next Cup coffee shop sells a special blend of coffee for $2.60 per mug. The shop sells about 200 mugs per day at this price. Customer surveys show that for every $0.05 decrease in price, the shop would sell 10 more mugs per day.

(HINT: start with numbers and a data table to see what may be going on)

(d) Determine the REVENUE that the coffee shop makes initially, given the price per mug and the amount of mugs sold.
(e) Since we are making changes in the pricing & revenues of the coffee shop, we need to decide upon an INDEPENDENT variable to use in modeling a change in the revenues ➔ so we need an R(x) equation …..
(f) Determine the MAXIMUM daily revenue from coffee sales and the price per mug in order to earn this revenue.
(g) Write an equation in both standard form and vertex form to model this problem. Then sketch the graph.